Interaction of Complex Scalar Fields and Electromagnetic Fields in Klein-Gordon-Maxwell Theory in Cosmological Inertial Frame

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ABSTRACT

We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame.

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Key words: Klein-Gordon-Maxwell Theory;
          Cosmological Inertial Frame;
          Complex Scalar fields;
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1. Introduction

The Lagrangian $L$ of complex scalar fields $\phi, \phi^*$ and Electromagnetic fields $F_{\mu\nu}, F_{\mu\nu}$ is Klein-Gordon-Maxwell theory in special relativity theory,

$$L = (\partial_{\mu} \phi + ieA_{\mu} \phi)(\partial^{\mu} \phi^* - ieA^{\mu} \phi^*) - \frac{m^2 c^2}{\hbar^2} \phi \phi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

$\phi^*$ is $\phi$'s adjoint scalar, $m$ is the mass of scalar fields $\phi, \phi^*$

$$F_{\mu\nu} = \partial_{\mu} A^{\nu} - \partial_{\nu} A^{\mu}, F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$$

2. Equations of Interaction of Complex Scalar Fields and Electromagnetic Fields in Cosmological Inertial Frame

The Lagrangian $L$ of interaction of complex scalar fields and Electromagnetic fields is Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$L = (\bar{\partial}_{\mu} \phi + ie\bar{A}_{\mu} \phi)(\bar{\partial}^{\mu} \phi^* - ie\bar{A}^{\mu} \phi^*) - \frac{m^2 c^2}{\hbar^2} \phi \phi^* - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$$

(2-1)

We consider Type A of wave function and Type B of expanded distance,[1],[2],[3],[4]

Type A of wave function: $r \rightarrow r \sqrt{\Omega(t_0)}, t \rightarrow \frac{t}{\sqrt{\Omega(t_0)}}$.

Type B of expanded distance: $r \rightarrow r\Omega(t_0), t \rightarrow t$

$$\bar{\partial}_{\mu} = (\sqrt{\Omega(t_0)} \frac{\partial}{c \dot{\Omega}(t)} \sqrt{\Omega(t_0)} \bar{\nabla}), \bar{\partial}^{\mu} = (\sqrt{\Omega(t)} \frac{\partial}{c \dot{\Omega}(t)} \sqrt{\Omega(t)} \bar{\nabla})$$

$$\bar{A}_{\mu}^* = (\phi, \bar{A}(t_0)), \bar{A}^{\nu} = (\phi, -\bar{A}(t_0)), \bar{F}_{\mu\nu} = F_{\mu\nu} \Omega(t_0), \bar{F}_{\mu\nu}^{\nu} = F_{\mu\nu} \Omega(t_0)$$

$t_0$ is the cosmological time. $\Omega(t_0)$ is the expanding ratio of universe in the cosmological time $t_0$.

(2-2)

Complex scalar field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_{\mu} \frac{\partial L}{\partial (\bar{\partial}_{\mu} \phi)} - \frac{\partial L}{\partial \phi} = (\bar{\partial}_{\mu} - ie\bar{A}_{\mu} \phi)(\bar{\partial}^{\mu} \phi^* - ie\bar{A}^{\mu} \phi^*) + \frac{m^2 c^2}{\hbar^2} \phi^* = 0$$

(3)

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

$$\bar{\partial}_{\mu} \left( \frac{\partial L}{\partial (\bar{\partial}_{\mu} \phi)} \right) - \frac{\partial L}{\partial \phi} = (\bar{\partial}^{\mu} + ie\bar{A}^{\mu} \phi)(\bar{\partial}_{\mu} \phi + ie\bar{A}_{\mu} \phi) + \frac{m^2 c^2}{\hbar^2} \phi = 0$$

(4)

If operator $\bar{\partial}_{\mu}^*, \bar{\partial}^{\nu}$ are in cosmological inertial frame,[1],[2],[3],[4]
\[ \bar{\sigma}_\mu' = \left( \frac{\partial}{c \partial t}, \frac{1}{\Omega(t)} \vec{\nabla} \right), \bar{\sigma}^{\mu'} = \left( \frac{\partial}{c \partial t'}, -\frac{1}{\Omega(t)} \vec{\nabla} \right) \]

\[ \bar{F}^{\mu\nu'} = \bar{\sigma}^{\mu'} \bar{A}^{\nu'} - \bar{\sigma}^{\nu'} \bar{A}^{\mu'}, \bar{F}_{\mu\nu'} = \bar{\sigma}_\mu' \bar{A}_\nu' - \bar{\sigma}_\nu' \bar{A}_\mu' \]  

