Reinterpretation of Length Contraction Derivation from Lorentz Transformation and Derivation of Logical Relativistic Length

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Roy Weinstein's length contraction derivation process has been acknowledged by several researchers, from Gamov to Einstein. However, there are several problems in his derivation, and this study looked into them in detail. I have confirmed that if the problems found in the Weinstein derivation process are removed, the length contraction equation is not derived, but rather the length expansion equation is derived. I also looked at the fact that some experimental facts support length expansion.

The length contraction phenomenon has been treated as important since the birth of the theory of relativity. However, key experimental evidence that this is correct has yet to be found. Except for the ether theory, the concept of a purely relativistic length begins with the derivation of Roy Weinstein and Hermann Bondi [1] [2]. Weinstein writes in his paper that many from Gamov to Einstein acknowledged the length contraction equations he derived. After that, many people accepted Weinstein's derivation process, and his derivation equation was recognized as the first length contraction equation derived from the Lorentz transformation. In other words, if there is a problem with the process derived by Weinstein, the problem of length contraction should be looked at again from the beginning.

Strangely, length contraction has not yet been experimentally discovered. We have detected gravitational waves that are very difficult to detect, so why can't we still find experimental evidence of length contraction? I think it's because the length contraction doesn't exist in the first place. In this study, I will show a fatal logical flaw in Weinstein's derivation process, and if we remove all those problems, we will know that length expansion is correct, not length contraction. If so, is there experimental evidence of length expansion? Of course, they exist. We will also examine some experimental evidence of length expansion.

I. PROBLEMS WITH WEISTEIN'S LENGTH CONTRACTION DERIVATION PROCESS

Classical theory presupposed the existence of an ether, but modern relativity said that there is no ether. Then, we do not need to examine the Lorentz-Fitzgerald length contraction hypothesis, which was born on the premise of Ether. Weinstein was the first to derive a length contraction using the Lorentz transformation in the pure theory of relativity rather than in the ether theory [1]. His description is as follows:

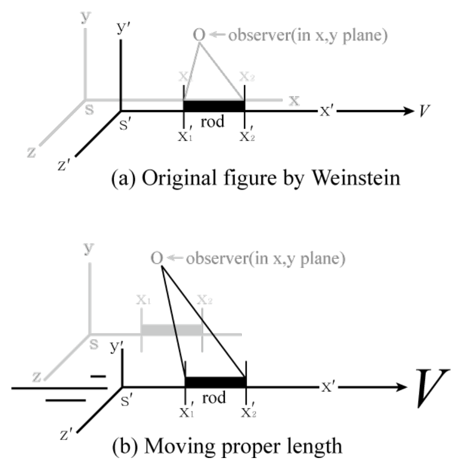

FIG. 1: Weinstein's length contraction derivation. The observer and the ruler are in different systems, and the ruler is moving. Therefore, the ruler cannot be the proper length or rest length.

$$L_o \equiv x_2^{'} - x_1^{'} \tag{1}$$

$$\Big(L \equiv x_2 - x_1\Big) \tag{2}$$

$$\left(x_1^{'} = \frac{x_1 - vt}{\sqrt{1-\beta^2}}, \quad x_2^{'} = \frac{x_2 - vt}{\sqrt{1-\beta^2}}\right) \tag{3}$$

$$= \frac{x_2 - vt}{\sqrt{1-\beta^2}} - \frac{x_1 - vt}{\sqrt{1-\beta^2}} \tag{4}$$

$$= \frac{x_2 - x_1}{\sqrt{1-\beta^2}} \tag{5}$$

$$= \frac{L}{\sqrt{1-\beta^2}} \tag{6}$$

$$\therefore L = L_o\sqrt{1-\beta^2} \tag{7}$$

Weinstein assumed that both ends of the rod were measured simultaneously. The assumption that both ends of the rod are measured simultaneously will also be followed in the derivation process for other lengths presented later in this article($\beta = v/c$).

In Figure 1, the observer O is at rest in the x-y plane (S system). The rod is attached to the $S^{'}$ system, and he explains that this $S^{'}$ system is moving. Weinstein's logic in this description contains two mistakes. One concerns the observer's perspective, and the other concerns the proper length. First, let us look at the problem of perspective. Weinstein divided everything into two cases. The system is the non-prime S-system and the primed $S^{'}$-system. There is an observer O in the S system and an observer $O^{'}$ in the $S^{'}$ system. He proceeded with his logic from only two perspectives. Unfortunately, the situation is not as simple as Weinstein suggests. There are two observers, but there are four points of view.

