Interaction of Complex Scalar Fields and Electromagnetic Fields in
Klein-Gordon-Maxwell Theory in Cosmological Inertial Frame

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ABSTRACT
We found equations of complex scalar fields and electromagnetic fields on interaction
of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory
from Type A of wave function and Type B of expanded distance in cosmological inertial
frame.

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1. Introduction

The Lagrangian \( L \) of complex scalar fields \( \phi, \phi^* \) and Electromagnetic fields \( F_{\mu\nu}, F_{\mu\nu} \) is Klein-Gordon-Maxwell theory in special relativity theory,

\[
L = (\partial_\mu \phi + i e A_\mu \phi)(\partial^\mu \phi^* - i e A^\mu \phi^*) - \frac{m^2 c^2}{\hbar^2} \phi \phi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}
\]

\( \phi^* \) is \( \phi \)'s adjoint scalar, \( m \) is the mass of scalar fields \( \phi, \phi^* \)

\[
F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}, F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}
\]

(1)

2. Equations of Interaction of Complex Scalar Fields and Electromagnetic Fields in Cosmological Inertial Frame

The Lagrangian \( L \) of interaction of complex scalar fields and Electromagnetic fields is Klein-Gordon-Maxwell theory in cosmological inertial frame,

\[
L = (\ddot{\phi} + i e \ddot{A}_\mu \phi)(\ddot{\phi}^* - i e \ddot{A}^\mu \phi^*) - \frac{m^2 c^2}{\hbar^2} \phi \phi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}
\]

We consider Type A of wave function and Type B of expanded distance,[1],[2],[3],[4]

Type A of wave function: \( r \rightarrow r \sqrt{\Omega(t_0)} \) \( t \rightarrow \frac{t}{\sqrt{\Omega(t_0)}} \)

Type B of expanded distance: \( r \rightarrow r \Omega(t_0), t \rightarrow t \)

\[
\ddot{\phi} = (\sqrt{\Omega(t_0)} \frac{\partial}{c \dot{t}}, \frac{1}{\sqrt{\Omega(t_0)}} \ddot{\phi}), \ddot{\phi}^* = (\sqrt{\Omega(t_0)} \frac{\partial}{c \dot{t}}, -\frac{1}{\sqrt{\Omega(t_0)}} \ddot{\phi})
\]

\[
\ddot{A}_\mu = (\phi, \ddot{A} \Omega(t_0)), \ddot{A}_\mu = (\phi - \ddot{A} \Omega(t_0)), \ddot{F}_{\mu\nu} = F_{\mu\nu} \Omega(t_0), \ddot{F}_{\mu\nu} = F_{\mu\nu} \Omega(t_0)
\]

(2-1)

\( t_0 \) is the cosmological time. \( \Omega(t_0) \) is the expanding ratio of universe in the cosmological time \( t_0 \).

Complex scalar field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

\[
\ddot{\phi} - i e \ddot{A}_\mu \phi)(\ddot{\phi}^* - i e \ddot{A}^\mu \phi^*) + \frac{m^2 c^2}{\hbar^2} \phi \phi^* = 0
\]

(3)

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

\[
\ddot{\phi} = (\ddot{\phi} + i e \ddot{A}^\mu \phi)(\ddot{\phi}^* - i e \ddot{A}^\mu \phi^*) + \frac{m^2 c^2}{\hbar^2} \phi = 0
\]

(4)

If operator \( \ddot{\phi}, \ddot{\phi}^* \) are in cosmological inertial frame,[1],[2],[3],[4]
\[ \overline{\mathbf{F}}_{\mu}^\nu = \overline{\mathbf{\nabla}}_\nu - \overline{\mathbf{A}}_\nu - \overline{\mathbf{\nabla}}_\mu - \overline{\mathbf{A}}_\mu \]

Electromagnetic field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

\[ \overline{\mathbf{\nabla}}_\nu \left( \frac{\partial \mathbf{L}}{\partial (\overline{\mathbf{\nabla}}_\mu, \overline{\mathbf{A}}_\mu)} \right) - \frac{\partial \mathbf{L}}{\partial \overline{\mathbf{A}}_\mu} = \frac{1}{4} \overline{\mathbf{\nabla}}_\nu \left( \overline{\mathbf{\nabla}}_\mu, \overline{\mathbf{A}}_\mu \right) - \overline{\mathbf{\nabla}}_\mu \left( \overline{\mathbf{\nabla}}_\nu, \overline{\mathbf{A}}_\nu \right) - i e \phi \left( \overline{\mathbf{\nabla}}_\mu \phi^* - i e \overline{\mathbf{A}}_\mu \right) \]

\[ = \frac{1}{4} \overline{\mathbf{\nabla}}_\nu \left( \overline{\mathbf{\nabla}}_\mu \phi - i e \overline{\mathbf{A}}_\mu \right) = 0 \]

Hence,

\[ \overline{\mathbf{\nabla}}_\nu \left( \frac{\partial \mathbf{L}}{\partial \overline{\mathbf{\nabla}}_\mu} \right) - \frac{\partial \mathbf{L}}{\partial \overline{\mathbf{A}}_\mu} = \frac{4 \pi}{c} \overline{\mathbf{J}}^\mu \equiv 4 \pi \overline{\mathbf{\nabla}}_\nu \left( \overline{\mathbf{\nabla}}_\mu \phi^* - i e \overline{\mathbf{A}}_\mu \right) \]

\[ \overline{\mathbf{J}}^\mu = \frac{C}{\pi} i e \overline{\Phi} \left( \overline{\mathbf{\nabla}}_\mu \phi^* - i e \overline{\mathbf{A}}_\mu \right) \]

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

\[ \overline{\mathbf{\nabla}}_\nu \left( \frac{\partial \mathbf{L}}{\partial (\overline{\mathbf{\nabla}}_\mu, \overline{\mathbf{A}}_\mu)} \right) - \frac{\partial \mathbf{L}}{\partial \overline{\mathbf{A}}_\mu} = \frac{1}{4} \overline{\mathbf{\nabla}}_\nu \left( \overline{\mathbf{\nabla}}_\mu, \overline{\mathbf{A}}_\mu \right) - \overline{\mathbf{\nabla}}_\mu \left( \overline{\mathbf{\nabla}}_\nu, \overline{\mathbf{A}}_\nu \right) + i e \phi \left( \overline{\mathbf{\nabla}}_\mu \phi^* + i e \overline{\mathbf{A}}_\mu \right) \]

\[ = \frac{1}{4} \overline{\mathbf{\nabla}}_\nu \left( \overline{\mathbf{\nabla}}_\mu \phi + i e \overline{\mathbf{A}}_\mu \right) = 0 \]

Hence,

\[ \overline{\mathbf{\nabla}}_\nu \left( \frac{\partial \mathbf{L}}{\partial \overline{\mathbf{\nabla}}_\mu} \right) - \frac{\partial \mathbf{L}}{\partial \overline{\mathbf{A}}_\mu} = \frac{4 \pi}{c} \overline{\mathbf{J}}^\mu \equiv - 4 \pi \overline{\mathbf{\nabla}}_\nu \left( \overline{\mathbf{\nabla}}_\mu \phi + i e \overline{\mathbf{A}}_\mu \right) \]

\[ \overline{\mathbf{J}}^\mu = - \frac{C}{\pi} i e \overline{\Phi} \left( \overline{\mathbf{\nabla}}_\mu \phi + i e \overline{\mathbf{A}}_\mu \right) \]

3. Conclusion
We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame.

References


