# Goldbach's conjecture and the double density of occupation by the union of the series of multiples of primes

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**Abstract**

The union of arithmetic progressions of primes reflected over a point at any distance from the origin, results the double density of occupation of integer positions by the series of multiples of primes. It is shown, that the number of free positions left by the double density of occupation has a lower limit function. These free positions represent equidistant primes satisfying Goldbach's conjecture. Herewith may be proved as well, that at any distance from the origin, within the section equal to the square root of the distance, there is a prime. Therefore the series of primes represent a continuum and may be integrated. Further it may be proved, that the number of any two primes, with a given even number as difference between them, is unlimited. Thus, the number of twin primes is unlimited as well.

## 1. The number of free positions left in case of the double density of occupation

The total number of the primes is the integral of the local logarithmic density of free positions evaluated by Riemann. The first approximation of the integral is the sum of the logarithmic density over all integers, in the following used as **sum over all integers**:

$$\pi(c) = \int_2^c \frac{1}{\ln(c)}\,dc \qquad \pi_{\ln\_appr}(c) \approx \sum_{n=2}^{c} \frac{1}{\ln(n)} \tag{1.1}$$

This above sum may be written as summing up first over all integers within the sections of the length ( $\sqrt{c}$ ) and then summing up over all the ( $\sqrt{c}$ ) sections of the length ( $\sqrt{c}$ ). Taking the average value over each section and summing up over the sections is an approximation, in the following used as **sum over all sections.**

$$\sum_{n=2}^{c} \frac{1}{\ln(n)} = \sum_{j=2}^{\mathrm{floor}(\sqrt{c})} \left[ \sum_{n=\mathrm{ceil}[(j-1)\cdot\sqrt{c}]}^{\mathrm{floor}(j\cdot\sqrt{c})} \frac{1}{\ln(n)} \right] \approx \sum_{j=1}^{\mathrm{floor}(\sqrt{c})} \frac{\sqrt{c}}{\ln(j\cdot\sqrt{c})} = \pi_{appr}(c) \tag{1.2}$$

The well proven prime-number-formula (PNF) results as simplification of the above approximation taking for all sections the largest value ( $j = \sqrt{c}$ ):

$$\sum_{j=1}^{\mathrm{ceil}(\sqrt{c})} \frac{\sqrt{c}}{\ln(j\cdot\sqrt{c})} > \sum_{j=1}^{\mathrm{ceil}(\sqrt{c})} \frac{\sqrt{c}}{\ln(\sqrt{c}\cdot\sqrt{c})} = \frac{c}{\ln(c)} = \pi_{PNF}(c) \tag{1.3}$$

The results of the PNF are for all distances smaller, then the best estimate value resulting from the approximation (1.2), because all terms of the addition are smaller:

$$\frac{\sqrt{c}}{\ln(j\cdot\sqrt{c})} > \frac{\sqrt{c}}{\ln(\sqrt{c}\cdot\sqrt{c})} \tag{1.4}$$

The PNF is well proven, therefore the method of addition over sections (1.3) is proven as well, because it results as low limit the PNF.

The series of multiples (arithmetic progression) of any prime ( $P_{(n)}$ ) together with the same series reflected over any point ( $C$ ) at the distance ( $c$, $\mathrm{mod}[c, P_{(n)}] > 0$ ) from the origin, cover a subset of all integer positions. The positions covered by the straight and by the reflected series of multiples of a prime are mutually exclusive: Together they cover the ( $2 / P_{(n)}$ )-th part of all integer positions. The union of the first ( $R(c) = \sqrt{c} / \ln(\sqrt{c})$ ) series of multiples of primes and of their reflected series results the **double density of occupation.** Pairs of the remaining, symmetrically placed free positions - left by the double density of occupation - compose the set of **central triads** around the point of reflexion at the distance ( $c$ ). This methode may be regarded as a special case of the sieve proposed by Chen Ref. [1].

