Centrifugal Explanation of Dark Energy

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Abstract

We shall use centrifugal forces to provide an explanation of dark energy by demonstrating that every orbit for a body has a centrifugal term in the radial equation of motion that balances gravity. Using the Schwarzschild solution of Einstein's field equations, we replace the cosmological constant in the Friedman equations for the expansion of the universe with a term related to the ratio of rotational energy density to the rest mass energy density. The new equations explain how a flat universe can be expanding with a positive acceleration by spending internal rotational energy. This paper compliments the findings of several that have demonstrated serious problems with the analysis of experimental data justifying "dark energy"[1 2 3]. Instead of postulating dark energy, perhaps we should be taking a more careful look at the validity of the standard cosmological assumption of a homogenous and isotropic universe.

Keywords: Dark energy, gravity, centrifugal, angular momentum, acceleration of universe, Schwarzschild solution, Friedman equation

I. Newtonian two body problem with an extra force:

Consider the two-body problem where one mass M is dominant, and the other mass m is infinitesimal so that we only need to consider the motion of the small mass m. In spherical coordinates, the motion of our particle is given by this equation[4]:

$$\begin{aligned}\vec{F} = m\ddot{\vec{r}} = m\left(\ddot{r} - r\dot{\varphi}^2 - r\sin^2\varphi\,\dot{\theta}^2\right)\hat{r}\\ + m\left(2\sin\varphi\,\dot{\theta}\dot{r} + 2r\cos\varphi\,\dot{\theta}\dot{\varphi} + r\sin\varphi\,\ddot{\theta}\right)\hat{\theta}\\ + m\left(2\dot{r}\dot{\varphi} + r\ddot{\varphi} - r\sin\varphi\cos\varphi\,\dot{\theta}^2\right)\hat{\varphi}\end{aligned}$$

Also, consider that there is an arbitrary additional force being applied to this test mass $\vec{F}_P$, and assume that the extra force is always applied perpendicularly to the velocity and the position vector.

$$\vec{F} = \vec{F}_G + \vec{F}_P = \left[-\frac{GMm}{r^2}\hat{r}\right] + \vec{F}_P, \qquad \vec{F}_P = \frac{f_{(\vec{r})}}{r^3}\left[\vec{r}\times\dot{\vec{r}}\right]$$

The velocity vector and extra force are given by:

$$\dot{\vec{r}} = \dot{r}\hat{r} + r\sin\varphi\,\dot{\theta}\hat{\theta} + r\,\dot{\varphi}\hat{\varphi}$$

$$\vec{F}_P = \frac{f_{(\vec{r})}}{r}\left[-\sin\varphi\,\dot{\theta}\hat{\varphi} + \dot{\varphi}\hat{\theta}\right]$$

Then the total force on our particle is given by:

$$\vec{F} = \vec{F}_G + \vec{F}_P = \left[-\frac{GMm}{r^2}\hat{r}\right] + f_{(\vec{r})}r^{-1}\left[-\sin\varphi\,\dot{\theta}\hat{\varphi} + \dot{\varphi}\hat{\theta}\right]$$

$$= m\left(\ddot{r} - r\dot{\varphi}^2 - r\sin^2\varphi\,\dot{\theta}^2\right)\hat{r} + m(\sim)\hat{\theta} + m(\sim)\hat{\varphi}$$

Then the equation of radial motion is:

$$\ddot{r} = -\frac{GM}{r^2} + r\left(\dot{\varphi}^2 + \sin^2\varphi\,\dot{\theta}^2\right)$$

Notice that even though we assumed an extra arbitrary force $\vec{F}_P$, because it was constrained perpendicularly to the velocity and position vectors, this extra force has absolutely no effect on the equation of motion for the $\hat{r}$ direction. The equations for the $\hat{\theta}$ and $\hat{\varphi}$ directions are very complicated, but that doesn't matter. Further, the angular momentum term naturally appears in the radial equation:

$$\ddot{r} = -\frac{GM}{r^2} + \frac{m^2r^4\left(\dot{\varphi}^2 + \sin^2\varphi\,\dot{\theta}^2\right)}{m^2r^3}$$
$$= -\frac{GM}{r^2} + \frac{\left(m\left[\vec{r}\times\dot{\vec{r}}\right]\right)^2}{m^2r^3} = -\frac{GM}{r^2} + \frac{L^2}{m^2r^3}$$