Case 1) Observer O may observe O himself,
Case 2) Observer O may observe the other party $O^{'}$,
Case 3) Observer $O^{'}$ may observe the other O,
Case 4) Observer $O^{'}$ may observe $O^{'}$ herself (Fig. 2).

Depending on who is observing whom, the type of length changes, and the coordinates also change. Without making this clear, the correct length cannot be derived. If the proper length is L_o and the observed length is L, there are four lengths.

$$Case\,1: \; O \rightarrow \; O \qquad L_o = x_2 - x_1 \tag{8}$$

$$Case\,2: \; O \rightarrow \; O^{'} \qquad L = x_2^{'} - x_1^{'} \tag{9}$$

$$Case\,3: \; O^{'} \rightarrow \; O \qquad L = x_2 - x_1 \tag{10}$$

$$Case\,4: \; O^{'} \rightarrow \; O^{'} \qquad L_o = x_2^{'} - x_1^{'} \tag{11}$$

If you look at the above, there are two coordinates with primes. They are Equations (9) and (11). Equation (9) $L = x_2^{'} - x_1^{'}$ is an expression of observing the other party, so it is possible to apply the Lorentz transformation equation, but since equation (11) $L_o = x_2^{'} - x_1^{'}$ is an equation that observes herself, the Lorentz transformation equation should not be applied. However, Weinstein applied the Lorentz transformation equation to equation (11) $L_o = x_2^{'} - x_1^{'}$. Why did he apply the Lorentz transformation equation to the expression representing her proper length? He should not apply that equation. Therefore, the expression he derives violates the rules, so the expression he derives is not correct from the beginning.

The second is the problem of proper length. He states that the equation $L_o = x_2^{'} - x_1^{'}$ in Equation (11) is in motion after assuming that it is a proper(rest) length. If you look closely at Figure 1, v is marked to indicate that a system with the proper length is moving. Weinstein said in his article that '*rest length*' is synonymous with the proper length. Is there such a thing as ***'moving proper length'*** or ***'moving rest length'***? Since this is impossible, of course, Weinstein's equation for length

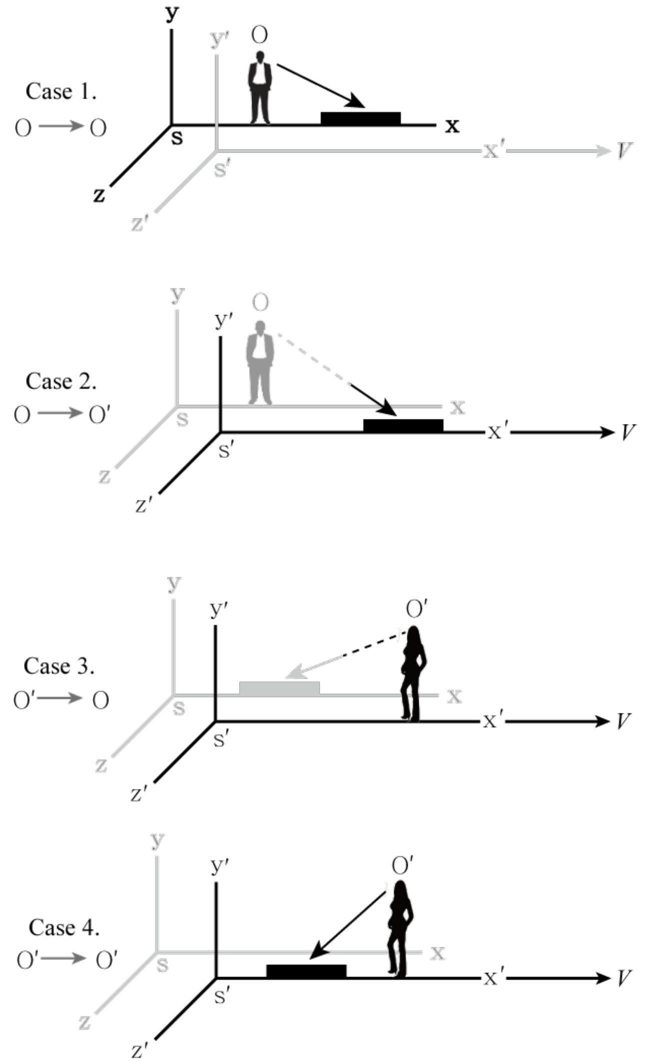

FIG. 2: Four perspectives by two observers

contraction is not correct. We are confused about who observed whom, so let's add a superscript to the length to indicate who observed whom.

