# THE GOLDBACH CONJECTURE –

# A DEFINITIVE PROOF.

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## Abstract.

This paper presents a proof of the Goldbach Conjecture by comparing the distribution of prime numbers with the inverse distribution of odd composite numbers.

**Contents.**

## 1.0 Introduction.

Christian Goldbach, 1690 – 1764 was a German mathematician who also studied law. On the 17th June 1742 he wrote a letter to Leonard Euler in which he proposed what is known today as the Goldbach Conjecture, [1]. The modern version of this conjecture states that :-

"Every even number, $N$, greater than 2, can be expressed as the sum of two primes."

Although there are many proposed proofs of this conjecture, mostly on the internet, it is still recognised today as unsolved, [1], [2].

It is the purpose of this paper to present a definitive proof of the conjecture, by comparing the distribution of primes in the Natural numbers, to the reverse distribution of odd composites.

## 2.0 Proof of the Goldbach Conjecture.

### 2.1 Construction of the Difference Matrix.

It is extremely easy to numerically prove the conjecture when $N$ is small, but becomes impossible as $N \to \infty$. Consequently, it is necessary to devise a generalised method of proof from a solvable example. Therefore, to initiate this proof, a difference matrix is constructed of all the odd numbers, within which, a solvable example that can be generalised may be demonstrated. The initial part of this matrix is shown below in Fig. 2.1.

| Column Numbers | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **Row Numbers** | ***O*** | **3** | **5** | **7** | **9** | **11** | **13** | **15** | **17** | **19** | **21** | **23** | **25** | **27** | **29** | **31** | **33** | **35** | **37** | **39** | **41** | **43** |
| 1 | **3** | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 | 40 |
| 2 | **5** | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 |
| 3 | **7** | | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 |
| 4 | **9** | | | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 |
| 5 | **11** | | | | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 | 32 |
| 6 | **13** | | | | | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 | 30 |
| 7 | **15** | | | | | | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 | 28 |
| 8 | **17** | | | | | | | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 | 26 |
| 9 | **19** | | | | | | | | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 | 24 |
| 10 | **21** | | | | | | | | | | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 | 22 |

**Fig. 2.1 – Difference Matrix of All Odd Numbers.**

The matrix shows the difference of the odd numbers in each column to those in each row. Where a difference number is in a yellow cell, it is the difference between two primes, i.e. the difference between 23 in column 11 to 13 in row 6, is 10.

### 2.2 Example for $N$ = 44.

To illustrate the process to be generalised, consider the number $N$ = 44. To determine if this number meets the conjecture, first halve it.

$$\frac{N}{2} = 22 \tag{2.1}$$

Now subtract one and add one

$$\frac{N}{2}-1=21, \quad \frac{N}{2}+1=23 \tag{2.2}$$

The difference is obviously

$$\left(\frac{N}{2}+1\right)-\left(\frac{N}{2}-1\right)=2 \tag{2.3}$$

This is shown in Fig.2.1 at the conjunction of 23 in column 11 and 21 in row 10. The cell is not yellow as 21 is not prime.

Now subtract two from $\left(\frac{N}{2}-1\right)$ and add two to $\left(\frac{N}{2}+1\right)$ to produce 19 and 25, shown in Fig. 2.1 as the conjunction of 25 in column 12 and 19 in row 9. Continuing in this fashion produces the following table

| ***n*** | $\frac{N}{2}+n$ | $\frac{N}{2}-n$ | **Difference** | **Meets Conjecture** |
|---|---|---|---|---|
| 1 | 23 | 21 | 2 | No |
| 3 | 25 | 19 | 6 | No |
| 5 | 27 | 17 | 10 | No |
| 7 | 29 | 15 | 14 | No |
| 9 | 31 | 13 | 18 | Yes |
| 11 | 33 | 11 | 22 | No |
| 13 | 35 | 9 | 26 | No |
| 15 | 37 | 7 | 30 | Yes |
| 17 | 39 | 5 | 34 | No |
| 19 | 41 | 3 | 38 | Yes |

