A Static Universe Explanation for the Redshift-Distance Relation as an Effect of Gravitational Time Dilation

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Abstract: The redshift-distance relation is explained in the concordance model by the metric expansion of space, which is described by the Friedmann solution to Einstein's field equations. Cosmological redshift is proportional to the cosmic scale factor, the relation between redshift and distance is non-linear, and the Hubble constant determines the proper velocity of space expansion. Edward A. Milne developed a competing Newtonian expanding model in static Euclidean space that is consistent with the Friedmann equation. It describes the universe as a conservative gravitational system which includes special relativity. In this model cosmic redshifts are explained as relativistic Doppler redshifts, the Hubble constant determines the peculiar velocity of matter expansion, and the relation between redshift and distance is non-linear. This paper describes a static model of the universe as a conservative gravitational system. This static model explains cosmic redshifts by time dilation in a universal gravitational field, which results in a linear redshift-distance relation that matches Hubble's 1929 discovery. The uniformity of the cosmic microwave background temperature is also explained by relativity effects in a universal gravitational field. In this model the square of the Hubble constant is the gravitational constant of cosmic gravity. The results of several different tests designed to determine whether or not space expansion is real are all consistent with the static gravitational model.

Introduction

The concordance model explains cosmological redshift by the metric expansion of space. Galaxies are carried along by the universal outward flow of expanding space, and their receding velocities are termed proper velocities. These proper velocities do not cause Doppler redshifts ($z = v/c$). Galactic velocities due to any other cause are termed peculiar velocities. The receding peculiar velocity of a galaxy causes a Doppler redshift. This raises the question: Why does peculiar receding velocity cause a Doppler redshift, but proper velocity does not?

The Doppler effect is a simple consequence of motion. The receding velocity of a body causes the energy of its emitted photons to be less when observed, redshifted. Space expansion results in a proper velocity of recession between us and a distant galaxy, but this velocity does not decrease the observed energy of its photons. This is analogous to the sound of a train approaching, passing, and receding from us remaining constant, instead of changing from a higher to a lower pitch, due to the Doppler effect. This logically leads to the conclusion that proper velocity is apparent and not real, which raises doubts about the reality of space expansion.

The expanding model gained credibility by explaining the redshift-distance relation discovered by Edwin Hubble. This empirical relation is interpreted as the strongest evidence supporting the hypothesis of space expansion. But there were doubts about the reality of space expansion from the beginning. Besides contradicting the intuitive idea of Euclidean space, this revolutionary hypothesis only applies on large cosmic scales. Hubble initially interpreted galactic redshifts as velocity redshifts in his 1929 paper and described it as the velocity-distance relation. [1] But within a few years he concluded the redshift-distance relation was not caused by space expansion, based upon his analysis of the relation between apparent magnitude and redshift. He supposed it must be explained by "a new principle of nature" in a static universe. [2]

Hubble eventually settled on an idea similar to that proposed by Fritz Zwicky just six months after his 1929 paper. Zwicky's paper described the characteristics of such a "new principle." This was subsequently referred to as the "tired light" hypothesis for the redshift-distance relation in a static universe. Photons interact with gravity in general relativity and they could lose energy to matter during their transit due to gravitational interactions. He characterized this idea as "a new effect of masses upon light … which is sort of a gravitational analogue of the Compton effect." He thought this "new effect" must be consistent with general relativity, but he did not describe the physical theory behind it. [3] Since doubts about space expansion have not been resolved to the satisfaction of all, numerous alternative theories for his "tired light" hypothesis have been proposed.

In principle, doubts about space expansion can be empirically resolved. A half dozen tests have been designed to discriminate between an expanding and a static universe. These include the Tolman surface brightness test (1930) and the Hoyle angular size test (1959). A 1987 review of such tests by Allan Sandage found their results had not provided "proof or not that the redshift is a true expansion." [4] In 2014 Martin Lopez-Corredoira reviewed 29 different implementations of these tests undertaken since the 1980s. He found their overall results were equivocal and reached the same general conclusion as Sandage: Substantial proof of the reality of space expansion is still lacking. [5]

This paper describes a static universe in which the redshift-distance relation is explained with conventional physics. The paper is organized in six sections. Section §1 briefly reviews the explanation for cosmological redshift in the expanding model. Section §2 considers E. A. Milne's Newtonian expanding model, which explains these redshifts with the Doppler mechanism. Section §3 presents a static model that explains the redshift-distance relation as a relativity effect of gravitational time dilation. Section §4 explains the uniform temperature of the CMB radiation by relativity effects in a gravitational field. Section §5 compares the static and expanding models and proposes that the square of the Hubble constant is the gravitational constant for cosmic gravity. Section §6 discusses the relative merits of this static gravitational model and the concordance ΛCDM model.

1. Cosmological Redshift Explained by the Metric Expansion of Space

Einstein presented a relativistic cosmological model in 1917 that describes a static universe of curved spacetime, in which matter is uniformly distributed. In the same year Willem de Sitter found a solution to the field equations of general relativity for an expanding universe of flat spacetime that is devoid of matter. While an empty universe is somewhat unrealistic, this expanding model attracted attention, because of the "de Sitter effect." The de Sitter model relates spectral shifts to velocity and distance, which made it applicable to astronomical objects. There is no similar relation in Einstein's static model. This effect offered a possible solution to the "redshift problem," which arose from the work of Vesto Slipher. Between 1912 and 1922 Slipher measured the spectra of forty-one spiral nebulae. Five were blueshifted by -300 km/s or less and 36 were redshifted by $+1800$ km/s or less. [6] The overwhelming preponderance of redshifts clearly implied the existence of some systematic cause.

At the very end of 1924 Edwin Hubble announced his proof that the Andromeda nebula was not a nearby star field but a distant galaxy. In 1929 Hubble reported his discovery that the redshift z of a galaxy divided by its distance r equals a constant k: There is a linear relation between redshift and distance $z = kr$. Redshift can be simply explained as a Doppler effect equal to a receding velocity over the speed of light $z = v/c$. This changes the relation $z = kr$ to $v = (ck)r$, where (ck) is the Hubble constant H_0. Hubble interpreted the redshift-distance relation as a velocity-distance relation $v = H_0 r$, since this was consistent with the de Sitter effect. [1] The exponential expansion in the de Sitter metric yields the pseudo-Doppler expression $v = cz$ for all z, and the velocity-distance relation is equivalent to the redshift-distance relation in this metric. [7] Hubble noted that in de Sitter's model redshift arises from both time dilation and the increase in the velocity of expansion with distance.

In the early 1930s the de Sitter model was superseded by the more realistic Friedmann model, in which matter is uniformly distributed in the universe. In 1924 Alexander Friedmann found an expanding solution to the field equations of general relativity, which describes a universe that increases in size due to the metric expansion of space. Friedmann's expanding model gave a more convincing explanation of Hubble's redshift-distance relation.

In the Friedmann model space expansion causes galaxies to recede with velocities given by the velocity-distance law $v = H_0 D$, where D is the current co-moving distance of a galaxy; that is, its distance at the time we observe its light. Cosmological redshift occurs over the whole period of time between emission and observation. A Doppler explanation is not plausible in this model, since Doppler redshift occurs at the moment of emission. The expansion velocity is proportional to the co-moving distance, but this velocity is not proportional to cosmological redshift. The velocity-distance law is more fundamental than the redshift-distance relation, because it is derived from the Friedmann solution and is strictly linear at all distances. [7]

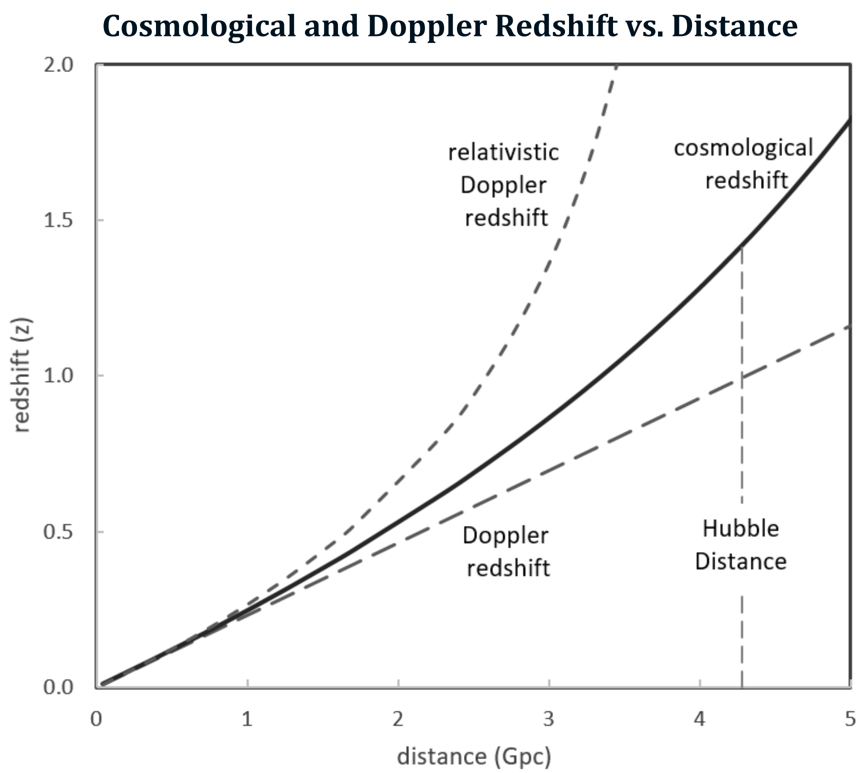

Figure 1: For $H_0 = 70$ km/s/Mpc, expansion velocity equals c at the Hubble distance D_H of 4257 Mpc ($D_H = c/H_0$) or 13.8 billion light-years.