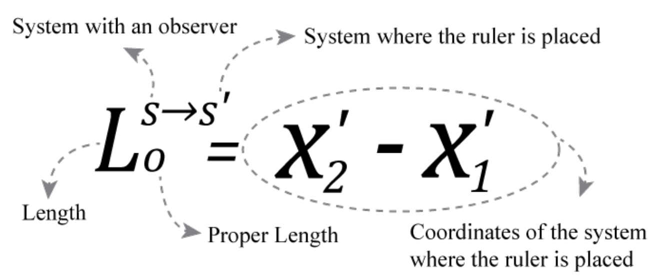

FIG. 3: A notation that clearly distinguishes the system that the observer belongs to and the system that the ruler belongs to.

1. (L) L stands for length and T stands for time.
2. (L_o) Subscript $\rightarrow$ Proper length.

(L) No subscript $\rightarrow$ Observed length

3. $\left(L^{s\rightarrow s'}\right)$ This is the length of the S' system observed by the observer of the S system.
4. $\left(L^{s\rightarrow s'} = x_2' - x_1'\right)$ The coordinates are x_1', x_2'.

$$Case\,1:\; O\rightarrow\; O \qquad L_o^{s\rightarrow s} = x_2 - x_1 \tag{12}$$

$$Case\,2:\; O\rightarrow\; O' \qquad L^{s\rightarrow s'} = x_2' - x_1' \tag{13}$$

$$Case\,3:\; O'\rightarrow\; O \qquad L^{s'\rightarrow s} = x_2 - x_1 \tag{14}$$

$$Case\,4:\; O'\rightarrow\; O' \qquad L_o^{s'\rightarrow s'} = x_2' - x_1' \tag{15}$$

Cases 1 and 4 are observations of his own ruler, and Cases 2 and 3 are observations of the other ruler. Now, let us rewrite the equation that Weinstein derived. He wrote $L_o = x_2' - x_1'$ and applied the Lorentz transformation to it. If we find this expression above, it is $L_o^{s'\rightarrow s'} = x_2' - x_1'$. Since he has clearly written L_o in the left term, this is obviously a proper length. This is the length O' observed O' herself. It is not logical to apply the Lorentz transformation equation to observe her own length. Therefore, this cannot be the correct relativistic length, and all logic must start again from the origin. So, what equation should we use? We just need to use Equations (13) and (14), which are the lengths of observation of the other party. If we use Equation (13), we can apply the Lorentz transformation equation, and if we use equation (14), we can use the inverse Lorentz transformation equation. Let us derive it from equation (13). γ is Lorentz factor.

$$Case\,2:\; O\rightarrow\; O' \quad L^{s\rightarrow s'} = x_2' - x_1' \tag{16}$$

$$\left(L_o^{s\rightarrow s} = x_2 - x_1 \quad proper\,length\right) \tag{17}$$

$$\left(x_2' = \gamma(x_2 - vt), \qquad x_1' = \gamma(x_1 - vt)\right) \tag{18}$$

$$L^{s\rightarrow s'} = \gamma(x_2 - x_1) \tag{19}$$

$$L^{s\rightarrow s'} = \gamma L_o^{s\rightarrow s} \tag{20}$$

$$\therefore\; L = \gamma L_o \tag{21}$$

This is a length expansion equation. Now let us use Equation (14).

$$Case\,3:\; O'\rightarrow\; O \quad L^{s'\rightarrow s} = x_2 - x_1 \tag{22}$$

$$\left(L_o^{s'\rightarrow s'} = x_2' - x_1' \quad proper\,length\right) \tag{23}$$

$$\left(x_2 = \gamma(x_2' + vt'), \qquad x_1 = \gamma(x_1' + vt')\right) \tag{24}$$

$$L_o^{s'\rightarrow s} = \gamma(x_2' - x_1') \tag{25}$$

$$L^{s'\rightarrow s} = \gamma L_o^{s'\rightarrow s'} \tag{26}$$

$$\therefore\; L = \gamma L_o \tag{27}$$

This, of course, is length expansion. Regardless of who observes whom, the expression of observing the other is always observed as length expansion, not length contraction.

