New Gravity Field Equation is derived by Einstein Field Equation in General Relativity Theory

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ABSTRACT
We found the 4-order curvature term satisfied the co-variant derivative. Einstein gravity field equation is consist of 2-order curvature terms. Hence, the 4-order curvature term and 2-order curvature terms make new gravity field equation. In this point, Einstein’s gravity field equation can be modified by new 4-order curvature term because gravity field equation’s term doesn’t have to be 2-order term. Indeed, Einstein himself was like that, 0-order term, the cosmological term. Therefore, our theory is based on legitimate facts.

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1. Introduction
We found the 4-order curvature term satisfied the co-variant derivative. Einstein gravity field equation is consist of 2-order curvature terms. Hence, the 4-order curvature term and 2-order curvature terms make new gravity field equation. In this point, Einstein’s gravity field equation can be modified by new 4-order curvature term because gravity field equation’s term doesn’t have to be 2-order term. Indeed, Einstein himself was like that, 0-order term, the cosmological term. Therefore, our theory is based on legitimate facts. If energy-momentum tensor is zero, Einstein gravity field equation is

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \]  

(1)

Or if energy-momentum tensor is zero, the equation is add the cosmological term (0-order term)

\[ R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda g_{\mu\nu} = 0 \]  

(2)

2. Derived 4-Order Curvature Term and New Gravity Field Equation
Einstein gravity field equation is satisfied by co-variant derivative.

\[ (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)_{,\rho} = 0 \]  

(3)

Einstein’s 2-order contra-variant gravitational equation is

\[ R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = 0 \]  

(4)

Also, if energy-momentum tensor is zero and we deal with the co-variant derivative of 2-order contra-variant gravitational equation, we get

\[ (R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R)_{,\rho} = 0 \]  

(5)

If Eq(1) multiply Eq(4), then

\[ (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R) \]

\[ = R_{\mu\nu} R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R_{\mu\nu} R - \frac{1}{2} g_{\mu\nu} R^{\mu\nu} R + \frac{1}{4} g_{\mu\nu} g^{\rho\sigma} R^2 \cdot g^{\mu\rho} g^{\nu\sigma} = R, g_{\mu\nu} R^{\mu\nu} = R, g_{\mu\nu} g^{\mu\nu} = 4 \]

\[ = R_{\mu\nu} R^{\mu\nu} - \frac{1}{2} R^2 - \frac{1}{2} R^2 + R^2 = R_{\mu\nu} R^{\mu\nu} \]  

(6)

New 4-order curvature term’s co-variant derivative is

\[ (R_{\mu\nu} R^{\mu\nu})_{,\rho} = R_{\mu\nu,\rho} R^{\mu\nu} + R_{\mu\nu} R^{\mu\nu}_{,\rho} \]

\[ = \{ (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)(R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R) \}_{,\rho} \]
\[
(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) \gamma_{\rho}(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) + (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)(R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R) = 0
\]  
(7)

Hence, new gravity field equation is consist of 2-order curvature terms and new 4-order curvature term.

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda ' g_{\mu\nu} R_{\rho\sigma} R^{\rho\sigma} = - \frac{8\pi G}{c^4} T_{\mu\nu}
\]  
(8)

Or new gravity field equation is consist of 0-order term (cosmological term) and 2-order terms and 4-order term

\[
\Lambda g_{\mu\nu} + R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + \Lambda ' g_{\mu\nu} R_{\rho\sigma} R^{\rho\sigma} = - \frac{8\pi G}{c^4} T_{\mu\nu}
\]  
(9)

3. Conclusion

We found the 4-order curvature term and new gravity field equation satisfied the co-variant derivative. In new gravity field equation, Schwarzschild solution and Kerr solution do not change.

References