(5)

Electromagnetic field equations are in Klein-Gordon-Maxwell theory in cosmological inertial frame,

\[ \bar{\nabla}_\nu \left( \frac{\partial L}{\partial (\bar{\nabla}_\nu \bar{A}_\mu')} \right) - \frac{\partial L}{\partial \bar{A}_\mu'} \]

\[ = \frac{1}{4} \bar{\nabla}_\nu^\prime (\bar{\sigma}^{\mu'} \bar{A}^{\nu'} - \bar{\sigma}^{\nu'} \bar{A}^{\mu'}) - \text{i}e\phi (\bar{\sigma}^{\mu'} \phi^* - \text{i}e \bar{A}^{\mu'} \phi^*) + \text{i}e\phi (\bar{\sigma}^{\nu'} \phi + \text{i}e \bar{A}^{\nu'} \phi) \]

\[ = \frac{1}{4} \bar{\nabla}_\nu^\prime \bar{F}^{\mu\nu'} - \text{i}e\phi (\bar{\sigma}^{\mu'} \phi^* - \text{i}e \bar{A}^{\mu'} \phi^*) + \text{i}e\phi (\bar{\sigma}^{\nu'} \phi + \text{i}e \bar{A}^{\nu'} \phi) = 0 \]  

(6)

Hence,[5],

\[ \bar{\nabla}_\nu^\prime \bar{F}^{\mu\nu'} = \frac{4\pi}{c} \bar{j}^\mu = \frac{4}{\pi} \text{i}e \phi (\bar{\sigma}^{\mu'} (\phi^* - \text{i}e \bar{A}^{\mu'} \phi)) - \phi^* (\bar{\sigma}^{\nu'} (\phi + \text{i}e \bar{A}^{\nu'} \phi)) \]

(7)

The other equation is in Klein-Gordon-Maxwell theory in cosmological inertial frame,

\[ \bar{\nabla}^{\nu'} \left( \frac{\partial L}{\partial (\bar{\nabla}^{\nu'} \bar{A}^{\mu'})} \right) - \frac{\partial L}{\partial \bar{A}^{\mu'}} \]

\[ = \frac{1}{4} \bar{\nabla}^{\nu'} (\bar{\sigma}^{\mu'} \bar{A}^{\nu'} - \bar{\sigma}^{\nu'} \bar{A}^{\mu'}) + \text{i}e\phi (\bar{\sigma}^{\mu'} \phi + \text{i}e \bar{A}^{\mu'} \phi) - \text{i}e\phi (\bar{\sigma}^{\nu'} \phi^* - \text{i}e \bar{A}^{\nu'} \phi^*) \]

\[ = \frac{1}{4} \bar{\nabla}^{\nu'} \bar{F}_{\mu\nu'} + \text{i}e\phi (\bar{\sigma}^{\mu'} \phi + \text{i}e \bar{A}^{\mu'} \phi) - \text{i}e\phi (\bar{\sigma}^{\nu'} \phi^* - \text{i}e \bar{A}^{\nu'} \phi^*) = 0 \]  

(8)

Hence,[5],

\[ \bar{\nabla}^{\nu'} \bar{F}_{\mu\nu'} = \frac{4\pi}{c} \bar{j}_\mu = \frac{4}{\pi} \text{i}e \phi (\bar{\sigma}^{\mu'} (\phi^* + \text{i}e \bar{A}^{\mu'} \phi)) - \phi^* (\bar{\sigma}^{\nu'} (\phi - \text{i}e \bar{A}^{\nu'} \phi)) \]

\[ = \frac{4}{\pi} \text{i}e \phi (\bar{\sigma}^{\mu'} \phi - \text{i}e \bar{A}^{\mu'} \phi) - \phi^* (\bar{\sigma}^{\nu'} \phi + \text{i}e \bar{A}^{\nu'} \phi) \]

(9)

3. Conclusion

We found equations of complex scalar fields and electromagnetic fields on interaction of complex scalar fields and electromagnetic fields in Klein-Gordon-Maxwell theory from Type A of wave function and Type B of expanded distance in cosmological inertial frame.

References