Where L^2 is the square of the angular momentum. L^2 Is constant in any reference frame. Therefore, we can now imagine another observer $\widehat{r_o}$ in a rotated reference frame given by the rotation of θ about the z axis, and φ about the y axis:

$$\begin{aligned}\widehat{r_o} &= \hat{r}\,R_x(\theta)\,R_y(\varphi)\\ &= \hat{r}\begin{bmatrix}\cos\theta & -\sin\theta & 0\\ \sin\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\cos\varphi & 0 & \sin\varphi\\ 0 & 1 & 0\\ -\sin\varphi & 0 & \cos\varphi\end{bmatrix} = \hat{z}\end{aligned}$$

In this reference frame, the extra force we applied $\vec{F}_P$ and the values of θ and φ are completely meaningless to our observer, and the orbit is a one-dimensional problem completely controlled by the equation of motion in the radial direction:

$$\ddot{r} = -\frac{GM}{r^2} + \frac{L^2}{m^2r^3} \quad (1)$$

[1] (Hunt, 2010)
[2] (Jacques Colin, 2019)
[3] (Subir Sarkar, 2020)
[4] (Weisstein, 2021)

Assume that this hypothetical observer is completely ignorant of the existence of θ and φ dimensions of reality. They would observe that the motion of bodies is always governed by a balance between an attractive gravitational force and a repulsive centrifugal force. If this observer noticed that the rate of expansion of the universe was accelerating, would they not deduce that the centrifugal term was probably dominant?

II. Building on the Schwarzschild solution:

Having done a Newtonian derivation, how does this affect appear in General Relativity? We will start with the Schwarzschild solution[5] to Einstein's field equations since it correlates with the previous derivation:

$$\left(1-\frac{2GM}{c^2r}\right)c^2dt^2 = \frac{dr^2}{\left(1-\frac{2GM}{c^2r}\right)} + r^2(d\varphi^2 + \sin^2\varphi\, d\theta^2)$$

With this metric, the equation of motion for the radial direction is similar to (1) except with an extra attractive factor:

$$\ddot{r} = -\frac{GM}{r^2} + \frac{L^2}{m^2r^3} - \frac{3GML^2}{mc^2r^4} \quad (2)$$

Within this equation, we can see that the relativistic factor will dominate if the radius is less than the Schwarzchild radius $R_s = \frac{2GM}{c^2}$. Although these last two terms are often neglected, we shall include them in our rederivation of the Friedman equations using (2).

III. Adding centrifugal forces to the Freidman equation:

How does rotational energy impact the expansion rate of the Universe? We shall rederive the classical version of the Freidman equations with these centrifugal forces included[6] [7].

Therefore, consider an infinitesimal mass at the edge of a sphere with a radius of $R_{(t)}$ within a universe of homogeneous and isotropic material density. However, instead of considering that this is a point mass, we shall assume that it is a homogeneous and isotropic distribution of matter with a significant amount of rotational energy.

In order to be isotropic, the total angular momentum must be zero, but this does not mean that the total rotational energy must also be zero. This distribution of matter is held within randomly oriented rotating gravitational structures.

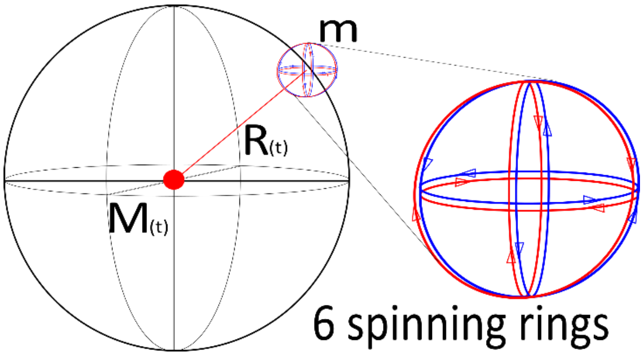

Figure 1: Replace point mass with rotating mass

We model it by assuming its rotational energy is equally split between 6 rings, with 2 in each of the three planes (Spin up and down for each axis). We can assume $r\omega$ represents the characteristic radius and angular velocity magnitudes and the rotational energy E_R is given by:

$$E_R = 6\frac{1}{2}\frac{m}{6}(r\omega)^2 = \frac{m}{2}\left(\frac{R_{(t)}mr\omega}{R_{(t)}m}\right)^2 = \frac{m}{2}\left(\frac{L}{R_{(t)}m}\right)^2 \quad (3)$$