The share of the integer positions left free by each of the series of multiples of primes is: ( $1 - \frac{2}{P_{(n)}}$ ). The density of free positions left by the series of multiples of the first prime is ( $1/2$ ). The density of free positions is multiplicative, therefore the density of free positions left by the double density of occupation of the first ( $R(c) = \sqrt{c} / \ln(\sqrt{c})$ ) primes, at the distance ( $c$ ) from the origin is:

$$\frac{1}{2}\cdot\left[\prod_{n=2}^{R(c)}\left(1-\frac{2}{P_n}\right)\right] \quad (1.5)$$

This again under the condition that the distance of the point of reflection ( $c$ ) is relative prime to all ( $P_{(n)}$ ) primes, meaning ( $c$ ) it is a prime. If the point of reflection ( $c$ ) is divisible by any of the ( $K$ )-th prime with ( $P_{(2)} < P_{(K)} < P_{(R(c))}$ ), then the reflected series of multiples of this prime do not cover any additional positions over the number already covered by the single series of multiples of the same prime. The local density of free positions is in this case:

$$\frac{1}{2}\cdot\left[\prod_{n=2}^{K-1}\left[1-\frac{2}{P_{(n)}}\right]\right]\cdot\left[1-\frac{1}{P_{(K)}}\right]\cdot\left[\prod_{n=K+1}^{R(c)}\left[1-\frac{2}{P_{(n)}}\right]\right] > \frac{1}{2}\cdot\left[\prod_{n=2}^{R(c)}\left[1-\frac{2}{P_{(n)}}\right]\right] \quad (1.6)$$

**Lemma 1.1:**

**The density of free positions left by the double density of occupation of primes has a minimum in case the point of reflection is a prime ( $c = P_{(n)}$ ).**

**Proof:** The series of multiples of any prime ( $P_{(n)}$ ) - together with its reflected series - occupy the ( $2/P_{(n)}$ )-th part of all integer positions, if the point of reflection ( $c$ ) is a relative prime to all primes ( $P_{(n)}$, $n \le R(c)$ ). Otherwise - for the prime dividends of ( $c$ ) - only a share of ( $1/P_{(n)}$ ). Therefore the share of the remaining free positions has a minimum, if the point of reflection is relative prime to all primes, meaning it is a prime itself, as stated in the lemma and **concluding the proof.**

**Definition of triads and diads**

If the point of reflection is a prime, than the positions left free by the double density of occupation represent primes, which are equidistantly placed around the point of reflection. Two primes equidistantly placed around the prime at the point of reflexion compose a **central triad**. The distance of the primes at the center of the triads is the mean value of the distances of the two other primes from the origin. In case the point of reflection is not a prime, then the density of free positions at this point is the density of **central diads.**

If the distance of the point of reflexion ( $c$ ) is equal to a prime, than the local density of free positions left by the straight series of multiples at the distance ( $d$, $2 < d < c$ ) from the point of reflexion is ( $\frac{1}{\ln(c-d)}$ ). The density of the reflected series is ( $\frac{1}{\ln(c+d)}$ ). The combined local density of free positions is:

$$D_{diad}(c,d) = \frac{\delta_2}{\ln(c-d)\cdot\ln(c+d)} \quad (1.7)$$

Here the factor of correction ( $\delta_2$ ) had to be introduced in order to enable the use of the square of the inverse of the logarithm of the distance for the double density of occupation at the distance ( $c$ ):

$$\frac{1}{2}\cdot\lim_{c\to\infty}\left[\prod_{n=2}^{R(2\cdot c)}\left[1-\frac{2}{P_{(n)}}\right]\right] = \frac{\delta_2(c)}{4}\cdot\lim_{c\to\infty}\prod_{n=2}^{R(2\cdot c)}\left[1-\frac{1}{P_{(n)}}\right]^2 = \delta_2(c)\cdot\lim_{c\to\infty}\prod_{n=1}^{S(c)}\left[1-\frac{1}{P_{(n)}}\right]^2 = \frac{\delta_2}{\ln(c)^2} \quad (1.8)$$

The value of the constant quickly converges to ( $\delta_2 := 1.3203348166$ ). It is evaluated in annex A1. Replacing the distance of the center of reflexion in (1.7) by the distance from the origin ( $j\cdot\sqrt{c}$ ), respectively from the distance ( $2\cdot c - j\cdot\sqrt{c}$ ) gives the density of diads:

$$D_{diad}(c,j) := \frac{\delta_2}{\ln\left(j\cdot\sqrt{c}\right)\cdot\ln\left(2\cdot c - j\cdot\sqrt{c}\right)}$$

With lemma 1.1 the the number of triads corresponds to the minimum of the number of diads. Therefor in the following the minimum, the number of triads will be evaluated.