II. SPACE EXPANSION AND LENGTH EXPANSION

In relativity, there is a thought experiment called Bell's spaceship paradox. This is an important issue of relativity, first raised by Dewan and Beran and later widely publicized by Bell [3] [4]. It is a thought experiment by connecting a thin string between two identical spacecraft and seeing what happens to that string when it moves at relativistic speeds. In the process of solving this problem, Dewan and Beran divided the length into two concepts.

(a) *The distance between two ends of a connected rod*

(b) *The distance between two objects which are not connected but each of which independently and simultaneously moves with the same velocity for an inertial frame.*

One is the length of the rod and the other is the distance between the two points. In general, we call the interval between the ends of the rod the length, The interval between two points is called the distance. Let us call the description in (a) above the length, and the description in (b) the distance. According to the conclusion of most researchers, the length of the rod-like structure contracts and the distance expands. Here is the distance

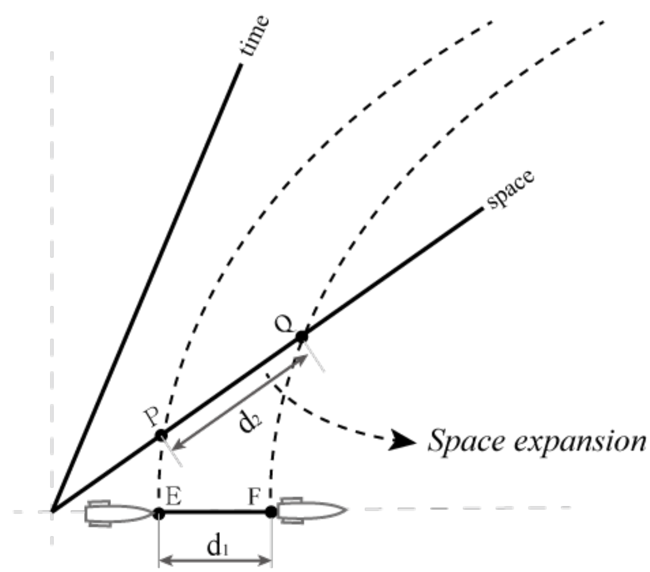

FIG. 4: Space expansion Derived from the space-time diagram

between the two points derived by Petkov and Franklin [5] [6].

$$x_p = \gamma(x_E - vt) \quad (28)$$

$$x_Q = \gamma(x_E + d_1 - vt) \quad (29)$$

The distance between the two spaceships in the moving system is given by

$$d_2 = (x_P - x_Q) = \gamma d_1 \quad \therefore d_2 = \gamma d_1 \quad (30)$$

If this is written in general symbols, it is as follows (L_o is the proper length, L is the observed length, γ is the Lorentz factor)

$$\therefore L = \gamma L_o \quad (31)$$

We need to distinguish this equation (31) from the length contraction equation. Let us call this temporarily *'space expansion'*. This equation is derived by Petkov and Franklin. Is it possible that we essentially distinguish between the length of the rod and the distance in space? A common interpretation of Bell's spaceship paradox is that length contracts and distance expands. Suppose that a rod, which they claim to shrink in length when moving at relativistic speeds, is made up of billions of carbon atoms. Now we're going to cut the rod and make it smaller and smaller. Let's assume that the atoms that make up the rod are reduced by one million, one hundred, ten, and even more, down to two carbon units(ethane). If so, is the interval between these two carbon atoms a length or a distance? If the interval between two carbon atoms should be called length and not distance, do you have a criterion for the number of atoms by which you judge it? In Figure 5, does the distance between the carbon atoms decrease as a result of length contraction or increase as a result of space expansion? If you choose one or the other, can you provide a rationale for your choice? This will be a very difficult choice. In fact, it is a well-known fact that the interior of an atom or molecule is mostly empty space. Therefore, it is meaningless to distinguish between length and distance. The distance equation(31) derived by Petkov and Franklin is the same as the length expansion equation (21), (27) above $L = \gamma L_o$. All of them are length expansions. Weinstein derived an equation for length contraction by using the concept vaguely or by destroying the correct concept, but if he uses the correct concept of length, he can never derive the equation for length contraction.

