Description of interaction applying representation of physical quantity at finite temperature from changed distribution function of quantum statistics

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Abstract

So far, we have proposed a method of expressing finite temperature in the form of ground state and pure state by changing the shape of the distribution function and using the step function.

Then, in the second quantized system in this form, the Hamiltonian and the method of giving its eigenvalues are presented.

We did this in the form that the creation and annihilation operators, whose matrix elements changed as the statistics changed, acted on the Hamiltonian in the ground state.

We also discussed the basics of the parastatistics method used for that.

In this paper, we apply this finite temperature expression method, express the eigenvalues of the Hamiltonian of the interacting system, and try to express the energy given by the interaction as heat.

1 Introduction

In our previous report [1], we expressed finite temperature energy in absolute zero form statistics by changing the shape of the distribution function.

Furthermore, we introduced a creation and annihilation operator that incorporates changes in this distribution function, and attempted to express a finite temperature field in the form of a pure state by acting on the Hamiltonian in the ground state and performing second quantization [2].

In this paper, we consider the expression that the energy due to the interaction is regarded as heat by expressing the Hamiltonian of the interacting system in this form.
Reprint of second quantization of finite temperature field using statistical change

First, I will reprint how to make the Hamiltonian in the ground state, which is represented in the pure state, act on the creation and annihilation operator, which represents the state in which the statistics are changed. This expresses a finite temperature.

Now, we assume the following ground state temperatures.

\[ T \rightarrow T_0 \simeq 0 \]  

(1)

Then assume the following distribution function.

\[ \frac{1}{\exp(E - \mu)/k_B T_0 + \kappa} \]  

(2)

Here, in order to express the temperature of this system depending on \( \kappa \) in this function, a function of the following shape is prepared.

\[ \kappa(\mu, T) = \frac{1}{\pi} \tan^{-1} \left( f \left( \frac{\mu}{k_B T} \right) \right) + \frac{1}{2} \]  

(3)

Here, \( \kappa(\mu, T) \) in the form that the integral of the distribution function in the form of the conventional Fermi distribution and the integral of the distribution function in the form of changing the statistics match as follows.

\[ n_0 = \int_0^\infty \frac{1}{\exp(\alpha + \beta \epsilon) + 1} \frac{4\pi p^2}{\hslash^3} dp = \int_0^\infty \frac{1}{\exp(\epsilon - \mu)/T_0 + \kappa} \frac{g4\pi p^2}{\hslash^3} dp = \frac{8\pi}{3\hslash^3 \kappa} p_f^3 \]  

(4)

\[ \alpha = -\frac{\mu}{k_B T}, \beta = \frac{1}{k_B T}, \]  

(5)

However, here as well, the change in Fermi energy due to temperature was ignored for the sake of simplicity.

The eigenvalues of any energy level according to this statistic are expressed using the Hamiltonian in the ground state below.

\[ \hat{H} = -\frac{\hslash^2}{2m} \frac{d^2}{dx^2} + V(x) \]  

(6)

Here, in the following distribution function,

\[ \frac{1}{\exp(E - \mu)/k_B T_0 + \kappa}, T = \text{Actual Temperature}, T_0 = 0 \]  

(7)

when the value of the energy eigenvalue is \( E < \mu \), the number of quantum states that can be occupied by one particle is \( 1/\kappa \), which is a new quantum system. In order to perform the second quantization on this, we consider a change in the creation and annihilation operator displayed in a matrix as shown below.
\[
\hat{a}^\dagger = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{a} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \quad \Rightarrow \quad \hat{\alpha}^\dagger = \begin{pmatrix} 0 & 1/\sqrt{\kappa} \\ 0 & 0 \end{pmatrix}, \quad \hat{\alpha} = \begin{pmatrix} 0 & 0 \\ 1/\sqrt{\kappa} & 0 \end{pmatrix}
\]

Here, the number density operator is as follows.

\[
N = \hat{\alpha}^\dagger \hat{\alpha} = \begin{pmatrix} 0 & 1/\sqrt{\kappa} \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1/\sqrt{\kappa} & 0 \end{pmatrix} = \begin{pmatrix} 1/\kappa & 0 \\ 0 & 0 \end{pmatrix}
\]

As a result, a system in which the minimum unit of the number of particles generated and extinguished is 1/\kappa is expressed.