Summing up the local density of diads over all ( $\sqrt{c}$ ) section, multiplied by the density of the prime in the centrum ( $1/\ln(c)$) results the best estimate number of triads up to the distance ( $c$ ):

$$\tau_{c\_appr\_}(c) := \frac{\sqrt{c}}{\ln(c)} \cdot \sum_{j=1}^{\text{ceil}(\sqrt{c})} \frac{\delta_2}{\ln(j\cdot\sqrt{c})\cdot\ln(2\cdot c - j\cdot\sqrt{c})} = \frac{\sqrt{c}}{\ln(c)} \cdot \sum_{j=1}^{\text{ceil}(\sqrt{c})} D_{diad}(c,j) \qquad (1.9)$$

The lower limit of the best estimate number of triads results with the density taken for all sections of the length ( $\sqrt{c}$ ) at the distance of the upper limit ( $j = \sqrt{c}$ ), yielding the **triads-number-formula (TNF)**:

$$\tau_{c\_appr\_low\_}(c) = \frac{1}{\ln(c)} \cdot \sum_{j=1}^{\text{floor}(\sqrt{c})} \frac{\delta_2\cdot\sqrt{c}}{\ln(c)\cdot\ln(2\cdot c - c)} = \frac{\delta_2\cdot\sqrt{c}}{\ln(c)^3}\left(\sum_{j=1}^{\text{floor}(\sqrt{c})} 1\right) = T_{c\_TNF}(c) := \frac{\delta_2\cdot c}{\ln(c)^3} = \frac{\delta_2}{4}\cdot\frac{R(c)^2}{\ln(c)} \qquad (1.10)$$

The method replacing the local density in the formula by the density at the upper limit of the distance in the formula of the best estimate number of triads is the same, which results the prime-number-formula as low limit of the best estimate number of primes. The fact, that the PNF is well proven, proves the TNF as well.

Similarly the best estimate number of triads with one component within the first section ( $\sqrt{c}$ ) - equal to the number of central triads with one component within the last section to ( $2\cdot c$ ) - is evaluated:

$$\tau_{c\_last\_appr\_}(c) := \frac{\sqrt{\sqrt{c}}}{\ln(c)} \cdot \sum_{j=1}^{\text{ceil}(\sqrt{\sqrt{c}})} \frac{\delta_2}{\ln(j\cdot\sqrt{\sqrt{c}})\cdot\ln(2\cdot c - \sqrt{j\cdot\sqrt{\sqrt{c}}})} \qquad (1.11)$$

The lower limit of the approximating function results - similarly to (1.10) - with the density taken at its upper limit ( $j = \sqrt{\sqrt{c}}$ ) for all sections of the length ( $\sqrt{\sqrt{c}}$ ), yielding the **triads-number-formula-last (TNFL)**:

$$\tau_{c\_last\_appr\_low\_}(c) = \frac{1}{2\cdot\ln(\sqrt{c})} \cdot \sum_{j=1}^{\text{floor}(\sqrt{\sqrt{c}})} \frac{\delta_2\cdot\sqrt{\sqrt{c}}}{\ln(\sqrt{c})\cdot\ln(2\cdot c - \sqrt{c})} = \frac{\delta_2\cdot\sqrt{\sqrt{c}}}{2\cdot\ln(\sqrt{c})^2\cdot\ln(2\cdot c - \sqrt{c})}\left(\sum_{j=1}^{\text{floor}(\sqrt{\sqrt{c}})} 1\right) >$$

$$> T_{c\_TNFL}(c) := \frac{\delta_2\cdot\sqrt{c}}{2\cdot\ln(\sqrt{c})^2\cdot\ln(2\cdot c)} = \frac{\delta_2}{8}\cdot\frac{R(\sqrt{c})^2}{\ln(2\cdot c)} \qquad (1.12)$$

The effective number of triads ( $\tau_{c\_eff}(c)$) and ( $\tau_{c\_last\_eff}(c)$) are evaluated in annex A2.They are compared with the corresponding best estimate approximations.