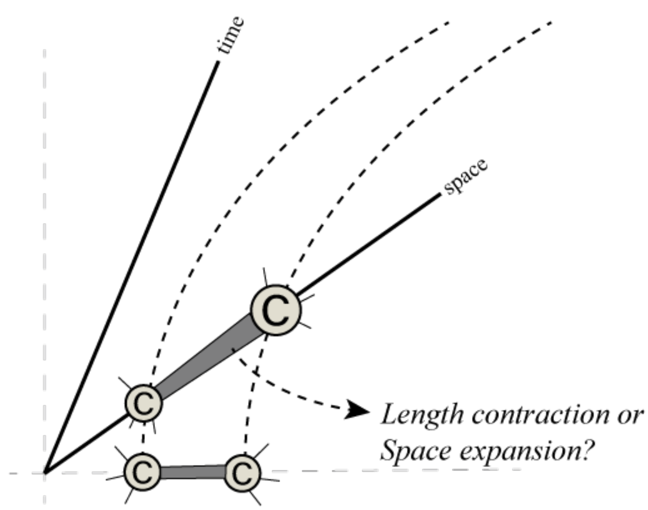

FIG. 5: A rod composed of only two carbon units

III. SPACE-TIME SYMMETRY

In the special theory of relativity, space-time is always symmetric. If this symmetry is broken, we have to look again to see if something is wrong.

$$t' = \gamma(t - \frac{vx}{c^2}) \quad (32)$$

$$x' = \gamma(x - vt) \quad (33)$$

$$y' = y \quad (34)$$

$$z' = z \quad (35)$$

$$\left(\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}\right) \quad (36)$$

These are generally known as Lorentz transformation, but if you add some conditions and write only time and one dimension of space, the above expression becomes the following expression

$$ct = x_o, \quad ct' = x'_o, \quad \beta = v/c \quad (37)$$

$$Spacetime\ symmetry \begin{cases} x'_o = \gamma(x_o - \beta x_1) \\ x'_1 = \gamma(x_1 - \beta x_o) \end{cases} \quad (38)$$

Looking at equation (38), time and space are perfectly symmetric with each other. Therefore, if Weinstein's derivation method is correct, relativistic time should also be derived according to the process in which the length contraction is derived. This is possible because space-times are symmetric to each other. The following process is a rewrite of Weinstein's length contraction derivation process.

$$L_o \equiv x_2^{'} - x_1^{'} \tag{39}$$

$$\Big(L = x_2 - x_1\Big) \tag{40}$$

$$\left(x_1^{'} = \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \qquad x_2^{'} = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}}\right) \tag{41}$$

$$L_o = \frac{x_2 - vt}{\sqrt{1 - v^2/c^2}} - \frac{x_1 - vt}{\sqrt{1 - v^2/c^2}} \tag{42}$$

$$= \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}} \tag{43}$$

$$= \frac{L}{\sqrt{1 - v^2/c^2}} \tag{44}$$

$$\therefore L = L_o\sqrt{1 - v^2/c^2} \tag{45}$$

Since space-time is symmetrical, the following method is justified if the length contraction is correct.

$$T_o \equiv t_2^{'} - t_1^{'} \tag{46}$$

$$\Big(T = t_2 - t_1\Big) \tag{47}$$

$$\left(t_1^{'} = \frac{t_1 - vx/c^2}{\sqrt{1 - v^2/c^2}} \qquad t_2^{'} = \frac{t_2 - vx/c^2}{\sqrt{1 - v^2/c^2}}\right) \tag{48}$$

$$T_o = \frac{t_2 - vx/c^2}{\sqrt{1 - v^2/c^2}} - \frac{t_1 - vx/c^2}{\sqrt{1 - v^2/c^2}} \tag{49}$$

$$= \frac{t_2 - t_1}{\sqrt{1 - v^2/c^2}} \tag{50}$$

$$= \frac{T}{\sqrt{1 - v^2/c^2}} \tag{51}$$

$$\therefore T = T_o\sqrt{1 - v^2/c^2} \tag{52}$$

If the Weinstein method is correct, time also derives the time contraction, not the time dilation. This is an obvious error. Conversely, if the length is derived in the same process as the time dilation is derived, length expansion is derived, not length contraction. The following process is the process in which Schröder derives the time dilation using the inverse Lorentz transformation [7].