Cosmological redshift depends on the cosmic scale factor. A receding galaxy emits light at a past time t and a proper distance r. This light is observed now at time t_0, when the galaxy is at a co-moving distance D. During the time between t and t_0, the scale size of the universe expands by the ratio of D/r, due to space expansion. Expansion increases the length of a "standard measuring rod," to use Einstein's analogy, by a factor of D/r. This results in the "stretching" of light waves by this factor. The longer observed wavelength λ_{obs} divided by the shorter emitted wavelength λ_{emit} equals the cosmological redshift plus one.

$$1 + z \equiv \frac{\lambda_{obs}}{\lambda_{emit}} = \frac{D}{r} \tag{1}$$

One plus the cosmological redshift is the redshift factor, which equals the ratio of the co-moving distance D over the proper distance r. There is a non-linear relationship between these two distances, since the rate of expansion varies over time.

The velocity-distance law does not result in a linear redshift-distance relation. However, if an expansion velocity is much less than the speed of light ($v \ll c$), dividing it by the speed of light gives an approximation of the cosmological redshift ($z \cong v/c$). Dividing the expansion velocity at 5 billion light-years (1.6 Gpc) by the speed of light gives a Doppler redshift of $z = 0.36$. This is ten percent less than the cosmological redshift of $z = 0.40$, as calculated in the expanding model. The divergence of cosmological and Doppler redshifts is shown graphically in Figure 1.

The current scale is commonly designated $a(t_0)$, where t_0 is the current age of the universe since the big bang. The scale of the universe in the past is designated $a(t)$, where t is the previous age of the universe. The ratio $a(t_0)/a(t)$ is the cosmic scale factor, which equals the redshift factor.

$$1 + z \equiv \frac{\lambda_{obs}}{\lambda_{emit}} \equiv \frac{a(t_0)}{a(t)} \tag{2}$$

Cosmological redshift is related to the proper velocity of expansion by the Hubble parameter. The Hubble parameter H is defined as the rate of change of the scale factor – its time derivative $\dot{a}$ (a-dot) – divided by the current value of the scale factor.

$$H(t) \equiv \frac{\frac{d\,a(t)}{dt}}{a(t)} \equiv \frac{\dot{a}}{a} \tag{3}$$

The redshift factor equals the scale factor, and the Hubble parameter equals the time derivative of the scale factor ($H = \dot{a}/a$). The expansion velocity equals the Hubble parameter multiplied by the co-moving distance: $v = HD$. The Hubble parameter is an element in the Friedmann equation which describes the expanding model.

$$H^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} \tag{4}$$

Conceptually, the square of the Hubble parameter represents the tensor energy of space expansion. In the first term on the right hand side, G is Newton's gravitational constant and ρ (*rho*) is the uniform mass density of the universe, which is a postulate proposed by Einstein. Conceptually, this first term represents the tensor energy of space contraction. In the second term on the right, k is the curvature constant, which can be positive, zero, or negative. This term represents the net total energy; the difference between the energies of expansion and contraction.

The curvature constant k determines the type of space which results from these expansive and contractive tensor energies. If $k = +1$, space is "closed" and the universe has a net negative total energy. In a closed universe the expansion velocity decreases to zero and then the universe starts contracting. If $k = -1$, space is "open," the universe has a net positive total energy, and the universe never stops expanding. If $k = 0$, space is "flat" and the net total energy equals zero. In a spatially flat universe, the velocity of expansion decreases asymptotically toward zero, until the universe reaches a terminal state of stasis. Flat spacetime is sometimes informally referred to as Euclidean space, since the straight line is a common geometric element in both. Formally, these two spaces are incommensurable, because the standard unit of length does not change in Euclidean space, while it does change in flat expanding space.

The Friedmann equation describes the current expanding model. It also describes a universe in which the matter of the universe is contained in a sphere that expands into static Euclidean space. This type of model was described by Edward A. Milne in the 1930s. Milne was able to demonstrate that the Friedmann equation can be derived for this Newtonian expanding model without any reliance on the theory of general relativity.

2. Cosmic Redshifts as Doppler Redshifts in Milne's Newtonian Expanding Model

The publication of the theory of general relativity in 1915 solved a decades old problem. Urbain Le Verrier measured the magnitude of the precession of the perihelion of Mercury's orbit in 1859 and found it differed by 43 arcseconds per century from the prediction of Newtonian mechanics. General relativity explained this discrepancy. Einstein's theory predicted that light passing near the sun would be deflected by twice the angle predicted by Newtonian theory, which was confirmed by Arthur Eddington in 1919 during a total eclipse of the sun. The theory also predicted that the gravity of the sun would cause the light emitted at its surface to be redshifted by about two parts in one million. This gravitational redshift is not predicted by Newtonian theory, but it was qualitatively confirmed in 1925 by Walter S. Adams from the spectrum of Sirius B.

These three classical tests of general relativity convinced most physicists of its validity. These successes and Hubble's discovery of the velocity-distance relation made Friedmann's expanding model the favored one in the early 1930s. The astrophysicist Edward A. Milne believed these tests validated general relativity on a local scale, but he was not persuaded they justified the radical idea of universal space expansion. He argued that the hypothesis of space expansion was unnecessary, since galactic redshifts could be explained as relativistic Doppler redshifts caused by their receding velocities in static Euclidean space.

Milne developed his "Newtonian expanding universe" model as a direct challenge to the expanding model. In a 1934 paper he demonstrated that the Friedmann equation can be derived from Newtonian theory and special relativity alone. A central premise of his paper is: "Moving particles in a static space will give the same observable phenomena as stationary particles in 'expanding' space." [8] In a 1965 paper C. Callan, Robert H. Dicke, and P. J. E. Peebles conclude the Milne model is sufficient for "a completely correct discussion of the dynamics of expansion in a region where both general relativity and Newtonian mechanics are equally valid." [9] The region in which the Milne model and the expanding model give similar predictions extends out to several billion light-years. [10] A 2004 review of this Newtonian derivation by J. Dunning-Davies shows that it is can be extended to include a universe with a non-zero pressure, where Milne only considered a zero pressure universe. His paper concludes: "The final equations derived by utilizing purely Newtonian methods are identical in form with those resulting from the more modern relativistic techniques." [11]

In Milne's expanding model cosmological redshift is explained as a relativistic Doppler shift caused by the receding velocity of galaxies through static space, instead of by the metric expansion of space. The observational equivalence of these two mechanisms is pointed out: "It follows that the local properties of the universes in expanding spaces of positive, zero or negative curvatures are observationally the same as in Newtonian universes with velocities respectively less than, equal to, or greater than the parabolic velocity of escape." [12] In Milne's model the escape velocity from a gravitational potential is the critical velocity which determines whether the trajectories of galaxies through static space are elliptical ($k = +1$), parabolic ($k = 0$) or hyperbolic ($k = -1$).

Milne begins his derivation with Einstein's postulate that the universe has a uniform mass density. Within a radius r of some observer, the total mass M equals the spherical volume defined by this radius times the mass density. By the first part of Newton's shell theorem (Book I, Proposition LXX), uniformly distributed matter located outside this spherical surface exerts no net gravitational force upon a particle within this surface; the presence of this matter is equivalent to empty space outside this surface. The second part of this theorem (Book I, Proposition LXXI) proves that the net result of the mutual attractions between all particles within a spherical surface is that a particle at or beyond its surface in empty space is acted upon by a centripetal force that is proportional to the inverse of the square of the distance to the center of the sphere.

The total energy of a gravitational system is conserved. The system energy equals the kinetic energy of a test particle of mass m plus its potential energy at a distance r from the center of a mass M. (Proposition LXXI)

$$E = KE + PE = \frac{1}{2}mv^2 - \frac{GMm}{r} \tag{5}$$

In Milne's model the radius of a sphere containing the total mass M of the universe increases over time, due to the outward velocity caused by the expansion of matter in static Euclidean space. A test particle on the surface of this expanding sphere of matter has a distance r_0 now at time t_0. It had a distance $r(t)$ in the past at time t. The scale factor at time t is then:

$$\frac{a(t)}{a(t_0)} = \frac{r(t)}{r_0} \qquad (6)$$

The current scale factor is defined as $a(t_0) = 1$. Rearranging equation (6) and differentiating both sides by time gives the velocity as the time derivative of the scale factor.

$$r(t) = a(t) r_0 \quad \rightarrow \quad \frac{d\, r(t)}{dt} = \frac{d\, a(t)}{dt} r_0 \qquad (7)$$

$$v = \dot{r} = \dot{a} r_0 \qquad (8)$$

Substituting $\dot{r}$ (r-dot) for v in the total system energy equation (5):

$$E = \frac{1}{2} m \dot{r}^2 - \frac{GMm}{r} \qquad (9)$$

The uniform mass density ρ equals the total mass divided by the volume of the sphere, so $(4\pi\rho r^3)/3$ can be substituted for M in equation (9). Making this substitution and dividing by the test particle mass m gives the total energy density ε (*epsilon*) of the system ($\varepsilon = E/m$ has dimensions of $\mathrm{L}^2/\mathrm{T}^2$).

