A fresh hope of proving Goldbach conjecture(1+1)

Li Ke

Institute of Chemical Industry of Forest Products. CAF
Nanjing, China, MI 210042
E-mail address: liketaiping@163.com

Abstract

This paper provide a new way of proving Goldbach conjecture - LiKe sequence (This method was published in 2019). And briefly introduces the proof process of this method: by indirect transformation, Goldbach conjecture is transformed to prove that, for any prime sequence $(3,5,7,\ldots,P_n)$, there must have no LiKe sequence less than $3\times P_n$. This method only studies prime numbers and composite numbers, which is very important for the study of Goldbach conjecture.

Key words: Goldbach conjecture; number theory; LiKe sequence; sequence
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0. Introduction

Goldbach conjecture is a mathematical puzzle known all over the world. At present, the research methods mainly include almost prime, exception set, three prime theorem and almost Goldbach problem. But none of them solved the problem. But I find a new way, in contrast to previous methods, it transforms goldebach conjecture to only study the relationship between prime and composite numbers. I named it LiKe sequence. This method is feasible in theory. This paper briefly introduces the proof process of this method: by indirect transformation, Goldbach conjecture is transformed to prove that the LiKe sequence of prime sequence (3,5,7,…,Pn) must be greater than $2\times(P_{n+1}-1)$. The problem is proved by proving the LiKe sequence$>3\times P_n>2\times P_{n+1}>2\times(P_{n+1}-1)$ step by step. Through this article, we can understand that if we can prove there is no LiKe sequence less than $3\times P_n$ for any prime sequence$(3,5,7,\ldots,P_n)$, the Goldbach conjecture will be true.

1. The definition of LiKe sequence

Let 2N represents even numbers and half of it is N, all odd primes less than N as the prime sequence $(3,5\ldots, P_n)$, only use these prime factors to represent composite numbers $(Y|Y\in R,\ R=3^x5^y\ldots P_{n-1}{}^iP_n{}^j)$. In these composite numbers, if there is a new sequence $(Y_1,Y_2,\ldots,Y_n)$, make the inverse interval of the corresponding items is equal to the sequential interval of the items of the prime sequence $(Y_1-Y_2=2,\ldots,Y_{n-1}-Y_n=P_n-P_{n-1})$. We called $(Y_1,Y_2,\ldots,Y_n)$ is the corresponding LiKe sequence of the prime sequence $(3,5\ldots,P_n)$.

Table 1: Prime sequence and the corresponding LiKe sequence

Prime sequence	(3)	(3,5)	$(3,5\ldots, P_n)$
LiKe sequence	(9), (27), …, (3^n)	(27, 25)	$(Y_1,Y_2,\ldots,Y_n)$
Character	3^n	$27=3^3$, $25=5^2$; 27-25=5-3	$Y_n=3^x5^y\ldots P_{n-1}{}^iP_n{}^j$; $Y_1-Y_2=2,\ldots,Y_{n-1}-Y_n=P_n-P_{n-1}$
Deduction	$9>2\times(3+1)$	$25>2\times(5+1)$	$Y_n>2\times(P_n+1)$

2. Why is it equivalent to goldbach conjecture?

As we all known, Golbach conjecture states that all even numbers greater than or equal to 4 (The following even numbers apply to this condition) can be represented as the sum of two prime numbers.

It has an equivalent proposition: For any number N(≥2), it's either a prime or there is a number x, make both N-x and N+x are prime numbers.

That is to say, see table 2, for any even number 2N (that is N-x+N+x), all odd primes no greater than N as the prime sequence (3,5..., P_n), there must have a prime in (2N-3, 2N-5, …, 2N-P_n) at least.

