A Proof of Collatz Conjecture

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Abstract

First, let us expound certain basic concepts relating to Collatz conjecture.

Next, list the mathematical induction that proves the conjecture.

Then again, prepare several judging criteria, which are solely used to determine whether each such operational result fits the conjecture.

After that, we sort positive integers successively and prove directly one of certain sorts after each sorting, until the last two sorts are proved bidirectionally.

The bidirectional proofs mean that for these two sorts of integers, on the one hand, start with several proven kinds of integers to expand successively the scope of proven kinds of integers, up to all kinds of integers. On the other, each unproven kind of integers is operated by the operational rule to find an integer expression that is less than the unproven kind of integers, from this, it meets a judging criterion, such that the unproven kind of integers is proved to fit the conjecture.

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1. Introduction

The Collatz conjecture is also called the *3x+1* mapping, *3n+1* problem, Hasse's algorithm, Kakutani's problem, Syracuse algorithm, Syracuse problem, Thwaites conjecture and Ulam's problem, etc.

But it remains a conjecture that has neither been proved nor disproved ever since named after Lothar Collatz in 1937; [1] and [4].

2. Certain of Basic Concepts

The Collatz conjecture states that take any positive integer n, if n is an even number, divide it by 2; if n is an odd number, multiply it by 3 and add 1, and repeat the process indefinitely, then, no matter which positive integer you start with, you are always going to end up with 1; [2] and [5].

We consider aforesaid operational stipulations as the operational rule.

If you start with any positive integer/integer expression to operate continually by the operational rule, then it will form a series of positive integers/integer expressions, where the symbol "/" means "or", and the same below.

Such being the case, we regard such a series of positive integers/integer expressions with codirectional arrows among these positive integers/ integer expressions as an operational route.

Next, let us use a capital letter with the subscript "ie" to express an integer expression, such as P_{ie}, C_{ie} etc.

In addition, we can call an operational route that contains a certain integer expression "the operational route via the integer expression".

In general, integer expressions on an operational route have a variable or many variables which can be converted into a common variable.

3. The Mathematical Induction that Proves the Conjecture

Let us prove the conjecture by the mathematical induction; [3] and [6]:

(1) From $1\to4\to2\to1$; $2\to1$; $3\to10\to5\to16\to8\to4\to2\to1$; $4\to2\to1$; $5\to16\to8\to4\to2\to1$; $6\to3\to10\to5\to16\to8\to4\to2\to1$; $7\to22\to11\to34\to17\to52\to26\to13\to40\to20\to10\to5\to16\to8\to4\to2\to1$; $8\to4\to2\to1$ and $9\to28\to14\to7\to22\to11\to34\to17\to52\to26\to13\to40\to20\to10\to5\to16\to8\to4\to2\to1$, you can be seen that each positive integer that is not more than *9* fits the conjecture.

(2) Suppose that an integer n fits the conjecture, where n is not less than 9.

(3) Prove that $n+1$ fits the conjecture likewise.

4. Several Judging Criteria

A certain result of operations which start with each sort / kind of positive integers is judged by one of following three criteria.

Theorem 1 . On an operational route via P_{ie}, if there is an integer expression that is less than P_{ie}, and $n+1\in P_{ie}$, then all integer expressions on the operational route fit the conjecture.

For example, let $P_{ie}=31+3^2\beta$ with $\beta\ge0$, and $n+1\in P_{ie}$, then from $27+2^3\beta\to82+3\times2^3\beta\to41+3\times2^2\beta\to124+3^2\times2^2\beta\to62+3^2\times2\beta\to\mathbf{31}+3^2\beta>\mathbf{27}+2^3\beta$, we get that all integer expressions on the operational route fit the conjecture.

In addition, let $P_{ie}=5+2^2\mu$ with $\mu\ge0$, and $n+1\in P_{ie}$, then from $\mathbf{5}+2^2\mu\to16+3\times2^2\mu$

$\rightarrow 8+3\times 2\mu \rightarrow \mathbf{4}+3\mu<\mathbf{5}+2^2\mu$, we get that all integer expressions on the operational route fit the conjecture.

Proof. Suppose that there is C_{ie} on an operational route via P_{ie}, and $C_{ie}<P_{ie}$, then when their common variable is equal to a certain fixed value, such that $P_{ie}=n+1$ and $C_{ie}=m$. So there is $m<n+1$, then m fits the conjecture, according to second step of the mathematical induction.