The dispersion of the effective values of the number of triads around the approximating functions divided by the number of primes at the square root of the distance ( $\pi_{appr}(\sqrt{c})$) results the relative dispersion. The following properties of the relative dispersion are evaluated in A3:

It is found, that the standard deviation of the relative dispersion is approaching for the triads up to ( $c$ ) and for the triads within the last section up to ( $2\cdot c$ ) - for large distances - a constant value:

$$SD_{\Delta\tau\_rel}(c) = \sqrt{\frac{1}{c}\cdot\sum_{c}\left(\frac{\tau_{c\_appr}(c) - \tau_{c\_eff}(c)}{R(c)}\right)^2} \;;\; F_{SD\_\Delta\tau\_rel} = SD_{\Delta\tau\_rel}(c_{lim}) = 0.018035 \qquad (1.13)$$

$$SD_{\Delta\tau\_last\_rel}(c) = \sqrt{\frac{1}{c}\cdot\sum_{c}\left(\frac{\Delta\tau_{c\_last}(c)}{R(\sqrt{c})}\right)^2} \;;\; F_{SD\_\Delta\tau\_last\_rel} = SD_{\Delta\tau\_last\_rel}(c_{lim}) = 0.0176597$$

As illustrated in annex 3, figure (A3.2) the constancy of this factor is only valid for large distances: there is an initial effect present for smaller distances. This effect is the reason for the slight difference between the factor for the standard deviation of the dispersion of the number of central triads up to the distance ( $c$ ) and of the same factor up to the distance ( $\sqrt{c}$ ). For distances rising without limit the standard deviation of the dispersion approaches a value very close to the first value:

$$\lim_{c\to\infty} SD_{\Delta\tau\_rel}(c_{last\_rel}) = \lim_{c\to\infty} SD_{\Delta\tau\_rel}(c) = \lim_{c\to\infty} SD_{\Delta\tau\_rel}(c_{lim}) = 0.018035 \qquad (1.14)$$

The constancy of the standard deviation of the relative dispersion of the primes over the distance results from the fact, that the density of primes decreases with the inverse of the logarithmic of the distance. Therefore the distance between the primes - on the average - increases with the logarithm of the distance. With the logarithmically increasing distance the dispersion is increasing proportionally to the - with the distance rising - number of primes ( $R(c)$).

From the constancy of the standard deviation of the relative dispersion follows the **congruency of the relative dispersion** of the effective number of primes around their best estimate value, over the distance.

Further it is found, that the difference between the best estimate value and the value resulting from the TNF is proportional to the TNF, divided by the logarithm of the distance. The factor of proportionality is:

$$F_{\Delta\tau\_TNF} = \left(\tau_{c\_appr}(c) - \tau_{c\_TNF}(c)\right)\cdot\frac{\ln(c)}{\tau_{c\_appr}(c)} = \left(1 - \frac{\tau_{c\_TNF}(c)}{\tau_{c\_appr}(c)}\right)\cdot\ln(c) = F_{\Delta\tau\_TNF} := 0.697952 \qquad (1.15)$$

The TNF from (1.10) corrected correspondingly to the difference between the best estimate number of primes and the TNF gives the corrected value:

$$T_{c\_TNF\_corr}(c) = T_{c\_TNF}(c) + F_{\Delta\tau\_TNF}\cdot\frac{\delta_2\cdot c}{\ln(c)^4} = \frac{\delta_2\cdot c}{\ln(c)^3} + F_{\Delta\tau\_TNF}\cdot\frac{\delta_2\cdot c}{\ln(c)^4} = T_{c\_TNF\_corr}(c) := \frac{\delta_2\cdot c}{\ln(c)^3}\cdot\left(1 + F_{\Delta\tau\_TNF}\cdot\frac{1}{\ln(c)}\right)$$