$$t_2 = \gamma\Big(t_2^{'} + \frac{vx^{'}}{c^2}\Big) \tag{53}$$

$$t_1 = \gamma\Big(t_1^{'} + \frac{vx^{'}}{c^2}\Big) \tag{54}$$

$$x_1^{'} = x_2^{'} = x^{'} \tag{55}$$

$$t_2 - t_1 = \gamma(t_2^{'} - t_1^{'}) \tag{56}$$

$$\therefore \Delta T = \frac{\Delta T_o}{\sqrt{1 - v^2/c^2}}, \quad \Delta T^{'} = \Delta T_o \tag{57}$$

If this process is applied to the length as it is, length expansion can be obtained as in Equation (62).

$$x_2 = \gamma(x_2^{'} + vt^{'}) \tag{58}$$

$$x_1 = \gamma(x_1^{'} + vt^{'}) \tag{59}$$

$$t_1^{'} = t_2^{'} = t^{'} \tag{60}$$

$$x_2 - x_1 = \gamma(x_2^{'} - x_1^{'}) \tag{61}$$

$$\therefore \Delta L = \frac{\Delta L_o}{\sqrt{1 - v^2/c^2}}, \quad \Delta x^{'} = \Delta x_o \tag{62}$$

IV. EXPERIMENTAL EVIDENCE OF LENGTH EXPANSION

There are several experimental proofs of length expansion. The first proof is the transverse Doppler effect. This is an observation of the frequency or wavelength of an object moving across in front of the observer. Suppose that an excited hydrogen atom passes in front of the observer at a relativistic speed. If so, the frequency of the hydrogen atom can be described as follows [8].

$$\textit{Transverse Doppler Effect} \quad \nu = \nu_o\sqrt{1 - \beta^2} \tag{63}$$

Although the frequency of light emitted from fast-moving hydrogen has decreased, the speed of light emitted is constant, so, naturally, the relationship is $c = \nu\lambda = \nu_o\lambda_o$. Then we can see that Equation (67) holds, and if this is converted to a general length rather than a wavelength, it can be written as Equation (68). ($\beta = v/c, \nu_o$: proper frequency, ν: relativistic frequency)

$$c = \lambda\nu \tag{64}$$

$$= \Big(\lambda_o\frac{1}{\sqrt{1 - \beta^2}}\Big)\Big(\nu_o\sqrt{(1 - \beta^2}\Big) \tag{65}$$

$$= \lambda_o\nu_o \tag{66}$$

$$\therefore \lambda = \frac{1}{\sqrt{1 - \beta^2}}\lambda_o \tag{67}$$

$$\therefore L = \frac{1}{\sqrt{1 - \beta^2}}L_o \tag{68}$$

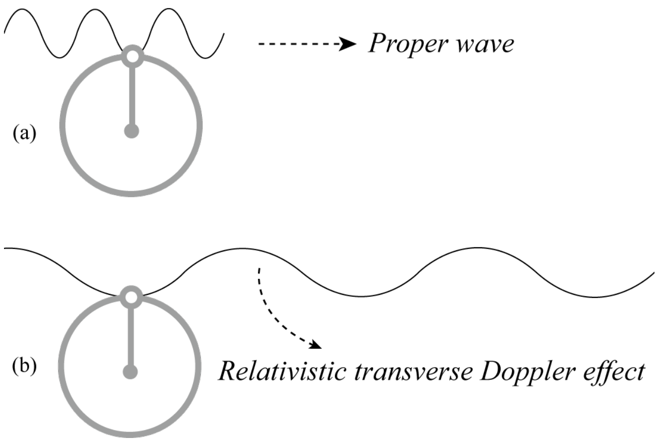

FIG. 6: Comparison of the classical Doppler effect and the relativistic transverse Doppler effect

If we accept the transverse Doppler effect as relativistic experimental evidence, it is inevitably admitted that length expansion is also correct. The transverse Doppler effect is generally expressed as Equation (63). This is only the transverse Doppler effect expressed in terms of frequency and can be expressed in terms of wavelength, as shown in (67). If the transverse Doppler effect is not expressed as a frequency but as a wavelength, it can be confirmed that the length expansion is correct immediately (68). Several more examples can experimentally prove the length expansion.