So from $n+1$ can operate to m, or from m can operate to $n+1$, and that it can continue to operate to 1 via m, such that $n+1$ fits the conjecture.

When their common variable is equal to its own each value, each integer that every integer expression contains is operated to 1, because each such integer matches an integer of C_{ie}, and the operations of the former go through the operations of the latter to attain 1.

As thus, all integer expressions on the operational route fit the conjecture.

Theorem 2. If an operational route via Q_{ie} and an operational route via P_{ie} intersect, and $n+1\in P_{ie}$, also an integer expression on the operational route via Q_{ie} is less than P_{ie}, then all integer expressions on these two operational routes fit the conjecture.

For example, let $Q_{ie}=71+3^3\times 2^5\varphi$ and $P_{ie}=63+3\times 2^8\varphi$ where $\varphi\geq 0$, then from

$\mathbf{63}+3\times 2^8\varphi \rightarrow 190+3^2\times 2^8\varphi \rightarrow 95+3^2\times 2^7\varphi \rightarrow 286+3^3\times 2^7\varphi \rightarrow 143+3^3\times 2^6\varphi \rightarrow 430+3^4\times 2^6\varphi \rightarrow$

$215+3^4\times 2^5\varphi \rightarrow 646+3^5\times 2^5\varphi \rightarrow 323+3^5\times 2^4\varphi \rightarrow 970+3^6\times 2^4\varphi \rightarrow 485+3^6\times 2^3\varphi \rightarrow 1456+3^7\times 2^3\varphi$

$\rightarrow 728+3^7\times 2^2\varphi \rightarrow 364+3^7\times 2\varphi \rightarrow 182+3^7\varphi \rightarrow \ldots$

$\uparrow 121+3^6\times 2\varphi \leftarrow 242+3^6\times 2^2\varphi \leftarrow 484+3^6\times 2^3\varphi \leftarrow 161+3^5\times 2^3\varphi \leftarrow 322+3^5\times 2^4\varphi$

$\longleftarrow 107+3^4\times 2^4\varphi \longleftarrow 214+3^4\times 2^5\varphi \longleftarrow \mathbf{71}+3^3\times 2^5\varphi \longleftarrow 142+3^3\times 2^6\varphi \longleftarrow \mathbf{47}+3^2\times 2^6\varphi < \mathbf{63}+3\times 2^8\varphi$, we get that all integer expressions on these two operational routes fit the conjecture.

***Proof*.** Suppose that there is D_{ie} on an operational route via Q_{ie}, and $D_{ie}<P_{ie}$, and that the operational route via Q_{ie} and an operational route via P_{ie} intersect at A_{ie}, so when their common variable is given a certain fixed value, such that $P_{ie}=n+1$, $A_{ie}=\xi$ and $D_{ie}=\mu$. Then, there is $\mu<n+1$, and μ fits the conjecture, according to second step of the mathematical induction.

Since ξ and μ are on an operational route, and μ fits the conjecture, so all integers including ξ on the operational route fit the conjecture, because an operation for each of them must through μ, then continue to operate to 1.

For the same reason, $n+1$ and ξ are on an operational route, and ξ fits the conjecture, so all integers including $n+1$ on the operational route fit the conjecture.

When the common variable is equal to its own each value, each integer which every integer expression contains on these two operational routes is also operated to 1, because each such integer matches an integer of D_{ie}.

So all integer expressions on the two operational routes fit the conjecture.

Lemma 1. If there is a proven integer expression on an operational route, then all integer expressions on the operational route fit the conjecture.

Lemma 2. If there is a proven integer expression on successive intersecting operational routes, then all integer expressions on these successive intersecting operational routes fit the conjecture.

By the way, two non-intersected operational routes within successive intersecting operational routes are in indirect connection.

Lemma 3. Each and every integer that any proven integer expression contains fits the conjecture.

To clarify, each Theorem or Lemma appearing in the following paragraphs refers to a corresponding Theorem or Lemma in this chapter.

5. Each Sorting of Positive Integers and a follow-up Proof

We sort positive integers successively, then $n+1$ could be included in any sort, thus each of all sorts must be proved to fit the conjecture.

Each Sorting and a Proof. Since in chapter 3, each positive integer that is not more than 9 has been proved to fit the conjecture, so we first divide integers which are more than 9 into even numbers and odd numbers.