We need to apply a factor of 2/3 due to the equal partition of the rotational energy in the 6 degrees of freedom for our mass and the fact that on average one plane of the rotating masses are tangential to the sphere, while the other two planes are perpendicular and contribute only half as much centrifugal force as the tangential plane.

$$\frac{1}{6}\left[(1+1) + \left(\frac{1}{2}+\frac{1}{2}\right) + \left(\frac{1}{2}+\frac{1}{2}\right)\right] = \frac{2}{3}$$

Combining this factor with equation (2) gives:

$$\vec{F}_m = m\left[-\frac{GM_s}{{R_{(t)}}^2} + \frac{2}{3}\frac{L^2}{m^2R_{(t)}^3} - \frac{2GM_sL^2}{mc^2R_{(t)}^4}\right]\hat{r}$$

Next, we can calculate the acceleration like this:

$$\frac{d^2R_{(t)}}{dt^2} = -\frac{GM_s}{{R_{(t)}}^2} + \frac{2}{3}\frac{L^2}{m^2R_{(t)}^3} - \frac{2GM_sL^2}{m^2c^2R_{(t)}^4}$$

$$\int \frac{d^2R_{(t)}}{dt^2}\frac{dR_{(t)}}{dt}dt = -\int \frac{GM_s}{R_{(t)}^2}\frac{dR_{(t)}}{dt}$$

$$+\frac{2}{3}\int \frac{L^2}{m^2R_{(t)}^3}\frac{dR_{(t)}}{dt}dt - \int \frac{2GM_sL^2}{c^2R_{(t)}^4}\frac{dR_{(t)}}{dt}dt$$

$$\frac{1}{2}\left(\frac{dR_{(t)}}{dt}\right)^2 = \frac{GM_s}{R_{(t)}} - 2\frac{L^2}{m^2R_{(t)}^2} + \frac{8GML^2}{m^2c^2R_{(t)}^3} + U$$

[5] (Schwarzschild, 1916)
[6] (Ryden, 2003, pp. 43-59)
[7] (Weinberg, 2008, pp. 34-40)

Next we use that the mass of the sphere remains constant as it expands or contracts, and we also express $R_{(t)}$ in the form:

$$M_s = \frac{4\pi}{3}\rho_{(t)} R_{(t)}^3, \qquad R_{(t)} = A_{(t)} R$$

Plugging in these equations gives a correction term to the classical version of the Freidman equation:

$$\left(\frac{\dot{A}}{A}\right)^2 = \frac{8\pi G\rho}{3} - \frac{4L^2}{m^2R^4A^4} + \frac{32\pi G\rho L^2}{3m^2c^2R^2A^2} + \frac{2U}{R^2A^2}$$

Replacing the angular momentum with equation (2) and rest energy $E_M = mc^2$ derives a more intuitive version of the Freidman equation:

$$\left(\frac{\dot{A}}{A}\right)^2 = \frac{8\pi G\rho}{3} - \frac{8\,E_R}{E_M}\left(\frac{c^2}{R^2A^2} - \frac{8\pi G\rho}{3}\right) + \frac{2U}{R^2A^2} \quad (4)$$

