Adding boundary terms to Anderson localized Hamiltonians leads to unbounded growth of entanglement

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Abstract

It is well known that in Anderson localized systems, starting from a random product state the entanglement entropy remains bounded at all times. However, we show that adding a single boundary term to an otherwise Anderson localized Hamiltonian leads to unbounded growth of entanglement. Our results imply that Anderson localization is not a local property. One cannot conclude that a subsystem has Anderson localized behavior without looking at the whole system, as a term that is arbitrarily far from the subsystem can affect the dynamics of the subsystem in such a way that the features of Anderson localization are lost.

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1 Introduction

In the presence of quenched disorder, the phenomenon of localization can occur not only in single-particle systems, but also in interacting many-body systems. The former is known as Anderson localization (AL) [1], and the latter is called many-body localization (MBL) [2,7]. In the past decade, significant progress has been made towards understanding AL and especially MBL.

A characteristic feature that distinguishes MBL from AL lies in the dynamics of entanglement. Initialized in a random product state, the entanglement entropy remains bounded at all times in AL systems [8], but grows logarithmically with time in MBL systems [9,11]. The logarithmic growth of entanglement can be understood heuristically [12,13] from a

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phenomenological model of MBL [14, 15]. Recently, it was rigorously proved that in MBL systems, the entanglement entropy obeys a volume law at long times [16].

Consider the random-field $XXZ$ chain with open boundary conditions

$$H_{XXZ} = \sum_{j=1}^{N-1} (\sigma^x_j \sigma^x_{j+1} + \sigma^y_j \sigma^y_{j+1} + \Delta \sigma^z_j \sigma^z_{j+1}) + \sum_{j=1}^N h_j \sigma^z_j, \quad (1)$$

where $\sigma^x_j, \sigma^y_j, \sigma^z_j$ are the Pauli matrices at site $j$, and $h_j$'s are independent and identically distributed uniform random variables on the interval $[-h, h]$. For $\Delta = 0$, this model reduces to the random-field $XX$ chain

$$H_{XX} = \sum_{j=1}^{N-1} (\sigma^x_j \sigma^x_{j+1} + \sigma^y_j \sigma^y_{j+1}) + \sum_{j=1}^N h_j \sigma^z_j. \quad (2)$$

Using the Jordan–Wigner transformation, $H_{XX}$ is equivalent to a model of free fermions hopping in a random potential. It is AL for any $h > 0$. The $\Delta$ term in Eq. (1) introduces interactions between fermions. $H_{XXZ}$ is MBL for any $\Delta \neq 0$ and sufficiently large $h$ [17–19].

In $H_{XXZ}$, the $\Delta$ term representing interactions between fermions is extensive in that it is the sum of $N-1$ local terms between adjacent qubits. Let

$$H_{XXb} = H_{XX} + \Delta \sigma^z_{N-1} \sigma^z_N = \sum_{j=1}^{N-1} (\sigma^x_j \sigma^x_{j+1} + \sigma^y_j \sigma^y_{j+1}) + \sum_{j=1}^{N} h_j \sigma^z_j + \Delta \sigma^z_{N-1} \sigma^z_N. \quad (3)$$

Without the last term, $H_{XXb}$ is AL. In this paper, we show that in the dynamics generated by $H_{XXb}$, the effect of this boundary term invades into the bulk: Starting from a random product state the entanglement entropy obeys a volume law at long times. For large $h$, the coefficient of the volume law is almost the same as that in the dynamics generated by $H_{XXZ}$. Our results imply that AL is not a local property. One cannot conclude that a subsystem has AL behavior without looking at the whole system, as a term that is arbitrarily far from the subsystem can affect the dynamics of the subsystem in such a way that the features of AL are lost.

We briefly discuss related works. Khemani et al. [20] showed nonlocal response to local manipulations in localized systems. This work considers time-dependent Hamiltonians, and is thus different from ours. Vasseur et al. [21] studied the revival of a qubit coupled to one end of an AL system, but the coupling is chosen such that the whole system (including the additional qubit) is a model of free fermions. This is in contrast to $H_{XXb}$.

2 Results

Definition 1 (entanglement entropy). The entanglement entropy of a bipartite pure state $\rho_{AB}$ is defined as the von Neumann entropy

$$S(\rho_A) := -\text{tr}(\rho_A \ln \rho_A) \quad (4)$$

of the reduced density matrix $\rho_A = \text{tr}_B \rho_{AB}$.
Figure 1: Dynamics of the half-chain entanglement entropy for $H_{XXb}$ (blue), $H_{XXZ}$ (green), and $H_{XX}$ (red).

We initialize the system in a Haar-random product state.

**Definition 2** (Haar-random product state). In a system of $N$ qubits, let $|\Psi\rangle = \bigotimes_{j=1}^{N} |\Psi_j\rangle$ be a Haar-random product state, where each $|\Psi_j\rangle$ is chosen independently and uniformly at random with respect to the Haar measure.

For our numerical results, we choose $h = 10$ and $\Delta = 1$, and average over 1000 disorder realizations. We choose $N = 10$ in Figure 1 and in the left panel of Figure 2.

Figure 1 shows the dynamics of the entanglement entropy between the left and right halves of the system for $H_{XXb}$, $H_{XXZ}$, and $H_{XX}$. We clearly see that the last term in Eq. (3) leads to slow entanglement growth.

Figure 2 shows that the entanglement entropy at long times obeys a volume law for $H_{XXb}$ and $H_{XXZ}$, and the coefficient of the volume law is very close to $1/2$. This is consistent with the analytical prediction of Ref. [16], which assumes that the spectrum of the Hamiltonian has non-degenerate gaps.

**Definition 3** (non-degenerate gap). The spectrum $\{E_j\}$ of a Hamiltonian has non-degenerate gaps if the differences $\{E_j - E_k\}_{j \neq k}$ are all distinct, i.e., for any $j \neq k$,

$$E_j - E_k = E_{j'} - E_{k'} \implies (j = j') \text{ and } (k = k'). \quad (5)$$

Indeed, we have numerically verified that the spectra of both $H_{XXb}$ and $H_{XXZ}$ almost surely have non-degenerate gaps.

In the right panel of Figure 2, we observe a constant correction to the volume law. This is expected, for such corrections also exist in other contexts [22, 27].
Figure 2: Left panel: The entanglement entropy between the first $j$ and the last $N-j$ qubits at long times for $H_{XXb}$ (blue) and $H_{XXZ}$ (green). The black lines are $S = \min\{j, N-j\}/2$.
Right panel: Finite-size scaling of the half-chain entanglement entropy at long times for $H_{XXb}$ (blue) and $H_{XXZ}$ (green). The black line is $S = N/4 - 1/2$.

3 Discussion

We have numerically shown that adding a single boundary term to an otherwise AL Hamiltonian leads to entanglement growth. Starting from a random product state the entanglement entropy obeys a volume law at long times, and the coefficient of the volume law is consistent with the analytical prediction of Ref. [16].

Here are some interesting problems that deserve further study.

- Can we prove that the spectrum of $H_{XXb}$ almost surely has non-degenerate gaps? A positive answer to this question would allow us to rigorously prove some of the numerical results in this paper.

- Can we develop an analytical understanding of how the entanglement entropy grows with time for $H_{XXb}$ by adapting the phenomenological model of MBL [14, 15]?

- How does $H_{XXb}$ scramble local information as measured by the out-of-time-ordered correlator [28, 34]?

- It was argued that MBL is less stable in two and higher spatial dimensions [35]. To what extent a single boundary term delocalizes an AL system in higher dimensions?

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References