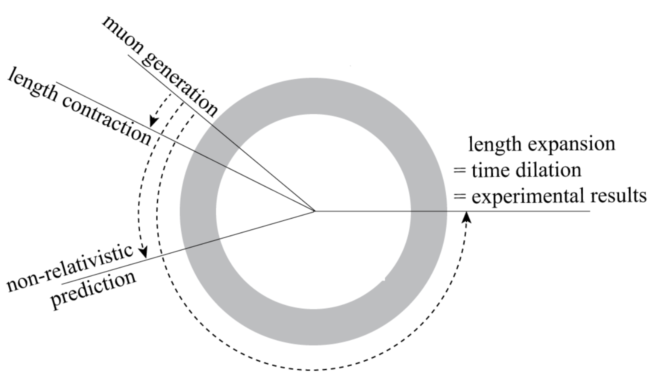

FIG. 7: Long-distance flight of muon in a particle accelerator

Second, in a particle accelerator, muon particles travel farther than is classically expected. According to Bailey's experiment, the high-speed muon particles in the particle accelerator flew about 30 times longer than the classically predicted value [9]. From a non-relativistic interpretation, the flight distance of a muon particle is short, but the actual flight distance is longer. This is not evidence of length contraction, but evidence of length expansion. This experiment has many differences in interpretation of the observation of David, the muon that falls from the sky [10]. Unlike the experiment, it is impossible to claim that the observation was made from muon's point of view. The third evidence is the special relativistic effect of GPS satellites. GPS satellites are flying at an altitude of 20,000 km above sea level at a speed of 4 km/sec per second. They flew about 10 meters in a year longer than classically predicted [11]. Since the satellite has flown farther than the classical prediction, this is not evidence of length contraction, but evidence of length expansion. Ashby passed over the effect of length contraction in the process of finding the special relativity effect of GPS [12]. This is natural. Since the length contraction phenomenon does not exist from the beginning, it is impossible to confirm this experimentally. Anyone can easily find the effect of length expansion instead of the length contraction effect. Therefore, the satellite's long-distance flight is additional evidence of length expansion.

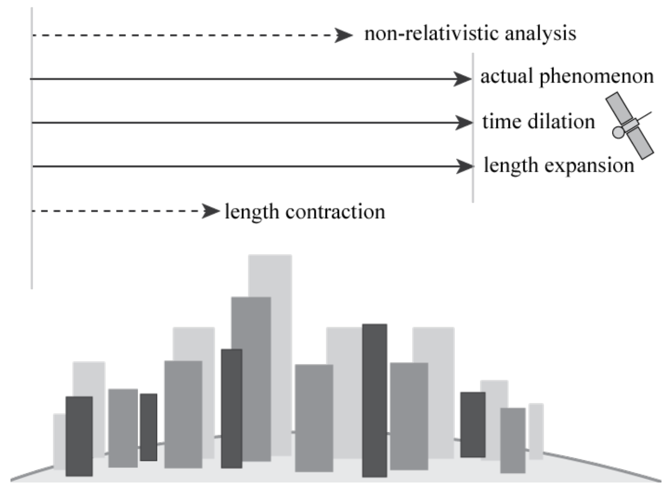

FIG. 8: Long-distance flight of muon in particle accelerator

The fourth proof of length expansion is the constancy of the speed of light. If the length contraction is correct, the speed of light is never constant (69). If the constancy of the speed of light is correct (70), equation (71) is also correct.

$$c = \frac{l_o}{t_o} = \frac{l}{t} = \frac{(1/\gamma)l_o}{\gamma t_o} = \frac{1}{\gamma^2}\frac{l_o}{t_o} \neq c \tag{69}$$

$$c = \frac{l}{t} = \frac{\gamma l_o}{\gamma t_o} = \frac{l_o}{t_o} = c \tag{70}$$

$$\therefore L = \gamma L_o \tag{71}$$

Therefore, the length should expand, not contract. When we admit that length expansion is correct, the constancy of the speed of light is no longer a mystery.

V. CONCLUSIONS

Although length contraction is recognized as a major phenomenon in the theory of relativity today, the theoretical basis and experimental evidence for length contraction are poor. Weinstein's derivation of length contraction, which is the core of length contraction, started from the wrong logic. Therefore, it should now be discarded. It was also confirmed that length expansion, not length contraction, was derived if we correctly applied the Lorentz transformation. Strangely, we cannot find definitive experimental evidence for length contraction with the advanced science and technology of today. However, if we look at it from the point of view that length expansion is correct, not length contraction, it is natural that no experimental evidence of length contraction is found. It has not been discovered because it does not exist. Many relativistic experimental results found to date support length expansion. The speed of light, GPS, muons, and the Doppler effect all support length expansion. The length contraction should be remembered only as a reference in the development of relativity. If there are people who think length contraction is right, they should logically answer the problems pointed out in this article.

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