What exactly is this term $\frac{E_R}{E_M}$? It is the average density of rotational energy per unit volume per average density of rest mass energy per unit volume. To estimate the value of this term, one would need to perform the following calculation over a representative sample of the universe where it is fairly homogenous. Using our derivation above:

$$\frac{E_R}{E_M} = \left(\frac{1}{mc^2}\right)\frac{1}{2}\frac{L^2}{mR_{(t)}^2}, \quad E_M = c^2 \oint \rho\left(\overrightarrow{R_{(t)}}\right) dv = \rho_{(t)} V c^2$$

$$E_R = \frac{1}{2}\oint \frac{dL^2\, dv}{dm\, R_{(t)}^2}, \qquad d\vec{L} = dm\left(\overrightarrow{R_{(t)}} \times \dot{\overrightarrow{R_{(t)}}}\right)$$

$$E_R = \frac{1}{2}\oint \frac{dm^2\left(\overrightarrow{R_{(t)}} \times \dot{\overrightarrow{R_{(t)}}}\right)^2}{dm\, R_{(t)}^2}$$

$$= \frac{1}{2}\oint \rho\left(\overrightarrow{R_{(t)}}\right)\frac{\left|\overrightarrow{R_{(t)}} \times \dot{\overrightarrow{R_{(t)}}}\right|^2}{R_{(t)}^2} dv$$

$$(2) \quad \frac{E_R}{E_M} = \frac{1}{2V\rho_{(t)}c^2}\oint_0^V \rho\left(\overrightarrow{R_{(t)}}\right)\frac{\left|\overrightarrow{R_{(t)}} \times \dot{\overrightarrow{R_{(t)}}}\right|^2}{R_{(t)}^2} dv$$

From this equation, we can see that $\frac{E_R}{E_M}$ clearly depends upon the local distribution of matter in the universe. We can use the relativistic Friedman equation to replace the cosmological constant with the centrifugal term like this:

$$\left(\frac{\dot{A}}{A}\right)^2 = \frac{8\pi G\epsilon}{3c^2} - \frac{k\,c^2}{R^2A^2} + \frac{\Lambda}{3}, \ \Lambda = -\frac{8\,E_R}{E_M}\left(\frac{3c^2}{R^2A^2} - \frac{8\pi G\epsilon}{c^2}\right)$$

$$\left(\frac{\dot{A}}{A}\right)^2 = \frac{8\pi G}{3c^2}\left(1 + 8\frac{E_R}{E_M}\right)\epsilon - \frac{c^2}{R^2A^2}\left(k + 8\frac{E_R}{E_M}\right) \ (5)$$

If we use our centrifugal term for the cosmological factor Λ then also apply the fluid equation[8] we can derive the equation for the acceleration factor. We take the derivative, and divide both sides by $2A\dot{A}$:

$$\frac{d}{dt}\dot{A}^2 = \frac{d}{dt}\left[\frac{8\pi G}{3c^2}\epsilon A^2 - \frac{8\,E_R}{3E_M}\left(\frac{3c^2}{R^2} - \frac{8\pi G\epsilon}{c^2}\epsilon A^2\right) - \frac{k\,c^2}{R^2}\right]$$

$$\frac{2\ddot{A}\dot{A}}{2A\dot{A}} = \frac{1}{2A\dot{A}}\left[\frac{8\pi G}{3c^2}\left(\dot{\epsilon}A^2 + 2\epsilon A\dot{A}\right) - \frac{8c^2}{R^2}\left[\frac{d}{dt}\left(\frac{E_R}{E_M}\right)\right] + \frac{64\pi G E_M}{3c^4}\left[\frac{d}{dt}\left(\frac{E_R}{E_M}\right)\epsilon A^2 + \frac{E_R}{E_M}\left(\dot{\epsilon}A^2 + 2\epsilon A\dot{A}\right)\right]\right]$$

$$using\ fluid\ equation: \dot{\epsilon} + 3\frac{\dot{A}}{A}(\epsilon + P) = 0$$

$$\frac{\ddot{A}}{A} = -\frac{4\pi G}{3c^2}\left(1 + 8\frac{E_R}{E_M}\right)(\epsilon + 3P) + 4\left[\frac{8\pi G}{3c^2}\epsilon - \frac{c^2}{R^2A^2}\right]\left[\frac{d}{dt}\left(\frac{E_R}{E_M}\right)\right]\frac{A}{\dot{A}} \ (6)$$

$$\frac{\ddot{A}}{A} = -\frac{4\pi G}{3c^2}\left(1 + 8\frac{E_R}{E_M}\right)(\epsilon + 3P) + \frac{4\left[\frac{8\pi G}{3c^2}\epsilon - \frac{c^2}{R^2A^2}\right]\left[\frac{d}{dt}\left(\frac{E_R}{E_M}\right)\right]}{\sqrt{\frac{8\pi G\epsilon}{3c^2}\left(1 + 8\frac{E_R}{E_M}\right) - \frac{c^2}{R^2A^2}\left(k + 8\frac{E_R}{E_M}\right)}} \ (7)$$