$$\varepsilon = \frac{1}{2} \dot{r}^2 - \frac{4\pi G \rho r^2}{3} \qquad (10)$$

The total energy density ε equals the kinetic energy density (KE/m) minus the gravitational potential energy density (PE/m). Using the relations $\dot{r} = \dot{a} r_0$ (eq. 8) and $r = a r_0$ (eq.6) to make substitutions for $\dot{r}$ and r:

$$\varepsilon = \frac{1}{2} r_0^2 \dot{a}^2 - \frac{4\pi G \rho r_0^2 a^2}{3} \qquad (11)$$

Dividing both sides by r_0^2 and a^2 and rearranging:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G \rho}{3} + \frac{2\varepsilon}{r_0^2 a^2} \qquad (12)$$

The left hand term $\left(\dot{a}/a\right)^2$ is the kinetic energy density divided by $r_0^2 a^2$. The first term on the right hand side is twice the gravitational potential energy density divided by $r_0^2 a^2$. The second term is twice the total energy density divided by $r_0^2 a^2$. If the kinetic energy density of a test particle happens to equal its potential energy, the net total energy of the gravitational system equals zero by equation (5). Rearranging this equality shows that the square of the velocity of the test particle equals twice the gravitational potential of the test particle.

$$\frac{1}{2} m v^2 = \frac{GMm}{r} \quad \rightarrow \quad v^2 = \frac{2GM}{r} \qquad (13)$$

The square root of v^2 is the escape velocity of a test particle from a gravitational system. In the Milne model a body with the escape velocity has a parabolic trajectory, and its velocity decreases asymptotically toward zero and a final state of stasis. This is analogous to a spatially flat universe in the expanding model ($k = 0$). A particle with less than the escape velocity has an elliptical trajectory ($k = +1$) and a hyperbolic trajectory ($k = -1$) for more than the escape velocity. In the Friedmann equation, $-kc^2 = 2\varepsilon/r_0^2$. Making this substitution in equation (12):

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} - \frac{kc^2}{a^2} \qquad (14)$$

This equation is identical in form to the Friedmann equation derived from general relativity, but the terms have altered meanings. The H^2 term is the square of the Hubble parameter and represents the kinetic energy of the gravitational system, instead of the tensor energy of space expansion. The $\dot{a}/a$ term represents the scale factor for a sphere of matter expanding in static Euclidean space, instead of for expanding space. The $8\pi G\rho/3$ term represents the gravitational potential energy of the gravitational system, instead of the tensor energy of space contraction. The kc^2/a^2 term represents the net total energy of the gravitational system. The Hubble parameter H determines the peculiar velocities of galaxies in static space, instead of the proper velocity of expanding space. Cosmic redshifts are relativistic Doppler redshifts resulting from peculiar receding velocities, instead of cosmological redshifts determined by the scale factor.

The Friedmann equation can be simplified, based upon high-precision observations of the cosmic microwave background (CMB) radiation. There is now a high level of confidence that the space of the observable universe is flat. The latest evidence for this conclusion is presented in the 2018 results from the ESA Planck mission, which found the universe is spatially flat to within a standard deviation of $\pm 0.2\%$. [13] Given this determination of flat space, the k-term in the Friedmann equation can be set to zero and dropped. This yields the critical density equation.

$$H^2 = H_0^2 = \frac{8\pi \mathrm{G}\rho}{3} \qquad (15)$$

In flat space the square of the Hubble parameter determined by the uniform mass density equals the square of the Hubble constant, which is measured empirically. This simplified cosmological equation is valid for both the expanding model of general relativity and Milne's Newtonian expanding model, which incorporates special relativity. The Newtonian derivation of the Friedmann equation establishes the equivalence of a relativistic model of flat expanding spacetime and a Newtonian model of expanding matter in static Euclidean space.

3. Gravitational Time Dilation Explanation for the Redshift-Distance Relation

The critical density equation describes both the concordance model and Milne's Newtonian expanding model. The Milne model is a conservative gravitational system in static Euclidean space. The kinetic energy of matter expansion equals the gravitational potential energy of matter contraction when the total system energy equals zero ($k = 0$).

$$KE = -PE + E \quad \rightarrow \quad \frac{1}{2}mv^2 = \frac{GMm}{r} + E \tag{16}$$

Dividing both sides by a test particle mass m and multiplying both sides by 2:

$$v^2 = \frac{2GM}{r} + \frac{2E}{m} \tag{17}$$

Since the total energy equals zero, the last term can be dropped from equation (17). This leaves the equation for the square of the escape velocity for a particle at a distance r from the center of a gravitational field, which is its interpretation in the Milne model. But matter is not universally receding from an observer in the static gravitational model, so v^2 cannot be interpreted as the square of the escape velocity. A valid alternative interpretation is that it is a kinetic energy density (KE/m) equal to the energy density of a gravitational potential (PE/m). Equation (17) describes a static universe model in terms of kinetic and potential energy densities in a conservative gravitational system.

This energy density interpretation is consistent with energy-mass equivalence $E = mc^2$; the energy divided by the relativistic mass equals the speed of light squared $c^2 = E/m$. The speed of light is the limiting velocity in an inertial frame by the second postulate of special relativity. Its square c^2 can be interpreted as the limiting energy density (dimensions of L^2/T^2) in an inertial frame. All photons have a velocity c, but their energies vary with frequency f according to Einstein's photoelectric equation $E = hf$, where h is Planck's constant. The total energy E of photons is purely kinetic, since they are massless. Dividing both sides of the photoelectric equation by the relativistic m gives the kinetic energy density of a photon, $c^2 = hf/m$, which is a variation of the Compton frequency equation, $f = mc^2/h$. The kinetic energy of photons is proportional to their frequencies, but all photons have the same kinetic energy density.

The square of the speed of light is a universal constant of energy density. The kinetic and potential energy densities in equation (17) can, therefore, be divided by c^2 to relativize the equation.

$$\frac{v^2}{c^2} = \frac{2GM}{rc^2} \tag{18}$$

The left hand term in this equation is the ratio of the kinetic energy density over the universal constant of energy density. This ratio is identical to the variable term in the Lorentz factor γ (*gamma*). In special relativity the Lorentz factor translates units of time from a stationary inertial frame to a moving one which has a relative velocity v.

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{19}$$

A duration t_0 measured by a clock in a stationary frame is translated to a duration t measured by an identical clock in a moving frame.

$$t = \gamma t_0 \quad \rightarrow \quad \frac{t_0}{t} = \sqrt{1 - \frac{v^2}{c^2}} \tag{20}$$

The time t in a moving frame dilates – passes more slowly – relative to the time t_0 in a stationary frame. Atomic clocks measure time by a defined number of hyperfine transitions of the cesium 133 atom in one second (9,192,631,770 Hz at 0 °K). One second measured by a moving atomic clock is longer than one second as measured by a stationary atomic clock; the fixed number of cycles is the same, but the duration of each cycle in the moving clock is longer than it is in the stationary clock. As velocity increases, time in a moving frame increasingly dilates, which causes all physical processes, like the hyperfine transition cycles of cesium atoms, to slow down relative to a stationary frame.

The first qualitative confirmation of time dilation caused by velocity was obtained by the 1940 Rossi-Hall experiment, which measured the lifetimes of muons created by cosmic rays in the upper atmosphere. A more precise confirmation was found by the 1963 Frisch-Smith experiment. This experiment demonstrated that the lifetimes of muons are about ten times longer than they are on the earth's surface, because of their near-light velocities.

The Lorentz factor applies to inertial frames with relative velocities. Since it varies with the square of the velocity over the square of the speed of light, it is equally valid to say that time dilation increases as the ratio of the moving frame's kinetic energy density v^2 over the universal constant of energy density c^2 increases.

The right hand term in equation (18) is twice the Newtonian gravitational potential divided by the universal constant of energy density. This ratio is identical to the variable term in the general relativity equation for gravitational time dilation shown in equation (21).

$$\frac{t_0}{t} = \sqrt{1 - \frac{2GM}{rc^2}} \tag{21}$$

The rate of time t measured by a clock at a distance r from the center of a spherical mass is slower than the time t_0 measured by an identical clock at an infinite distance r_∞, where there is no gravitational potential. The time dilation ratio is the gravity-free un-dilated time at infinity divided by the dilated time in a gravitational field at a distance r from a center of mass: $t_0(r_\infty)/t(r)$. Equation (21) is derived from the static Schwarzchild solution to Einstein's field equations. It quantifies the gravitational time dilation in the empty space outside the surface of a sphere of matter. Since $v^2 = 2GM/r$, equation (21) is equivalent to equation (20). Gravitational time dilation increases as the ratio of gravitational potential over the universal constant of energy density increases.

This equivalence was used by Einstein in his initial derivation of gravitational time dilation from the theory of special relativity. At the end of a 1907 paper summarizing special relativity, he considers time dilation in a uniformly accelerated frame. [14] He interprets v^2 as a gravitational potential Φ, instead of as a squared velocity, that is equal to a uniform acceleration times a distance: $\Phi = ad$. To a first order of approximation, special relativity shows there is a time dilation that equals $(1 + \Phi/c^2)$. Using this equation Einstein predicted that time is dilated at the surface of the sun, which causes light emitted from its surface to be redshifted by about two parts per million. Atoms generate characteristic spectral lines and can be considered "clocks." Time dilation causes a redshift in the frequency/wavelength of spectral lines generated by these "clocks."