For even numbers $2k$ with $k>4$, from $2k \to k<2k$, we get that if $n+1 \in 2k$, then $2k$ and $n+1$ fit the conjecture according to Theorem 1 and Lemma 3.

For odd numbers which are more than 9, divide them into $11+4k$ and $13+4k$, where $k \geq 0$. For $13+4k$, from $13+4k \to 40+12k \to 20+6k \to 10+3k<13+4k$, we get that if $n+1 \in 13+4k$, then $13+4k$ and $n+1$ fit the conjecture according to Theorem 1 and Lemma 3.

We continue to divide $11+4k$ into $11+12c$, $15+12c$ and $19+12c$, where $c \geq 0$. For $11+12c$, from $7+8c \to 22+24c \to 11+12c>7+8c$, we get that if $n+1 \in 11+12c$, then $11+12c$ and $n+1$ fit the conjecture according to Theorem 1 and Lemma 3.

Hereinafter, we will prove that 15+12c and 19+12c fit the conjecture.

6. Prove that 15+12c and 19+12c Fit the Conjecture

For the sake of avoiding confusion and conducive convenience, we substitute d, e, f, g, etc. for c on operational routes of 15+12c/19+12c.

We continue to operate 15+12c/19+12c by the operational rule, where c≥0.

Theoretically speaking, after operate 15+12c/19+12c, each and every kind of 15+12c/19+12c must find an integer expression that is less than the kind of 15+12c/19+12c, in order to meet one of judging criteria, such that the kind of 15+12c/19+12c fits the conjecture.

Firstly, we start with 15+12c to operate continuously by the operational rule, as listed below.

15+12c→46+36c→23+18c→70+54c→35+27c ♣

d=2e+1: 29+27e (**1**)　　e=2f: 142+486f→71+243f ♥
♣35+27c↓→c=2d+1: 31+27d↑→d=2e: 94+162e→47+81e↑→e=2f+1:64+81f (**2**)
c=2d: 106+162d→53+81d↓→d=2e+1:67+81e↓→e=2f+1:74+81f (**3**)
d=2e:160+486e♦　　e=2f: 202+486f→101+243f ♠

g=2h+1: 200+243h (**4**)　　**…**
♥ 71+243f↓→f=2g+1:157+243g↑→g=2h: 472+1458h→236+729h↑→ **…**
f=2g: 214+1458g→107+729g↓→g=2h+1: 418+729h↓→**…**
g=2h: 322+4374h→**… …**

g=2h: 86+243h (**5**)
♠101+243f↓→f=2g+1:172+243g↑→g=2h+1:1246+1458h→**…**
f=2g: 304+1458g→152+729g ↓→**…**
…

…
♦160+486e→80+243e↓→e=2f+1: 970+1458f→485+729f↑→ **…**　　**…**
e=2f:40+243f↓→f=2g+1:850+1458g→ 425+729g↑→ **…**
f=2g: 20+243g↓→g=2h: 10+243h (**6**)　　**…**
g=2h+1:790+1458h→395+729h↑→ **…**

Annotation:

(1) Each of letters c, d, e, f, g, h and otherwise on listed above operational routes expresses each of natural numbers and 0.

(2) There are ♣↔♣, ♥↔♥, ♠↔♠, and ♦↔♦ on listed above operational routes.
(3) Aforesaid two points are suitable to latter operational routes of 19+12c similarly.

First, let us define a term. That is, if an operational result is less than a kind of 15+12c/19+12c, and it appears first either on an operational route via the kind of 15+12c/19+12c or on another operational route relating to the operational route, then we call the operational result "first satisfactory operational result" about the kind of 15+12c/19+12c.

Accordingly, on the above bunch of operational routes of 15+12c, we first conclude following 3 kinds of 15+12c derived from first satisfactory operational results to fit the conjecture.

1). From c=2d+1 and d=2e+1 to get c=2d+1=2(2e+1)+1=4e+3, then there are $15+12c=51+48e=51+3\times2^4e\rightarrow154+3^2\times2^4e\rightarrow77+3^2\times2^3e\rightarrow232+3^3\times2^3e\rightarrow116+3^3\times2^2e\rightarrow58+3^3\times2e\longrightarrow29+27e$ where the mark (**1**).

Due to 29+27e<51+48e, we get that if there is $n+1\in 51+48e$, then 51+48e and $n+1$ fit the conjecture according to Theorem 1 and Lemma 3.