Therefore, including the centrifugal forces in the Friedman equations shows that the acceleration of the universe is affected by the ratio of $\frac{E_R}{E_M}$. We can clarify these equations by substituting:

$$\frac{c^2}{R^2A^2} = \frac{1}{T^2}, \qquad T\ is\ time\ for\ light\ to\ travel\ RA$$

$$\frac{8\pi G\epsilon}{3c^2} = \left(\frac{2G}{c^2}\frac{4\pi R^3A^3}{3}\frac{\epsilon}{c^2}\right)\frac{c^2}{R^3A^3} = \frac{R_s}{R_{(t)}}\frac{1}{T^2}$$

Where R_s is the Schwarzchild radius of the black hole size for our universe. Then we can express the equations for the expansion rate and the acceleration rate like this:

$$\left(\frac{\dot{A}}{A}\right)^2 = \frac{1}{T^2}\left[\frac{R_s}{R_{(t)}}\left(1 + 8\frac{E_R}{E_M}\right) - \left(k + 8\frac{E_R}{E_M}\right)\right] \ (8)$$

$$\frac{\ddot{A}}{A} = \frac{1}{2T^2}\left[\frac{T\left[\frac{R_S}{R_{(t)}} - 1\right]\left[\frac{d}{dt}\left(1 + 8\frac{E_R}{E_M}\right)\right]}{\sqrt{\frac{R_S}{R_{(t)}}\left(1 + 8\frac{E_R}{E_M}\right) - \left(k + 8\frac{E_R}{E_M}\right)}} - \frac{R_S}{R_{(t)}}\left(1 + 8\frac{E_R}{E_M}\right)\left(1 + \frac{3P}{\epsilon}\right)\right] (9)$$

[8] (Ryden, 2003, p. 53)

Notice that equations (8) and (9) reduce to the normal Friedman equations without the cosmological constant when $\frac{E_R}{E_M} = 0$ and $\frac{d}{dt}\left(\frac{E_R}{E_M}\right) = 0$.

IV. Evaluation of new equations for expansion:

Now we shall evaluate equations (8) and (9) to see if they are able to explain the current observations of cosmology. To summarize these observations:

1. The universe is expanding[9]: $\frac{\dot{A}}{A} > 0$
2. The acceleration of expansion is positive[10] [11]: $\frac{\ddot{A}}{A} > 0$
3. The universe appears to be flat[12] [13]: $k = 0$
4. The universe is not inside a black hole[14]: $\frac{R_s}{R_{(t)}} < 1$

First, we use (9) to find three possible explanations for the positive acceleration of the universe. These possible solutions may be reasoned by plugging in that $k = 1, -1, 0$. For example, if we assume that $k = 1$, equation (9) simplifies to:

$$\frac{\ddot{A}}{A} = \frac{1}{2}\frac{1}{T^2}\left[T\sqrt{\frac{R_s}{R_{(t)}} - 1}\,\frac{\left[\frac{d}{dt}\left(1+8\frac{E_R}{E_M}\right)\right]}{\sqrt{1+8\frac{E_R}{E_M}}} - \frac{R_s}{R_{(t)}}\left(1 + 8\frac{E_R}{E_M}\right)\left(1 + \frac{3P}{\epsilon}\right)\right]$$

Then if we try to use this to explain observation #2, we arrive at the contradiction that:

$$\frac{d}{dt}\left(\frac{E_R}{E_M}\right) < 0$$

$$\frac{d}{dt}\left(\frac{E_R}{E_M}\right) > \frac{\left(1 + 8\frac{E_R}{E_M}\right)^{\frac{3}{2}}\frac{R_s}{R_{(t)}}\left(1 + \frac{3P}{\epsilon}\right)}{8\,T_{(t)}\sqrt{\frac{R_s}{R_{(t)}} - 1}} > 0$$

Therefore there is no solution for $k = 1$. Similarly, for $k = -1$, we can show that the only possible explanation for a positive acceleration requires that:

$$\frac{R_s}{R_{(t)}} > 1, \qquad \frac{d}{dt}\left(\frac{E_R}{E_M}\right) > 0$$

This solution applies to the situation where a universe is inside a black hole and the ratio of $\frac{E_R}{E_M}$ is increasing. Although this would explain observation #2, it fails to explain observation #1, #3, and #4.