The Schwarzchild equation (21) makes virtually the same prediction. Einstein's 1907 formula of $(1 + \Phi/c^2)$ consists of the first two terms in a binomial expansion of $1/\sqrt{1 - 2\Phi/c^2}$. This is the gravitational form of the Lorentz factor, since $2\Phi = 2GM/r = v^2$.

$$\frac{t}{t_0} = \frac{1}{\sqrt{1 - \frac{2\Phi}{c^2}}} \quad \rightarrow \quad \frac{t}{t_0} = 1 + \frac{\Phi}{c^2} + \frac{3\Phi^2}{2c^4} + \frac{5\Phi^3}{2c^6} + \cdots \tag{22}$$

The kinetic and gravitational forms of the Lorentz factor are equivalent. Setting the right hand sides of the time dilation equations for special relativity (eq. 20) and general relativity (eq. 21) equal to each other reproduces equation (18)

$$\sqrt{1 - \frac{v^2}{c^2}} = \sqrt{1 - \frac{2GM}{rc^2}} \tag{20, 21}$$

$$\frac{v^2}{c^2} = \frac{2GM}{rc^2} \tag{18}$$

Gravitational potential changes with distance, which generates a gravitational field gradient. The strength g of the gravitational field decreases from a negative maximum at the surface (r) to zero at infinity (r_∞). The gravitational field gradient ∇ (del operator or *nabla*) is negative and $g = -\nabla\Phi$; the direction of greatest increase in field strength g is toward the center of the field, which is opposite to the positive direction of increase in r.

The strength of a gravitational field is measured by its acceleration. In equation (17) v^2 is understood as the kinetic energy density equal to a gravitational potential $\Phi = 2GM/r$. Since $v^2 = \Phi = ar$, the equation for acceleration in this gravitational field can be found from equation (17).

$$a = \frac{2GM}{r^2} \tag{23}$$

Proceeding outward from the surface of a sphere to infinity, acceleration decreases inversely with the distance squared: $a \propto 1/r^2$. This inverse-square acceleration applies at the surface of a sphere and in the empty space outside its surface.

As applied to the static model, equation (17) describes the universe as a gravitational system with a total mass that is uniformly distributed throughout finite space. Einstein's postulate excludes empty space in the universe. The acceleration cannot be calculated from equation (23), since the total mass of the universe is unknown. However, uniformly distributed mass within a spherical volume has a uniform mass density ρ. And Newton's shell theorem (Proposition LXX) proves that the gravitational potential within a spherical surface is only determined by the mass encompassed by the surface; uniformly distributed mass outside this surface, like empty space, exerts no net gravitational forces on the mass particles within it. The mass M in equation (23) can be replaced by $(4\pi r^3 \rho)/3$ to find the acceleration at a distance r.

$$a = \frac{8\pi G\rho r}{3} \tag{24}$$

Equation (24) shows that the centripetal acceleration in a universe with a uniform mass density increases in direct proportion with the distance from the center of a universal gravitational field: $a \propto r$.

This linear acceleration-distance relation within a sphere of matter is completely different from the inverse-square acceleration-distance relation in the empty space outside the sphere. Instead of gravitational field strength decreasing inversely with the square of the distance from the center ($a \propto 1/r^2$), it increases in direct proportion with the distance from the center ($a \propto r$). The negative gravitational field gradient ($g = -\nabla\Phi$) surrounding the surface of a sphere becomes a positive gravitational field gradient ($g = +\nabla\Phi$) within its surface. The gradient of this interior field is positive, because the direction of its greatest increase in strength is in the same positive direction as the displacement r.

These interior and exterior gravitational field gradients are shown in Figure 2 relative to the absolute value of the maximum field intensity at the surface of a sphere with a uniform mass density in empty space. The field intensity increases in a linear way with distance from zero at the center to a maximum at the surface. It then decreases in an exponential way with distance from this maximum intensity to zero at infinity.

Gravitational Field Strength Normalized to a Spherical Surface

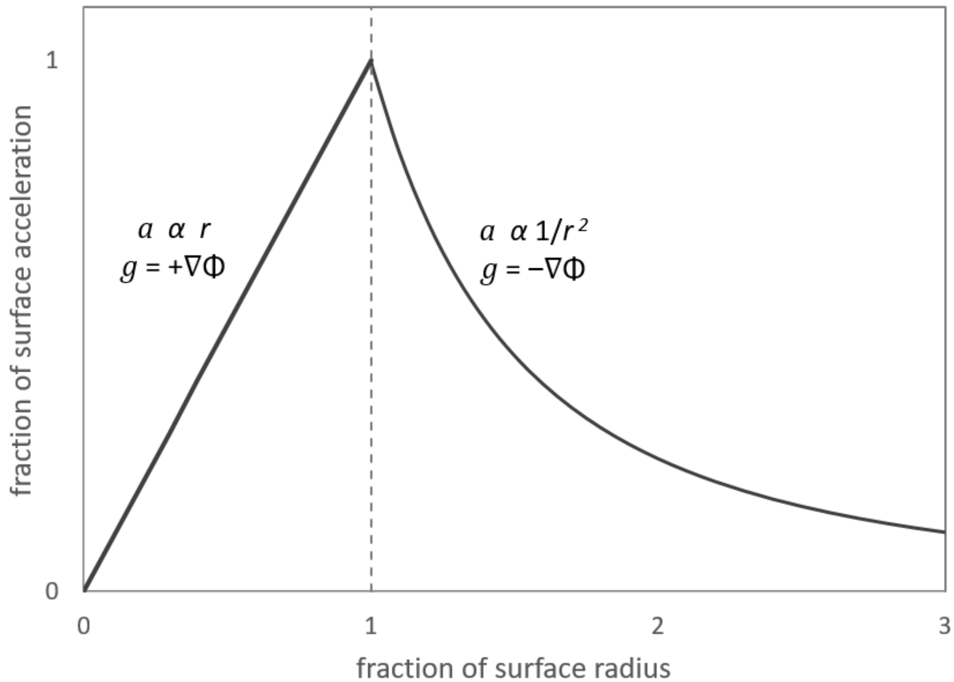

Figure 2: Centripetal acceleration inside/outside a sphere with a uniform mass density

The linear nature of gravitational force within a material sphere is analogous to an elastic force. This type of force is described by Hooke's law $F = kr$, where F equals a mass on the end of an elastic tether, like a spring, multiplied by an acceleration, and k is a force constant with units of mass over seconds squared. Dividing both sides by mass gives the acceleration $a = (k/m)r$. Both (k/m) and $(8\pi G\rho)/3$ have dimensions of T^{-2}, which makes the acceleration described by equation (24) analogous to that which causes an elastic force.

The ratio of time dilation increases as the gravitational field strength increases. In the negative gravitational field surrounding a body, the strength increases moving inward from an infinite distance, where the gravitational potential is zero and time is un-dilated. Light emitted at a height h above the earth's surface, which has a radius R, is blueshifted to a higher frequency when observed at the surface. Relative to no time dilation at infinity, there is less time dilation at $(R + h)$ than there is at the surface. Light emitted in the weaker gravitational potential at $(R + h)$ is blueshifted when observed in the stronger potential at the surface. Conversely, light emitted at the surface is redshifted when observed at $(R + h)$, since time dilation is greater at the surface than it is at $(R + h)$. Light emitted in a stronger gravitational potential is redshifted when observed in a weaker potential.

The spectral redshift and blueshift predicted by gravitational time dilation was first confirmed in 1959 by the Pound-Rebka experiment. The Global Positioning System is an ongoing test of time dilation. GPS satellites are in a weaker gravitational field and their atomic clocks run faster than they do on the earth's surface by 46 μs per day. If gravitational time dilation was not accounted for, calculated positions would be off by several kilometers at the end of a 24 hour period.

The situation is different in the *positive* gravitational field gradient inside a sphere with a uniform mass density. The field strength increases as the distance from the center of the sphere increases. The gravitational potential is greater at a distance *r* than it is at the center, where there is no potential. Since time dilation increases as gravitational field strength increases, light emitted at *r* is redshifted when it is observed at the center of a positive gravitational field.