**Table 2.1 – *N* = 44 Conjecture Analysis.**

The path of this simple example through the matrix of Fig. 2.1 is shown as the blue diagonal starting at the conjunction of 23 and 21 and ending at the conjunction of 41 and 3, and as shown in Table 2.1 meets the conjecture. Also, from this, it is clear that for $N$ = 44 to not meet the conjecture, it would be necessary for the reverse distribution of odd composites in the first row of Fig. 2.1, from 41 to 23, to be identical to the distribution of primes in the first column, from 3 to 21, thus

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Primes | 3 | 5 | 7 | 11 | 13 | 17 | 19 | 23 |
| Difference | 2 | 2 | 4 | 2 | 4 | 2 | 4 | |

**Table 2.2 – Distribution of Primes *N* = 3 to *N* = 23.**

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Composites | 41 | 39 | 37 | 33 | 31 | 27 | 25 | 21 |
| Difference | 2 | 2 | 4 | 2 | 4 | 2 | 4 | |

**Table 2.3 – Reverse Distribution of Composites *N* = 41 to *N* = 21 for *N* = 44 to Fail the Goldbach Conjecture.**

Therefore, for the Goldbach Conjecture to be false for $N$ = 44, numbers 41, 37 and 31 would need to be composite. The other three numbers, 35, 29 and 23 would consequently be the only primes.

Note that if $N$/2 at (2.1) were odd, then the first step at (2.2) would be to add and subtract 2.

## 2.3 Generalisation.

The results of the example above for $N$ = 44 can be generalised to say that, for any even number $N$ > 2, for Goldbach's Conjecture to be false would require the distribution of odd composites from $N$ – 3 to $\left(\frac{N}{2}+1\right)$, to be identical to the prime distribution from $N$ = 3 to $\left(\frac{N}{2}-1\right)$, (or vice-versa).

Now, the distribution of all odd composites was shown in [3] to be

$$O = 4m^2 + 4mn + 2n - 1 \tag{2.4}$$

where $m$ and $n$ vary independently from 1 to ∞. The distribution of primes cannot follow the same law, albeit in the reverse direction, because primes and odd composites together, make up a subset of the Natural numbers and their laws of distribution must be mutually exclusive. Consequently, Goldbach's Conjecture cannot be false for any even number > 2. The conjecture is therefore true.

## 2.4 The Inclusion or Exclusion of Unity as a Prime Number

With exclusion of unity as a prime number, there are two even numbers that can only be expressed as the sum of two identical primes, i.e. 4 = 2 + 2 and 6 = 3 + 3. Thus the sum of identical primes must be included in the conjecture. This means that there will be many other even numbers that can be so expressed, i.e. where $N$/2 is prime.

Alternatively, if unity is included as a prime number, as in Goldbach's original conjecture, then 4 becomes 1 + 3 and 6 becomes 1 + 5, and identical primes can be excluded from the conjecture. Note that inclusion of unity as a prime means that if $N$ – 1 is prime, then $N$ – 1 + unity is a valid solution.

## 3.0 Conclusions.

Goldbach's Conjecture has remained unproven for 278 years. The proof here cannot be claimed to be a fully analytical one because it depends upon the distribution of primes in the Natural numbers, and a law for that distribution is unknown. A fully analytical proof of the conjecture would only be possible if the law for the distribution of primes was discovered. However, it is clear that the law for odd composites, as stated above, fully precludes the distribution of primes being identical in the reverse direction, and this is a sufficient condition to prove the conjecture.

## APPENDICES.

### A.1 Distribution of Odd Composites for *N* = 41 to *N* = 23 from Eq.(2.4).

This is presented in the form of a table. The appropriate values of *m* and *n* are determined from [3], Eq.(2.8).

| ***m*** | ***n*** | **Odd Composites** |
|---|---|---|
| 1 | 6 | 39 |
| 2 | 2 | 35 |
| 1 | 5 | 33 |
| 1 | 4 | 27 |
| 2 | 1 | 25 |

**Table A.1 – Values of Odd Composites for**
***N* = 41 to *N* = 23 From Eq.(2.4).**

## References.


[1] Wikepedia, *The Goldbach Conjecture,* en.wikepedia.com.

[2] Quora, *The Goldbach Conjecture,* www.Quora.com.

[3] P.G.Bass, *An Algorithmic Version of the Sieve of Eratosthenes, Plus Other Solutions Associated With Prime Numbers, (Versions 1 and 2),* www.relativitydomains.com.