2). From c=2d+1, d=2e and e=2f+1 to get c=2d+1=4e+1=4(2f+1)+1=8f+5, then there are $15+12c=75+96f=75+3\times2^5f\rightarrow226+3^2\times2^5f\rightarrow113+3^2\times2^4f\rightarrow340+3^3\times2^4f\rightarrow170+3^3\times2^3f\longrightarrow85+3^3\times2^2f\longrightarrow256+3^4\times2^2f\longrightarrow128+3^4\times2^1f\longrightarrow64+81f$ where the mark (**2**).

Due to 64+81f <75+96f, we get that if there is $n+1\in 75+96f$, then 75+96f and $n+1$ fit the conjecture according to Theorem 1 and Lemma 3.

3). From c=2d, d=2e+1 and e=2f+1 to get c=2d=4e+2=4(2f+1)+2=8f+6, then there are $15+12c=87+96f=87+3\times2^5f\rightarrow262+3^2\times2^5f\rightarrow131+3^2\times2^4f\rightarrow394+3^3\times2^4f\rightarrow197+3^3\times2^3f\longrightarrow592+3^4\times2^3f\longrightarrow296+3^4\times2^2f\longrightarrow148+3^4\times2^1f\longrightarrow74+81f$ where the mark (**3**).

Due to 74+81f < 87+96f, we get that if there is $n+1\in 87+96f$, then 87+96f and *n+1* fit the conjecture according to Theorem 1 and Lemma 3.

In the same way, on the bunch of operational routes of 15+12c, each reader can also conclude other 3 kinds of 15+12c derived from first satisfactory operational results to fit the conjecture, as listed below.

4). Pursuant to c=2d+1, d=2e, e=2f, f=2g+1 and g=2h+1 to get c=32h+25, then there is 15+12c=315+384h derived from 200+243h where the mark (**4**);

5). Pursuant to c=2d, d=2e+1, e=2f, f=2g+1 and g=2h to get c=32h+10, then there is 15+12c=135+384h derived from 86+243h where the mark (**5**);

6). Pursuant to c=2d, d=2e, e=2f, f=2g and g=2h to get c=32h, then there is 15+12c=15+384h derived from 10+243h where the mark (**6**).

Secondly, we start with 19+12c to operate continuously by the operational rule, as listed below.

19+12c→58+36c→ 29+18c→ 88+54c→ 44+27c ♣

d=2e: 11+27e (**α**)   e=2f:37+81f (**β**)

♣ 44+27c↓→c=2d: 22+27d↑→d=2e+1:148+162e→74+81e↑→e=2f+1:466+486f ♥

c=2d+1: 214+162d→107+81d↓→d=2e:322+486e ♠

d=2e+1:94+81e↓→e=2f:47+81f (γ)

e=2f+1:526+486f ♦

g=2h: 119+243h (**δ**)   …

f=2g+1:238+243g↑→g=2h+1:1444+1458h→722+729h↑→…

♥466+486f→233+243f↑→f=2g: 700+1458g→350+729g↓→g=2h+1:3238+4374h↓

g=2h: 175+729h↓→…   …

…

g=2h+1:172+243h (**ε**)

f=2g: 101+243g↑→g=2h: 304+1458h→…

e=2f+1:202+243f↑→f=2g+1:1336+1458g→…

♠322+486e→161+243e↑→e=2f:484+1458f→…

♦526+486f→263+243f↓→f=2g: 790+1458g→…

f=2g+1: 253+243g↓→g=2h+1: 248+243h (ζ)

g=2h: 760+1458h→…

As listed above, on the bunch of operational routes of 19+12c, we first conclude following 3 kinds of 19+12c derived from first satisfactory operational results to fit the conjecture.

1). From c = 2d and d = 2e to get c = 2d = 4e, then there are 19+12c = 19+48e = $19+3\times2^4e \to 58+3^2\times2^4e \to 29+3^2\times2^3e \to 88+3^3\times2^3e \to 44+3^3\times2^2e \to 22+3^3\times2e \to 11+27e$ where the mark (**α**).

Due to 11+27e<19+48e, we get that if there is $n+1 \in$ 19+48e, then 19+48e and $n+1$ fit the conjecture according to Theorem 1 and Lemma 3.