Equation (21) $t_0/t = \sqrt{1 - 2GM/rc^2}$ describes time dilation in a negative gravitational field gradient. Moving inward from infinity (t_0), the strength of the negative gravitational field increases. Equation (21) can be rewritten in terms of mass density. If *r* is greater than the radius *R* of a sphere of matter with a uniform mass density ρ, then $(4\pi R^3\rho)/3$ can be substituted for *M*.

$$\frac{t_0}{t} = \sqrt{1 - \frac{8\pi G\rho R^3}{3rc^2}} \quad \text{where } r > R \tag{25}$$

As *r* decreases toward *R*, the ratio of un-dilated time at infinity (t_0) over dilated time (t) at *r* gets smaller. When *r* equals *R* and the result under the square root sign is positive, this equation reduces to:

$$\frac{t_0}{t} = \sqrt{1 - \frac{8\pi G\rho r^2}{3c^2}} \quad \text{where } r = R \tag{26}$$

The maximum negative gravitational potential for the external field occurs at the surface of the sphere. This coincides with the maximum positive potential for the internal gravitational field which also occurs at the surface. The time dilation zero point for the negative field gradient is at infinity, where time is un-dilated (t_0). The time dilation zero point for the positive field gradient is at the center of the field, where time is un-dilated (t_0). In a positive field gradient time dilation increases with the distance *r*, so the time dilation ratio is (t/t_0), which is the inverse of the ratio in a negative field gradient (t_0/t). Making this replacement in equation (26), changing the field gradient sign from negative to positive, and squaring both sides:

$$\frac{t}{t_0} = \sqrt{1 + \frac{8\pi G\rho r^2}{3c^2}} \quad \rightarrow \quad \frac{t^2}{t_0^2} - 1 = \frac{8\pi G\rho r^2}{3c^2} \tag{27a, b}$$

The right hand term in equation (27b) is equivalent to $2GM/rc^2$ in equation (18), so v^2/c^2 can be substituted for $(t^2/t_0^2 - 1)$ on the left hand side of equation (27b).

$$z^2 = \frac{v^2}{c^2} = \frac{8\pi G\rho r^2}{3c^2} \tag{28}$$

Redshift is the ratio of velocity over the speed of light; redshift squared equals the kinetic energy density over the universal constant of energy density. Taking the square root of both sides:

$$z = \sqrt{\frac{8\pi G\rho r^2}{3c^2}} \quad \rightarrow \quad \frac{z}{r} = \sqrt{\frac{8\pi G\rho}{3c^2}} \tag{29a, b}$$

Since all of the elements on the right hand side of equation (29b) are constants, gravitational time dilation in a positive gravitational field gradient results in a redshift that is directly proportional to the distance ($z \propto r$). This matches Hubble's redshift-distance relation, which is a strictly linear relation in the static model. In the expanding model this relation is only approximately linear within a few billion light-years. The redshift z caused by the time dilation at r equals $z = (t/t_0 - 1)$. Making this substitution for z in equation (29a) gives the time dilation equation in a positive gravitational field gradient.

$$1 + z = \frac{t}{t_0} = 1 + \sqrt{\frac{8\pi G\rho r^2}{3c^2}} \tag{30}$$

The redshift factor can be added, since $(1 + z) = t/t_0$. Moving outward from the center of a positive gravitational field, the ratio of time dilation increases relative to un-dilated time at the center. Equation (30) predicts that time dilation is proportional to $(1 + z)$ in the static gravitational model.

The expanding model predicts the same relation between cosmic time dilation and cosmological redshift of $(1 + z) = t/t_0$. In this relation t_0 is the un-dilated time at our location, which is the "cosmic time" for every fundamental observer in the expanding universe paradigm. In the static model cosmic redshift is a consequence of gravitational time dilation in a positive gravitational field gradient. In the expanding model the rate of change of the scale factor causes a time dilation ratio that equals the redshift factor. Cosmic time dilation due to the motion of space expansion is generic to general relativity models described by the Friedmann equation. The Milne expanding model is based on special relativity, but it makes the same prediction. Galaxies have receding peculiar velocities in static Euclidean space, and these velocities cause the same relation between cosmic time dilation and cosmological redshift. [15]

It was first proposed in 1939 that cosmic time dilation should be observed in the light curves of distant Type Ia supernovae (SN Ia). [16] These supernovae begin as white dwarfs which explode over a period of a month or two with a luminosity that is several billion times brighter than our sun. The change in their luminosity can be plotted over this time period to draw what is called a light curve. These light curves show their luminosities increase over a week or two to a peak luminosity and then decrease over about three times as many days. In the expanding model the duration of the light curve for a distant supernova should be longer (wider) than the light curve for nearby supernovae by a factor $(1 + z)$. This prediction was confirmed in a 2008 study, which examined the light curves of 22 low redshift SN Ia (z < 0.04) and 13 high redshift SN Ia (0.28 < z < 0.62). [15] The durations of the light curves for high redshift supernovae were longer by a factor of $(1 + z)$ to within an error of $\pm 10\%$. A 2001 study of 18 low redshift SN Ia (z < 0.11) and 35 high redshift SN Ia (0.30 < z < 0.70) reported a similar finding. [17] A supernova at redshift z has a light curve with a duration that is $(1 + z)$ times longer than that of a local supernova at $z \approx 0$. Cosmic time dilation causes all physical processes to slow down, including supernovae explosions.

These findings confirm the relation between the redshift factor and cosmic time dilation predicted by both the expanding and static models. In both models there is an orderly temporal structure in the universe. Proceeding outward, there are spherically concentric shells of uniformly dilated time, in which the duration of physical processes increases relative to their duration at our location. In the concordance model cosmic time dilation is the result of the rate of change of the scale factor due to space expansion. In the Milne model it is the result of peculiar velocities in static Euclidean space. In the static model cosmic time dilation is the result of the gravitational potential at different distances from the center of a positive gravitational field.

In the expanding model cosmic time is the time measured by a clock at rest in an expanding frame of reference; fundamental observers are at rest because they are co-moving with the expanding frame. The clock of every fundamental observer measures un-dilated cosmic time t_0. Relative to the inertial frame of each fundamental observer's clock, the time t measured by all other clocks is cosmically dilated by the redshift factor. Space expansion causes the time dilation between the clocks of any two fundamental observer's to increase over time. The temporal structure of the universe varies over time. In the static model the clock of every fundamental observer is at rest in a universal inertial frame and measures un-dilated cosmic time. The time of all other clocks is cosmically dilated by the redshift factor, but the time dilation between any two clocks is constant over time. The temporal structure of spherically concentric shells of uniformly dilated time in the universe is static.

The gravitational time dilation explanation for cosmic redshifts has an effect similar to that of Zwicky's "tired light" hypothesis, but there is no transfer of photon energy. The redshift-distance relation requires a decrease in the energy of photons with distance. The tired light idea is that photons systematically lose energy to matter with distance due to some sort of causal interaction during transit. In the static gravitational model the reduction in photon energy is due to gravitational time dilation, which is fundamentally different from any Compton-like effect between photons and matter. Cosmic redshift is the result of cosmic time dilation, which is a relativity effect of a differential in gravitational potential that does not involve the transfer of photon energy to anything else.

The time dilation caused by the negative gravitational field surrounding a body is necessarily a local phenomenon. The gravitational potential decreases rapidly with distance toward zero, and time dilation becomes negligible when the distance is much greater than the radius of a material sphere ($d \gg r$). Cosmic time dilation caused by the positive gravitational field in the static gravitational model is necessarily a universal phenomenon, since gravitational potential and time dilation consistently increase with distance.

4. The Uniform Temperature of the CMB Radiation in a Static Universe

The primary physical evidence for space expansion is the redshift-distance relation. The cosmic microwave background (CMB) radiation is interpreted as secondary evidence confirming this hypothesis. This radiation was predicted in the late 1940s as a consequence of a big bang event and was first clearly detected in 1964. It has since been observed in every direction in the sky and has a temperature of 2.72548 ± 0.00057 °K (Kelvin). This radiation is measured by temperature, instead of frequency (wavelength), because it is thermal radiation with a blackbody spectrum. The CMB radiation is spread across a spectrum that is mostly between 1 and 600 GHz (Gigahertz), which includes the microwave range of frequencies, as shown in Figure 3.

CMB Blackbody Spectrum

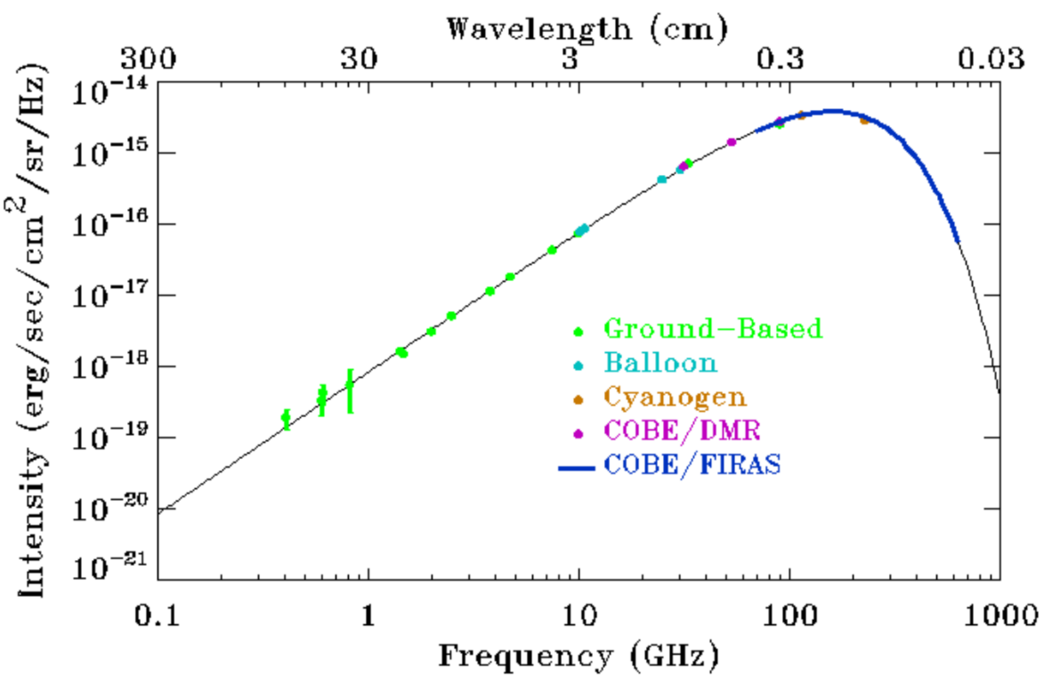

Figure 3: NASA: https://asd.gsfc.nasa.gov/archive/arcade/cmb_spectrum.html

An ideal blackbody theoretically absorbs and emits all frequencies of radiation. A blackbody is in thermodynamic equilibrium and emits radiation with a characteristic spectrum that has a peak frequency, at which the radiation intensity is greatest. The CMB radiation almost perfectly matches the theoretical spectrum of an ideal blackbody and has a peak frequency of 160.23 GHz. By Planck's law of blackbody radiation, the peak frequency f_{peak} is proportional to the temperature T measured in Kelvins and $f_{peak} = (\alpha k_B/h)T$, where the constants α, k_B, and h together equal 5.879 x 10^{10} Hz/K. (α is a proportionality constant, k_B is the Boltzmann constant, and h is the Planck constant.) The peak frequency of 160.23 GHz corresponds to a blackbody temperature with an equilibrium temperature of 2.725 °K.