Here, the field operator represented by the wave function \(\psi_i\) in an arbitrary ground state is expressed as follows using the new creation and annihilation operator shown above.

\[
\hat{\psi}(x) = \sum_i \psi_i(x) \hat{a}_i \quad \Rightarrow \quad \hat{\psi}'(x) = \sum_i \psi_i(x) \hat{\alpha}_i
\]

\[
\hat{\psi}(x) = \sum_i \psi^*_i(x) \hat{a}_i^\dagger \quad \Rightarrow \quad \hat{\psi}'(x) = \sum_i \psi^*_i(x) \hat{\alpha}_i^\dagger
\]

Using this, the eigenvalue of the Hamiltonian shown in the equation (6) at temperature T is that the same number of 1/\kappa particles are occupied by all the levels up to the Fermi level. Based on this approximation, it is calculated as follows.

\[
\hat{F}_1 = \int \hat{\psi}'^\dagger \hat{H}_1(x) \hat{\psi}'(x) dx = \sum_{ij} \left[ \psi^*_i(x) \hat{H}_1(x) \psi_j(x) \right] \hat{\alpha}_i^\dagger \hat{\alpha}_j
\]

\[
= \frac{1}{\kappa} \left[ \psi^*_i(x) \hat{H}_1(x) \psi_j(x) \right] \hat{a}_i^\dagger \hat{a}_j
\]

As described above, the eigenvalues of Hamiltonian at finite temperature could be expressed in the form of pure and ground states.

We named this form "hot absolute zero."

### 3 Representation of interaction field

Now, I would like to try to express the Hamiltonian of the field of interacting particles by using the expressions described so far.

Now, assume the Hamiltonian of the interacting particles as follows.

\[
\hat{H} = \hat{H}_0 + \hat{H}_{int}
\]

Then, here, it is assumed that an interaction with an external electric field or the like is added to the entire system represented by the distribution function (2).

It is assumed that the eigenvalue of the ground state free particle part \(\hat{H}_0\) is given by the ground state field operator shown in the previous chapter as follows.

\[
\hat{F}_0 = \int \hat{\psi}^\dagger \hat{H}_0(x) \hat{\psi}(x) dx
\]

Consider the eigenvalues of the entire Hamiltonian with the interaction.
Assume that $\kappa$ is given so that this value is equal to the eigenvalue when the field operator that changed the statistics is applied only to the $\hat{H}_0$ part.

\[ \tilde{F}_{0+int} = \int \psi^\dagger (\hat{H}_0(x) + \hat{H}_{int}(x)) \psi(x) dx = \int \psi^\dagger \hat{H}_0(x) \psi(x) dx + \sum_{ij} \left[ \psi^*_i(x) \hat{H}_0(x) \psi_j(x) \right] \hat{a}^\dagger_i \hat{a}_j \]

Here, the eigenvalues of the Hamiltonian $\hat{H}$ of the interacting system are shown by the method of the field operator that changed the statistics.

In this way, the interaction was expressed in a way different from the method of adding a perturbation term by the method of changing the operator.

From this, the interaction is expressed by the temperature-dependent parameter $\kappa(\mu, T)$. As a result, some classical motions appear in the quantum free particles due to the interaction, and the motions are interpreted as thermal motions.

4 Conclusion

In this report, we applied the previous research report of expressing a finite temperature field by second quantization by a distribution function that changed statistics, and tried to describe the field interaction in this format.

As a result, some classical motions appear in the quantum free particles due to the interaction, and the motions are interpreted as thermal motions.

Using this format, we aim to develop a method for thermodynamically calculating the energy of various interactions.

References

A New Representation of the Finite Temperature Distribution Function Using the Step Function

Derivation of Finite Temperature " hot Absolute Zero" in Pure State Form