2). From c=2d, d=2e+1 and e=2f to get c=2d=2(2e+1)=4e+2=8f+2, then there are $19+12c=43+96f=43+3\times2^5f \to 130+3^2\times2^5f \to 65+3^2\times2^4f \to 196+3^3\times2^4f \to 98+3^3\times2^3f \to 49+3^3\times2^2f \to 148+3^4\times2^2f \to 74+3^4\times2^1f \to 37+81f$ where the mark (**β**).

Due to 37+81f<43+96f, we get that if there is $n+1 \in$ 43+96f, then 43+96f and $n+1$ fit the conjecture according to Theorem 1 and Lemma 3.

3). From c=2d+1, d=2e+1 and e=2f to get c=2d+1=4e+3=8f+3, then there are $19+12c=55+96f=55+3\times2^5f \to 166+3^2\times2^5f \to 83+3^2\times2^4f \to 250+3^3\times2^4f \to 125+3^3\times2^3f \to 376+3^4\times2^3f \to 188+3^4\times2^2f \to 94+3^4\times2^1f \to 47+81f$ where the mark (**γ**).

Due to 47+81f < 55+96f, we get that if there is $n+1 \in$ 55+96f, then 55+96f and $n+1$ fit the conjecture, according to Theorem 1 and Lemma 3.

In the same way, on the bunch of operational routes of 19+12c, each reader can also conclude other 3 kinds of 19+12c derived from first satisfactory operational results to fit the conjecture, as listed below.

4). Pursuant to c=2d, d=2e+1, e=2f+1, f=2g+1 and g=2h to get c=32h+4, then there is 19+12c=187+384h derived from 119+243h where the mark (**δ**);

5). Pursuant to $c=2d+1$, $d=2e$, $e=2f+1$, $f=2g$ and $g=2h+1$ to get $c=32h+21$, then there is $19+12c=271+384h$ derived from $172+243h$ where the mark (**ε**);

6). Pursuant to $c=2d+1$, $d=2e+1$, $e=2f+1$, $f=2g+1$ and $g=2h+1$ to get $c=32h+31$ then there is $19+12c=391+384h$ derived from $248+243h$ where the mark (ζ).

So far, take into account what we and each reader have done, there are altogether 6 kinds of $15+12c/19+12c$ to fit the conjecture.

It follows that if *n+1* belongs in any kind of $15+12c/19+12c$ derived from first satisfactory operational result, then, that kind of $15+12c/19+12c$ and *n+1* fit the conjecture according to Theorem 1 and Lemma 3.

By now, we analyze two bunches of operational routes of $15+12c$ and $19+12c$ whether they are related, as described below.

Due to $c\geq1$, there are infinitely more odd numbers of $15+12c/19+12c$, whether they belong to infinite or finite more kinds, boil down to all kinds can only be on the bunch of operational routes of $15+12c/19+12c$.

For variables on operational routes of $15+12c/19+12c$, they are often seen as having many, however, in fact, they can be converted into a common variable, otherwise, any kind of $15+12c/19+12c$ is in the hide always.

Since there is a common variable, not only allows you to compare the size between a certain operational result and a kind of $15+12c/19+12c$, but also let you know that every two operational routes on the bunch of operational routes of $15+12c/19+12c$, they either directly intersect or indirectly connect, since they can be extended enough.

Besides, not only one kind of $15+12c/19+12c$ derives from first satisfactory operational result, but also there is the case where at least two kinds of $15+12c/19+12c$ derive from a satisfactory operational result, such as $15+12(4+2^{55}\times3^{2}y)$ and $15+12(8+2^{32}\times3^{17}y)$ are derived from $61+2^{3}\times3^{37}y$.

In some cases, an operational route of $15+12c$ and an operational route of $19+12c$ coincide partially or intersect from each other, such as start with $15+12(1+2^{57}y)$ to operate via five steps in a row, you get $19+12(1+2^{54}\times3^{2}y)$.

As stated, we have analyzed these two bunches of operational routes of $15+12c$ and $19+12c$, and that we are possessed of the enough evidences to prove that all unproven kinds of $15+12c/19+12c$ fit the conjecture.

Under these circumstances, let us prove all unproven kinds of $15+12c/$ $19+12c$ from each other's- opposite directions, in following paragraphs.

Firstly, we start with proven 6 kinds of $15+12c/19+12c$ to expand the scope of proven kinds of $15+12c/19+12c$ continuously.