Both above constant factors (1.14) and (1.15) represents **inherent properties of the double density of occupation, respectively of the prime numbers.**

## 2. The low limit of the number of triads

Up to now basically the phenomenology of the double density of occupation by the union of the series of multiples of primes was analyzed. Now the consequences will be summarized. First the following lemmas may be formulated:

**Lemma 2.1:**

**The difference between the best estimate value of the number of triads and the value of the TNF over the distance is always greater then the local value of the dispersion of the effective number of triads around the best estimate value: The TNF represents the low limit of the effective number of triads.**

**Proof:** The difference between the value of the best estimate of the effective number of triads and the value of the (TNF) is with (1.15) growing proportional to ( $\frac{R(c)^2}{4\ln(c)^2} = \frac{c}{\ln(c)^4}$ ). The standard deviation and therefore the width of the dispersion of the number of triads is growing proportionally to the number of the series of multiples ( $R(c)$ ), which are covering integer positions at the distance ( $c$ ), (illustrated in annex 3, figure (A3.2).

For any sufficiently large distance ( $c_{limit}(K) < c$ ) the difference between the value of the effective number of triads and the value of the (TNF) outgrows the width of the dispersion ( $K_{\Delta B}\cdot R(c)$ ), in case of any value of the constant ( $K_{\Delta B}$ ):

$$\frac{\tau_{c\_appr}(c) - T_{c\_TNF}(c)}{K_{\Delta B}\cdot R(c)} = \frac{\Delta\tau_{c\_TNF}(c)}{K_{\Delta B}\cdot R(c)} = \frac{F_{\Delta\tau.c\_T.c\_TNF}\cdot\frac{T(c)}{\ln(c)}}{K_{\Delta B}\cdot R(c)} > \frac{F_{\Delta\tau.c\_T.c\_TNF}}{4\cdot K_{\Delta B}}\cdot\frac{R(c)}{\ln(c)}$$

$$\lim_{v\to\infty}\left(\frac{F_{\Delta\tau.c\_T.c}}{4\cdot K_{\Delta B}}\cdot\frac{R(c)}{\ln(c)}\right) = \infty$$

From this follows, that the effective number of central triads has a lower limit function, the TNF divided by the logarithm of the distance, which is growing to infinity, **as stated in the lemma and concluding the proof.**

**Lemma 2.2:**

**The number of free positions left by the double density of occupation by the series of multiples of ( $R(2\cdot c)$ ) primes, within the first and last sections of the length ( $\sqrt{c}$ ) up to ( $2\cdot c$ ) is proportional to the number of the series of multiples of primes, which cover positions at ( $c$ ), multiplied by the density of primes at the distance ( $c$ ), is growing without limit with the distance.**

**Proof:** The triads-number-formula-last (1.12) - gives the low limit of the approximation of the number of triads at ( $c$ ), evaluated up to ( $\sqrt{c}$ ) and equal to the same number evaluated over the last section up to ( $2\cdot c$ ):

$$\tau_{c\_last\_appr\_low\_}(c) = \frac{1}{2\cdot\ln(\sqrt{c})}\cdot\sum_{j=1}^{\mathrm{floor}\left(\sqrt{\sqrt{c}}\right)}\frac{\delta_2\cdot\sqrt{\sqrt{c}}}{\ln(\sqrt{c})\cdot\ln(2\cdot c-\sqrt{c})} > \frac{\delta_2\cdot\sqrt{c}}{2\cdot\ln(\sqrt{c})^2\cdot\ln(2\cdot c)} = \frac{\delta_2}{8}\cdot\frac{R(\sqrt{c})^2}{\ln(2\cdot c)}$$

The prime-number-formula PNF for the distance from the origin ( $\sqrt{c}$ ) gives the number of primes up to ( $R(\sqrt{c})$ ). It is growing without limit. Its square, divided by ( $\ln(2\cdot c)$ ) is growing monotonously, without limit as well, as stated in the lemma and **concluding the proof.**

The congruency of the relative dispersion - the dispersion of the number of central triads around its best estimate value, divided by the number of the series of multiples ( $R(c)$ ), which are covering integer positions at the distance ( $c$ ) - over the distance is an important property of the set of primes.