For $k = 0$ there are two solutions that explain observation #2. One solution is like the previous solution for $k = -1$, but the other solution is consistent with our other observations. For $k = 0$, equation (9) simplifies to:

$$\frac{\ddot{A}}{A} = \frac{1}{2T^2}\left[\frac{T\left[\frac{R_s}{R_{(t)}} - 1\right]\left[\frac{d}{dt}\left(1+8\frac{E_R}{E_M}\right)\right]}{\sqrt{\frac{R_s}{R_{(t)}} + \left(\frac{R_s}{R_{(t)}} - 1\right)8\frac{E_R}{E_M}}} - \frac{R_s}{R_{(t)}}\left(1 + 8\frac{E_R}{E_M}\right)\left(1 + \frac{3P}{\epsilon}\right)\right] \quad (10)$$

Observation #2 is possible if:

$$\frac{R_s}{R_{(t)}} < 1, \qquad \frac{d}{dt}\left(\frac{E_R}{E_M}\right) < 0$$

Then if we check equation (8), this solution implies:

$$\frac{\dot{A}}{A} > 0 \;\; iff \;\; \left[\frac{R_s}{R_{(t)}} + \left(\frac{R_s}{R_{(t)}} - 1\right) 8\frac{E_R}{E_M}\right] > 0$$

This is also required by equation (9) since the same term appears in the radical. Therefore, we arrive at the requirement that:

$$\left(1 + 8\frac{E_R}{E_M}\right) < \left(1 - \frac{R_s}{R_{(t)}}\right)^{-1} \quad (11)$$

Therefore, this solution explains observation #1. The requirement for this solution assumes $k = 0$ and $\frac{R_s}{R_{(t)}} < 1$. Therefore, it is obviously consistent with observation #3 and #4. This theory explains the cosmological observations without any need for dark energy.

VI. Conclusion:

Could dark energy be explained by centrifugal forces? This seems like the most reasonable explanation because including centrifugal forces in the derivation of the Friedman equations removes some of the original reasons why dark energy was postulated[15] [16].

In addition to explaining the general observations of cosmology, this explanation also addresses several other problems with dark energy. For example, is it a coincidence that we find ourselves in a universe where "dark energy" and normal matter are approximately in balance[17]? No, because it is normal for systems to have a balance of kinetic and

[9] (Hubble, 1929)
[10] (Perlmutter, 1998)
[11] (AG Riess, 1998)
[12] (Balbi A, 2021)
[13] (de Bernardis, 2000)
[14] (Edwin Loh, 1986)
[15] (Weinberg, 2008, p. 54)
[16] (Smethurst, 2021)
[17] (O'Dowd, 2021)

rotational energy. Further, dark energy allowed for the universe to be at an unstable equilibrium. However, these equations allow for stable equilibriums where the acceleration rate can oscillate[18].

We must be careful to consider that energy is not only exchanged between gravitational potential energy and kinetic energy, but also exchanged with rotational energy. Equations (8) and (9) can most intuitively be thought of as equations that govern a universal trade between kinetic energy, rotational energy, and gravitational potential energy, governed by conservation of angular momentum and energy.

Combining our results with those of authors who did a critical analysis of the accelerated expansion data[19] [20] provides a more compelling argument against "dark energy" by undermining the theoretical need for the cosmological constant to balance the Friedman equations. Given that "dark energy" feels unintuitive, lacks reasonable explanation, and seems to violate conservation of energy: I have become convinced that "dark energy" is simply not real.

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[18] (Ryden, 2003, pp. 57-59)
[19] (Hunt, 2010)
[20] (Jacques Colin, 2019)