Thermal radiation arises from the kinetic energy of particles. It is emitted when charged particles are accelerated, which occurs whenever they collide with other particles. The temperature of the CMB radiation is related to the average kinetic energy $\overline{KE}$ of particles by the kinetic temperature equation.

$$\overline{KE} = \frac{1}{2} m\overline{v^2} = \frac{3}{2} k_B T \tag{31}$$

In this equation m is the mass of each free particle (or the average mass $\overline{m}$ of all particles), $\overline{v^2}$ is the average of the square of their velocities over a period of time, k_B is the Boltzmann constant (1.380649 x 10^{-23} J/K), and T is the absolute temperature in Kelvins (K).

The blackbody spectrum of the CMB radiation demonstrates it was emitted by a universe in thermodynamic equilibrium, since its temperature (peak frequency) is the same in all directions. In the concordance model the temperature of the universe decreases with time due to the expansion of space; as the volume of space increases, the temperature decreases as a result of adiabatic cooling. Since the CMB radiation has a highly uniform temperature, it must all have been emitted at the same time when the universe was in thermodynamic equilibrium at a much higher temperature. The highest temperature at which this radiation could have been emitted is about 3000 °K. Above this temperature protons and electrons are dissociated and form an ionized plasma that is relatively opaque to light; photons cannot travel very far in this plasma without being scattered. At about 3000 °K protons and electrons combine to form hydrogen atoms ("recombination epoch"). At this temperature the state of matter in the universe changes from an ionized plasma of subatomic particles to a hydrogen gas, which is relatively transparent to light.

In the expanding model the temperature of this radiation is proportional to the redshift factor: $T = T_0(1 + z)$, where T_0 is the temperature observed now and T was the temperature in the past at redshift *z.* Since the current observed temperature of this radiation is 2.725 °K and the theoretical temperature at emission was 3000 °K, the CMB radiation was emitted at a redshift of $z \approx 1100$. In the expanding model this redshift equates to a time that is 13.1 billion years in the past for a Hubble constant of $H_0 = 73$ km/s/Mpc. [18] The expansion of space over the last 13 billion years results in the original frequencies of the CMB blackbody spectrum being redshifted by a factor of 1100. This redshift corresponds to a current co-moving distance of 44 billion light-years. The spherical shell defined by this distance is referred to as the surface of last scattering.

In the static model considered here the universe has a uniform mass density equal to that in the expanding model at the present time. For a Hubble constant of 73 km/s/Mpc the critical density equation (15) gives a density of 1×10^{-26} kg/m^3. This is equivalent to the mass of about six hydrogen atoms. Since the universe is static, the uniform mass density and the average kinetic energy of particles do not change over time; the universe is in continuous thermodynamic equilibrium at a temperature of 2.725 °K. This equilibrium temperature means the velocity for a hydrogen atom ($m = 1.673 \times 10^{-27}$ kg) has a root mean square speed of 260 m/s by the kinetic temperature equation $\left(v_{rms} = \sqrt{3k_B T/m}\right)$. This is the average velocity for all free hydrogen atoms anywhere in a static universe at any time, as measured relative to our location.

In special relativity both time dilation and relativistic mass are quantified by the Lorentz factor. The relativistic mass equation $m = \gamma m_0$ has exactly the same form as the time dilation equation (20), where m_0 is the rest mass, m is the relativistic mass, and γ is the Lorentz factor. Relativistic mass is the sum of the invariant rest mass and the mass equivalent of the kinetic energy of the moving mass. Where t_0 and m_0 are the un-dilated time and rest mass in a stationary inertial frame, and t and m are the dilated time and relativistic mass in an inertial frame with a velocity v:

$$\frac{t_0}{t} = \frac{m_0}{m} = \sqrt{1 - \frac{v^2}{c^2}} \qquad (32)$$

The time dilation ratio is identical to the relativistic mass ratio in special relativity. This relation follows from the first postulate of special relativity, which states that physical laws are the same in all inertial frames. If time dilates, relativistic mass must increase by the same proportion for physical laws to be invariant. This relation between dilated time and relativistic mass must hold in general relativity, which is derived from special relativity. The v^2 in equation (32) can be interpreted as a kinetic energy density equal to $2GM/r$, which is a gravitational potential; that is, as the gravitational form of the kinetic Lorentz factor. An increase in gravitational potential causes an increase in the relativistic mass ratio that equals the increase in the time dilation ratio.

In the positive gravitational field of the static model, time dilation and relativistic mass both increase moving outward from the center of the field, which is the location of un-dilated time (t_0) and invariant rest mass (m_0). Equation (30) describes time dilation in a positive gravitational field. This equation can be expanded to include the rest mass and relativistic mass.

$$1 + z = \frac{t}{t_0} = \frac{m}{m_0} = 1 + \sqrt{\frac{8\pi G\rho r^2}{3c^2}} \tag{33}$$

The redshift factor equals equals the relativistic mass ratio. The relativistic mass of hydrogen atoms at r is greater than their invariant rest mass by a factor of $(1 + z)$. The kinetic temperature equation (31) gives an average speed of 260 m/s for free hydrogen atoms anywhere in a static universe. The mass of hydrogen atoms in this equation is the invariant rest mass m_0. While the velocities of hydrogen atoms do not change with their distance from us, their relativistic masses do. The stronger gravitational potential at a distance r results in a relativistic mass m that equals $m_0(1 + z)$. Substituting the relativistic mass for the rest mass in equation (31) and simplifying:

$$m_0(1 + z)\,\overline{v^2} = 3k_B T \quad \rightarrow \quad \frac{m_0\overline{v^2}}{3k_B} = \frac{T}{(1 + z)} = T_0 \tag{34}$$

The kinetic temperature generated by free hydrogen atoms in any local region of space is 2.725 °K. Considered with respect to our location at the center of a positive gravitational field, the relativistic kinetic temperature is $(1 + z)$ times higher. The right hand side of Equation (34) is the relativistic CMB temperature divided by the redshift factor, which equals the local non-relativistic temperature T_0 calculated using the rest mass m_0. This reproduces the temperature-redshift relation in the expanding model: $T = T_0(1 + z)$. Equation (33) can be expanded to include the ratio of relativistic temperature over local temperature.

$$1 + z = \frac{t}{t_0} = \frac{m}{m_0} = \frac{T}{T_0} = 1 + \sqrt{\frac{8\pi G\rho r^2}{3c^2}} \tag{35}$$

The local kinetic temperature T_0 of the CMB radiation is the same everywhere, since it is generated by the same local process of random collisions between particles with the same average velocity. Gravitational potential causes the relativistic mass of particles to increase in proportion with redshift, which results in a higher relativistic kinetic temperature of $T = T_0(1 + z)$. This higher temperature results in the frequencies f of the CMB blackbody spectrum at remote distances being blueshifted relative to locally generated spectrum frequencies f_0 by $f = f_0(1 + z)$. The blueshifted CMB frequency spectrum at remote distances is subject to gravitational redshifting by a factor of $(1 + z)$. This results in their observed frequencies being equal to the locally generated frequencies: $f_0 = f/(1 + z)$. The CMB blackbody spectrum is blueshifted by the increase in relativistic mass and then redshifted the same amount by cosmic time dilation.

The static model predicts that the relativistic temperature of the CMB radiation equals its local temperature multiplied by the redshift factor: $T = T_0(1 + z)$. The expanding model predicts the same thing. This relation between the CMB temperature and the redshift factor has been confirmed. The temperature of the CMB radiation at a redshift can be indirectly measured by the atomic fine-structure energy levels this thermal radiation excites. The CMB radiation field increases these energy levels from their ground-states in a predictable way. The fine-structure energy levels of different atomic elements associated with high redshift quasars ($z \geq 2$) show the CMB temperatures T at these redshifts are consistent with the prediction of $T_0(1+z)$. [19][20][21]

In the static model the locally measured CMB temperature is 2.725 °K everywhere and at all earlier epochs, since there is no space expansion. However, the relativistic kinetic temperature of free atoms is $T_0(1+z)$, due to the increase in relativistic mass of $m_0(1+z)$. This increase in relativistic mass (total energy) causes the atomic fine-structure energy levels to increase from their ground-states by a factor of $(1+z)$. In the static model the relativistic CMB temperature varies with distance. In the expanding model the CMB temperature varies with historical epoch.