According to Theorem 1 and Theorem 2, all integer expressions on at least one operational route via each of proven 6 kinds of $15+12c/19+12c$ are proved to fit the conjecture.

After that, all integer expressions which fit the conjecture are turned into proven integer expressions.

Next, according to Lemma 1 and Lemma 2, all integer expressions on successive intersecting operational routes via each of proven integer expressions are proved to fit the conjecture.

After that, all integer expressions which fit the conjecture are turned into proven integer expressions.

The rest can be deduced in batches by analogy. According to Lemma 1 and Lemma 2, proven integer expressions on the bunch's operational routes of 15+12c/19+12c are getting more and more.

In theory, by doing in this way and according to Lemma 1 and Lemma 2, the scope of proven integer expressions is successively extended, so after doing it an infinite number of times, all integer expressions on the bunch's operational routes of 15+12c/19+12c are proved to fit the conjecture.

Because the bunch of operational routes of 15+12c/19+12c consists of operational routes whose each starts with an integer expression of 15+12c/19+12c.

Now that all integer expressions on the bunch of operational routes of 15+12c/19+12c have seen proved to fit the conjecture, undoubtedly, all integer expressions of 15+12c/19+12c have seen proved too to fit the conjecture, since all integer expressions on the bunch's operational routes of 15+12c/19+12c contain all integer expressions of 15+12c/19+12c.

Secondly, we start with each unproved kind of 15+12c/19+12c to operate by the operational rule, until first satisfactory operational result of the unproved kind of 15+12c/19+12c appears on an operational route via the unproved kind of 15+12c/19+12c or on an operational route related to it.

First of all, how do you present an unproved kind of 15+12c/19+12c ?

Since there is only the variable c in 15+12c/19+12c, if the c value in some kind of 15+12c/19+12c is not equal to any of c values in all proven kinds of 15+12c/19+12c, then this kind of 15+12c/19+12c is exactly an unproved kind of 15+12c/19+12c.

In addition, before continuing to prove unproved kinds of 15+12c/19+12c, there have been 6 proven kind of 15+12c/19+12c. As proved above, they respectively are: when c=4e+3, 8f+5, 8f+6, 32h+25, 32h+10 and 32h, such that 15+12c have been proved to fit the conjecture; when c=4e, 8f+2, 8f+3, 32h+4, 32h+21 and 32h+31, such that 19+12c have been proved to fit the conjecture, so you don't have to worry about having no proven kind of 15+12c/19+12c.

Now that we can find unproved kinds of 15+12c/19+12c in turn, also are possessed of the operational rule and criteria for judging operational results, then we can prove each unproved kind of 15+12c/19+12c to fit the conjecture, so apply and can only apply one of the following three ways.

(1) On an operational route via an unproved kind of 15+12c/19+12c, if there is an integer expression that is less than the unproved kind of 15+12c/19+12c, then the unproved kind of 15+12c/19+12c is proved to fit the conjecture, according to Theorem 1.

(2) If an operational route via an unproved kind of 15+12c/19+12c and an operational route via a proven kind of 15+12c/19+12c intersect, then the unproved kind of 15+12c/19+12c is proved to fit the conjecture, according to Lemma 2.

(3) If an operational route via an unproved kind of 15+12c/19+12c and an operational route via a proven kind of 15+12c/19+12c are in indirect connection, then the unproved kind of 15+12c/19+12c is proved to fit the conjecture, according to Lemma 2.

As one kind of 15+12c/19+12c after another is proved, then proven kinds of 15+12c/19+12c are getting more and more, like so, it goes on infinitely, up to all unproved kind of 15+12c/19+12c are proved to fit the conjecture.

According to the bidirectional proofs which have been given above, such that all kind of 15+12c/19+12c have been proved to fit the conjecture.

Therefore, if $n+1$ belongs in a kind of 15+12c/19+12c, then $n+1$ is proved to fit the conjecture, accord to Lemma 3.

7. Make a Summary and Reach the Conclusion

To sum up, $n+1$ has been proved to fit the conjecture, whether $n+1$ belongs in which sort of odd numbers or which kind of odd numbers, or it is exactly an even number.

We can also prove integers $n+2$, $n+3$ etc. up to every integer that is greater than $n+1$ to fit the conjecture, in the light of the old way of doing thing.

The proof was thus brought to a close. As a consequence, the Collatz conjecture is tenable.

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