Table 2: LiKe Matrix

2N	2N-P_n							
6	**3**	0	0	0	0	0	0	0
8	**5**	0	0	0	0	0	0	0
10	**7**	**5**	0	0	0	0	0	0
12	9	**7**	0	0	0	0	0	0
14	**11**	9	**7**	0	0	0	0	0
16	**13**	**11**	9	0	0	0	0	0
18	15	**13**	**11**	0	0	0	0	0
20	**17**	15	**13**	0	0	0	0	0
22	**19**	**17**	15	**11**	0	0	0	0
24	21	**19**	**17**	**13**	0	0	0	0
26	**23**	21	**19**	15	**13**	0	0	0
28	25	**23**	21	**17**	15	0	0	0
30	27	25	**23**	**19**	**17**	0	0	0
32	**29**	27	25	21	**19**	0	0	0
34	**31**	**29**	27	**23**	21	**17**	0	0
36	33	**31**	**29**	25	**23**	**19**	0	0
38	35	33	**31**	27	25	21	**19**	0
40	**37**	35	33	**29**	27	**23**	21	0
…	O	O-2	…	O-(11-3)	…	…	…	…

Its negative statement is that all (2N-3, 2N-5, …, 2N-P_n) are composite numbers.

And, for all composite numbers less than 2N, the factors can only be the prime number less than N.

So, for all the prime sequence (3,5..., P_n), if there is no LiKe sequence less than $2\times(P_{n+1}-1)$, the Goldbach conjecture will be true.

3. What is the proof path?

To sum up, the method must be right, but how do we prove Goldbach conjecture in this way? Here are some of my simple derivations for your reference.

Theorem:

Given the odd primes sequence (3,5,7...,P_n), and the terms of its corresponding LiKe sequence can only be > $2(P_{n+1}-1)$.

Proof:

To prove this theorem, we must known the following two lemmas:

Lemma 1:

The minimum LiKe sequence of odd prime sequence (3,5,7,…,P_n) is greater than $3\times P_n$.

The LiKe sequence of (3) are (9), (27), …, (3^n). They all ≥3×3;

The smallest LiKe sequence of (3, 5) is (25, 27). 25 >3×5;

For all the prime sequence (3,5..., P_n), we can easily verify that there is no LiKe sequence less than $2\times(P_{n+1}-1)$. But how prove it? Take (3, 5, 7) for example.

The sequence of (3, 5, 7) has no LiKe sequence, the proof is easy:

Let (a,b,c) is a LiKe sequence of (3, 5, 7)

If u and v are made up of factor 3,5,7 and they are not relatively prime (v > u)

Suppose x is the greatest common divisor of u and v

x is divisible by at least one of 3,5, and 7

That is v-u can be divisible by x. But according to the questions v-u $\in \{2,4\}$

So a,b,c must prime to each other, They can only be a permutation of $(3^i, 5^j, 7^k)$

Because $5^j \bmod 25 = 0$, $7^k \bmod 25 \in \{24,18,1,7\}$

But LiKe sequence requires $|7^k - 5^j| \in \{2,4\} \not\subset \{24,18,1,7\}$, it's impossible.

So (3,5,7) has no LiKe sequence.

Similarly, it's not hard to prove there is no LiKe sequence of $(3,5,7,...,P_n)$ less than $3\times P_n$.

Lemma 2:

When N is large enough, there must be primes between N and 1.5N.

It's obvious true.

From the lemma 2 we can get:

When N is large enough, there must be: $(3\times P_n)/(2\times P_{n+1}) > 1$

that is $3\times P_n > 2\times P_{n+1}$

And because: $2\times P_{n+1} > 2\times (P_{n+1}-1)$

So: $3\times P_n > 2\times (P_{n+1}-1)$

From the lemma 1 we can get:

LiKe sequence $> 3\times P_n$

So: LiKe sequence $> 2\times (P_{n+1}-1)$

So the theorem is right and the Goldbach conjecture is true too.

Q.E.D

4. Conclusion

To sum up, the new method(LiKe sequence) not only can solve the Goldbach conjecture, but only need to prove for any prime sequence$(3,5,7,\ldots,P_n)$, there must have no LiKe sequence less than $3\times P_n$, the Goldbach conjecture will be true.

References:

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