The evolution of CMB temperature over the history of the universe creates a problem in the expanding model. This temperature is observed to be uniform to about one part in 10,000. This degree of uniformity cannot be explained by the original big bang model. When the CMB radiation was emitted about 370,000 years after the big bang, CMB photons separated by more than about 2° on the celestial sphere were too far apart to be in thermodynamic equilibrium: Photons separated by more than this angle were not causally connected at that time. But the high level of uniformity observed in the CMB temperature requires all areas on the celestial sphere to be causally connected when the CMB radiation was emitted. This is referred to as the "horizon problem" and was identified by Wolfgang Rindler in 1956. The *ad hoc* hypothesis of cosmic inflation was developed, in part, to explain how the whole universe was causally connected 370,000 years after the big bang. This problem does not occur in the static model, in which the universe is in perpetual thermodynamic equilibrium at 2.725 °K.

In the concordance model the uniform CMB temperature defines a universal co-moving frame of reference in a spatially flat universe. We are located at the center of the spherical shell of last scattering; just like every other fundamental observer, we are located at the single stationary point of reference in this expanding frame. In the static model the uniform CMB temperature defines a universal inertial frame of reference in static Euclidean space. This universal inertial frame is consistent with the presence of a universal gravitational field and the static temporal structure of the universe.

5. A Static Gravitational Model of the Universe

The Friedmann equation describes the expanding, Milne, and static models, which all postulate the universe has a uniform mass density. The evidence strongly supports the conclusion that the space of the observable universe is flat, so the critical density equation is valid for these models.

$$H_0^2 = \frac{8\pi G\rho}{3} \tag{15}$$

The proper expansion velocities in the concordance model equal the receding peculiar velocities in the Milne model. Galaxies recede at their escape velocities in the Milne model. The squares of these proper and peculiar velocities are numerically equal to the kinetic energy densities in the static gravitational model.

$$v^2 = \frac{2GM}{r} \quad \Leftrightarrow \quad v_\varepsilon^2 = \frac{2GM}{r} \tag{13a, b}$$

The kinetic energy density depends upon the total mass within a distance r. The postulate of uniform mass density permits a substitution of $(4\pi G\rho r^3)/3$ for M in equation (13b).

$$v_\varepsilon^2 = \frac{8\pi G\rho r^2}{3} \tag{36}$$

In the critical density equation the Hubble constant squared H_0^2 equals $(8\pi G\rho)/3$. Making this substitution in equation (36):

$$v_\varepsilon^2 = H_0^2 r^2 \tag{37}$$

The kinetic energy density equals a gravitational potential equal to the square of the Hubble constant times the square of the distance $(H_0^2 r^2)$. Since the uniform mass density does not change in a static universe, H_0^2 is constant over time. The Hubble parameter changes over time in the expanding and Milne models. Taking the square root of both sides of equation (37):

$$v_\varepsilon = H_0 r \quad \Leftrightarrow \quad v = H_0 D \tag{38a, b}$$

The square root of the kinetic energy density is an apparent velocity in the static model. There is a linear relation between this apparent velocity and distance in the static model (38a) which is identical in form to the velocity-distance law in the expanding model (38b). Since the expansion velocity equals the apparent velocity in the static model and the Hubble constant is the same, the co-moving distance D equals the distance r in the static model. In the static model $H_0 r$ is the square root of the gravitational potential.

The static model equation (29a) describes the linear redshift-distance relation discovered by Hubble. Cosmic redshift equals the square root of the gravitational potential $(H_0 r)$ divided by the speed of light.

$$z = \sqrt{\frac{8\pi G\rho r^2}{3c^2}} = \frac{H_0 r}{c} \tag{39}$$

Since $v_\varepsilon = H_0 r$ (eq. 38a) and $cz = H_0 r$ (eq. 39), there is an apparent Doppler relation $v_\varepsilon = cz$ between apparent velocity, cosmic redshift, and the speed of light. The redshift-distance law can be written as:

$$\frac{z}{r} = \frac{H_0}{c} \tag{40}$$

Cosmic redshift per unit distance (z/r) equals the Hubble constant divided by the speed of light. The right hand side of equation (40) is the reciprocal of the Hubble distance D_H in the expanding model, $D_H = c/H_0$. The co-moving distance D equals the fixed distance r in the static model, so $r_H = c/H_0$. In the expanding model the Hubble distance divided by the speed of light equals the Hubble time $D_H/c = 1/H_0$, the age of the universe since the big bang. The age of the universe is measured by un-dilated cosmic time (t_0) defined by an expanding co-moving frame of reference. In the static model the Hubble time $r_H/c = 1/H_0$ is the travel time for light to traverse the Hubble distance. The Hubble time defines un-dilated cosmic time (t_0) in a universal inertial frame of reference, instead of a co-moving one. Since $1/H_0 = t_0 = r_H/c$ and the time dilation ratio is $(1 + z) = t/t_0$, cosmic time dilation can be defined by the product of the redshift factor and the Hubble distance $t = (1 + z)r_H/c$.

The Hubble constant times the Hubble distance r_H equals an apparent velocity v_ε (eq. 38a) that is numerically equal to an expansion velocity (eq. 38a) of *c.* In the static model the cosmic redshift at the Hubble distance equals unity by the apparent Doppler relation $z = v_\varepsilon/c$. In the expanding model $z = 1.43$ at the Hubble distance.

This apparent Doppler relation for cosmic redshift $(z = v_\varepsilon/c = H_0 r/c)$ equals the time dilation ratio minus one $(t/t_0 - 1)$ and can be included in the equivalent relations in equation (35) which describe the static model.

$$1 + z = \frac{t}{t_0} = \frac{m}{m_0} = \frac{T}{T_0} = 1 + \frac{H_0 r}{c} = 1 + \sqrt{\frac{8\pi G \rho r^2}{3c^2}} \tag{41}$$

It is notable that cosmic redshift, cosmic time dilation, relativistic mass, relativistic temperature, apparent Doppler relation, and the square root of the gravitational potential divided by the square of the speed of light are all equal to each other. The static model gives a unified explanation for all of these relations as relativity effects in a universal gravitational field with a positive gradient.

The Hubble constant squared has the characteristics of a cosmic gravitational constant. Multiplying this constant by the square of the distance gives the gravitational potential $v_\varepsilon^2 = H_0^2 r^2$ (eq. 37). Multiplying H_0^2 by the distance gives the gravitational acceleration $a = H_0^2 r$ (eq. 24). This characterization is also consistent with the proportional relation between the Hubble constant and the Newtonian constant in the critical density equation. The Hubble constant depends upon the uniform mass density of the universe. The determination of the Newtonian constant also depends upon mass density.

The equation $a = GM/r^2$ describes the relation between acceleration, distance, and mass. The surface acceleration on the earth depends upon its mass. This depends upon the specific gravities of the different types of matter which make up the earth. Since the proportions of each type are not known, the mass of the earth can be replaced by its mass density, which changes the acceleration equation to $a = 4\pi G\rho r/3$. The value of the constant G can then be calculated from the acceleration, mass density, and distance.

$$G = \frac{3a}{4\pi r\rho} \tag{42}$$

Newton knew the earth's radius and surface acceleration to within one percent of their modern values, but its mass density was uncertain. The specific gravities for different types of matter were known, but not the proportion each contributed to the earth's total mass. Based on a series of suppositions, Newton estimated the density of the earth "may be five or six times" greater than water. (Book III, Proposition X) A density of 5,000-6,000 $\mathrm{kg/m^3}$ gives a working estimate of $6.1 - 7.3 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2}$ for the gravitational constant. This 1687 estimate was not improved upon until 1879, when Henry Cavendish found a value of 5,448 $\mathrm{kg/m^3}$ for the density of the earth. This is within one percent of the modern value. He determined this by measuring the force of attraction between small and large lead balls using a torsion balance. Since the force of attraction between the small ball and the earth was known, the mass density of the earth could be calculated from the ratio of these forces. This density yielded a gravitational constant of $6.673 \times 10^{-11}\ \mathrm{m^3/kg \cdot s^2}$, which is within one percent of the modern value.

There is a parallel here with the Hubble constant, which is related to the mass density of the universe by the critical density equation. The universe's total mass is very uncertain, which is similar to the situation that confronted Newton. Cavendish was able to determine the earth's mass density by measuring the gravitational force between two masses using a torsion balance. This is comparable to determining the uniform mass density of the universe from the Hubble constant by measuring cosmic redshifts and distances. Measuring the redshift at a distance is comparable to measuring the gravitational acceleration at a distance. There is a linear relation between redshift and acceleration, because redshift is proportional to distance by equation (29) and acceleration is proportional to distance by equation (24). In the static model the critical density equation (15) gives the square of the Hubble constant, which is analogous to equation (42) for the Newtonian gravitational constant.

The hypothesis that the Hubble constant squared is a cosmic gravitational constant is supported by the observed flatness of space. The Friedmann equation distinguishes between the Hubble parameter, the expansion rate calculated from the density, and the Hubble constant, the empirically measured expansion rate. The difference between the two determines the degree of space curvature. In the static model the empirical Hubble constant must equal the theoretical Hubble parameter, since the mass density of the universe is constant over time. A flat Euclidean space is the only possibility.

The observed flatness of space cannot be credibly explained by the original big bang model. In this model, there is no reason why the Hubble parameter should equal the Hubble constant, resulting in flat space. It is, in fact, supremely improbable that they should be equal. This "flatness problem" was first identified in 1969 by Robert Dicke. For space to be flat now, the mass density in the first 10^{-35} seconds after the big bang must have been equal to the critical density to within one part in about 10^{60}. At the end of the first second, it would have to be equal to within one part in about 10^{16}. If the original density was ever so slightly greater, the universe would have collapsed before any galaxies could form. If it was ever so slightly less, expansion would have been too rapid for any galaxies to form. It is statistically impossible that the mass density at this very early time should almost perfectly match the critical density by random accident.

To remain credible the original big bang model was modified to become the inflationary big bang model. This model incorporates the *ad hoc* hypothesis of cosmic inflation, which was initially developed by Alan Guth in 1980: If the scale of the universe expands by a factor of about 10^{26} in the first 10^{-32} seconds after the big bang, the resulting density should be virtually identical to the critical density. While this hypothetical process saves the big bang model, it is purely speculative. Cosmic inflation is not a scientific hypothesis, in the proper sense, since this process occurs just once in the history of the universe and, therefore, cannot be empirically tested and possibly refuted.

6. Discussion

The static model explains the linear redshift-distance relation as a relativity effect of time dilation in a positive gravitational potential. The square of the Hubble constant is the empirically determined gravitational constant of cosmic gravity. The relativity effects of gravitational potential also explain the highly uniform temperature of the CMB radiation. This model relies on conventional physics, has no tunable parameters, and requires no speculative hypotheses.

The big bang model cannot credibly explain the redshift-distance relation without the *ad hoc* hypothesis of cosmic inflation, since there would be no galaxies without it. Cosmic inflation is also required for a credible explanation of the uniform temperature of the CMB radiation. Further developments required the addition of the hypotheses of dark matter and dark energy to the inflationary big bang model for it to remain credible. This has resulted in the concordance ΛCDM (*lambda* cold dark matter) model. This model incorporates unconventional physics, has half a dozen tunable parameters, and requires speculative hypotheses.

Both models are consistent with the critical density equation and describe a spatially flat universe, although there is no "flatness problem" in the static model. The expansion velocity $v = H_0 D$ is numerically equal to the square root of the kinetic energy density $v_\varepsilon = H_0 r$. The expansion velocity is an apparent velocity in the static model, so there is no "receding velocity without Doppler redshift" problem. Both models give the same Hubble distance. They give the same Hubble time, but in the static model this defines the universal un-dilated clock rate of cosmic time, instead of the age of the universe. The predicted relations between the redshift factor and cosmic time dilation $(1+z) = t/t_0$ and the redshift factor and CMB temperature $(1+z) = T/T_0$ are the same in both. The uniformity of the CMB temperature is explained in both models, although there is no "horizon problem" in the static model.

The two models make the same numerical predictions, except where the redshift-distance relation is involved. This is non-linear in the concordance model, because redshift is interpreted as a mechanical effect of space expansion. It is linear in the static model considered here, because redshift is interpreted as a relativity effect of gravitational time dilation. Which is the better interpretation depends upon a resolution of the question of the reality of space expansion.

As noted above, M. Lopez-Corredoira [5] reviewed the results of different types of tests for space expansion. The CMB temperature versus redshift test results are a good fit for space expansion, while they do not fit static space. The cosmic time dilation versus redshift test results are a good fit for space expansion and do not fit static space. The Hubble diagram of apparent magnitudes versus redshift is not a good fit for space expansion, unless the luminosity of galaxies evolves with redshift or the hypothesis of dark energy is incorporated. The Hubble diagram test is a good fit for static space with no assumptions. The Tolman surface brightness versus redshift test results are not a good fit for space expansion, unless it is assumed there is a strong evolution in the luminosity of galaxies with redshift. The Tolman test results are a good fit for static space with no assumptions. The Hoyle angular size versus redshift test results are not a good fit for space expansion, unless it is assumed there is a very strong evolution in galaxy size with redshift. The Hoyle test results are a good fit for static space with no assumptions.

The cosmic time dilation and CMB temperature test results are good fits for the expanding model but not for the static model. However, the static gravitational model considered here is a good fit for both of these tests, since it predicts the same results as the expanding model. The Hubble diagram, surface brightness, and angular size test results are not good fits for the expanding model, unless certain unproven assumptions about galaxy evolution are adopted. They are good fits for the static model. The static gravitational model considered here is a good fit for these three tests. Where the overall results of these tests are equivocal with regards to the expanding and static models, all five tests are unequivocally consistent with the static gravitational model.

The static gravitational model is far simpler than the expanding model. All of its key elements are relativistically explained by a positive gravitational potential. This potential is determined by the Hubble constant squared multiplied by the distance squared. The square of the Hubble constant is the gravitational constant of cosmic gravity and is defined by the critical density equation.

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References:

[1] A Relation between Distance and Radial Velocity among Extra-galactic Nebulae, E. Hubble, Mar. 15, 1929, *Proceedings of the National Academy of Sciences (PNAS)*, Vol. 15(3)
https://www.pnas.org/content/15/3/168

[2] The Observational Approach to Cosmology, E. Hubble, 1937, *Oxford University Press*
https://ned.ipac.caltech.edu/level5/Sept04/Hubble/paper.pdf

[3] On the Redshift of Spectral Lines through Interstellar Space, F. Zwicky, Oct. 15, 1929, *Proceedings of the National Academy of Sciences (PNAS)*, Vol. 15(10)
https://www.pnas.org/content/pnas/15/10/773.full.pdf

[4] Observational Cosmology 1920-1985, Allan Sandage, Jan. 1987, IAU Symposium, Vol. 124
https://ui.adsabs.harvard.edu/abs/1987IAUS..124....1S

[5] Tests for the Expansion of the Universe, Martin Lopez-Corredoira, July 2014, Frontiers of Fundamental Physics 14
https://arxiv.org/pdf/1501.01487v1.pdf

[6] The Mathematical Theory of Relativity, Arthur Eddington, 1923, Cambridge University Press, pg. 288
https://www.gutenberg.org/files/59248/59248-pdf.pdf

[7] The Redshift-Distance and Velocity-Distance Laws, Edward Harrison, Jan. 1993, *Astrophysical Journal,* 403:28-31
http://adsabs.harvard.edu/full/1993ApJ...403...28H

[8] A Newtonian Expanding Universe, E. A. Milne, 1934, *Quarterly Journal of Mathematics*, Vol. 5, Issue 1
https://academic.oup.com/qjmath/article-abstract/os-5/1/64/1576447

[9] Cosmology and Newtonian Mechanics, C. Callan, R. H. Dicke, and P. J. E. Peebles, 1965, *American Journal of Physics* 33, 105

[10] An Interpretation of Milne Cosmology, Alasdair Macleod, 2005, Lews Castle College, https://arxiv.org/abs/physics/0510170

[11] E. A. Milne and the Universes of Newton and Relativistic Cosmology, J. Dunning-Davies, Feb. 2004, University of Hull
https://arxiv.org/abs/astro-ph/0402554

[12] Newtonian Universes and the Curvature of Space, W. H. McCrea and E. A. Milne, March 1934, *Quarterly Journal of Mathematics,* 5(17):73-80,

[13] Planck 2018 Results, Planck Collaboration Team, Sep. 2020, *Astronomy & Astrophysics,* Vol 641
https://arxiv.org/pdf/1807.06205.pdf

[14] Einstein's Comprehensive 1907 Essay on Relativity, English translation by H. M. Schwartz, June, Sep, Oct 1977, *American Journal of Physics,* Vol 45, No. 6, 9, 10 (Part III)
http://hermes.ffn.ub.es/luisnavarro/nuevo_maletin/Einstein_1907_Jahrbuch.pdf

[15] Time Dilation in Type Ia Supernova Spectra at High Redshift, S. Blondin, et al., 2008, *Astrophysical Journal*, Vol. 682 No. 2
https://arxiv.org/abs/0804.3595

[16] Possible Applications of Supernovae to the Study of the Nebular Red Shifts, Wilson, O. C., Nov. 1939, *Astrophysical Journal*, Vol. 90, p. 634

[17] Timescale Stretch Parameterization of Type Ia Supernova B-band Light Curves, G. Goldhaber, et al., Sep. 2001, *Astrophysical Journal*, Vol 558, No. 1
https://arxiv.org/abs/astro-ph/0104382

[18] A 2.4% Determination of the Local Value of the Hubble Constant, Adam G. Reiss, et al., 21 July 2016, *The Astrophysical Journal*, Volume 826, Number 1
https://arxiv.org/abs/1604.01424

[19] The Cosmic Microwave Background temperature at a redshift of 2.33771, R. Srianand, et al., Dec. 2000, *Nature*, Vol. 408, Issue 6815
https://arxiv.org/pdf/astro-ph/0012222.pdf

[20] A New Measurements of the Cosmic Microwave Background Radiation Temperature at z = 1.97, Jian Ge, et al., *Astrophysical Journal*, 1997, 472
https://arxiv.org/abs/astro-ph/9607145

[21] The Evolution of the Cosmic Microwave Background Temperature, P. Noterdaeme, et al., February 2011, *Astronomy & Astrophysics*, 526 L7
https://arxiv.org/abs/1012.3164