The number of central triads up to the distance ( $c$ ) grows with the distance to infinity, since it has a monotonously and continuously to infinity growing lower limit function.

In case the central element of a triad is not a prime, but any positive even number between neighboring primes, than with lemma 1.1 the number of equidistant primes is a greater number, then in case of any of the neighboring primes. Thus the number of diads grows to infinity as well. The monotonously growing low limit of the number of the diads is with lemma 2.1 equal to ( $\frac{\delta_2 \cdot R(c)}{2 \cdot \ln(c)}$ ), **proving Goldbach's conjecture.**

**Lemma 2.3:**

**Within the last section ( $\sqrt{c}$ ) at the distance ( $c$ ), the low limit of free positions left by the union of the series of multiples of the first ( $R\left(\frac{c}{2}\right)$ ) primes is - for large distances - equal to ( $\frac{2 \cdot \delta_2 \cdot R\left(\frac{c}{2}\right)^2}{\ln\left(\frac{c}{2}\right)}$ ).**

**Proof:** With lemma 2.1 and 2.2, in case of the double density of occupation, at the distance ( $c$ ), within the last section of the length ( $\sqrt{c}$ ), the low limit of the number of free positions left is equal to ( $\frac{2 \cdot \delta_2 \cdot R\left(\frac{c}{2}\right)^2}{\ln\left(\frac{c}{2}\right)}$ ).

But this is certainly the case within the section of the same length, at the same distance, in case of the single density of occupation by the straight series of multiples alone, **as stated in the lemma and concluding the proof.**

**Lemma 2.4:**

**In case of the union of the series of multiples of the first ( $R\left(\frac{c}{2}\right)$ ) primes the low limit of free positions left within the section of the length ( $\sqrt{c}$ ) following any distance ( $c$ ) is ( $\frac{2 \cdot \delta_2 \cdot R\left(\frac{c}{2}\right)^2}{\ln\left(\frac{c}{2}\right)}$ ).**

**Proof:** With lemma 2.3 within the interval of the length ( $\sqrt{c}$ ) at the distance ( $c$ ), the low limit of free positions left by the union of the series of multiples of ( $R\left(\sqrt{\frac{c}{2}}\right)$ ) primes is ( $\frac{2 \cdot \delta_2 \cdot R\left(\frac{c}{2}\right)^2}{\ln\left(\frac{c}{2}\right)}$ ). But this is the case within the next section of the same size following ( $c$ ) as well, because the first free position covered by the series of multiples of the smallest possible prime greater then ( $P_{(R(c))}$ ) - equal to ( $P_{(R(c)+1)}$ ) - is already greater then ( $c + \sqrt{c}$ ), as shown below:

The smallest prime, the series of which is covering positions following ( $c$ ) is ( $P_{(R(c)+1)} = P_{(R(c))} + 2$ ), in case ( $P_{(R(c))}$ ) and ( $P_{(R(c)+1)}$ ) are twin primes. In this case is ( $P_{(R(c))} \geq \sqrt{c} - 1$ ) and the square of this smallest prime is already outside of ( $c + \sqrt{c}$ ):

$$\left[P_{(R(c)+1)}\right]^2 = \left[P_{(R(c))} + 2\right]^2 = \left[P_{(R(c))}\right]^2 + 4 \cdot P_{(R(c))} + 4 \geq \left(\sqrt{c} - 1\right)^2 + 4 \cdot \left(\sqrt{c} - 1\right) + 4 =$$

$$= c - 2 \cdot \sqrt{c} + 1 + 4 \cdot \left(\sqrt{c} - 1\right) + 4 = c + 2 \cdot \sqrt{c} + 1 > c + \sqrt{c}$$

Herewith the low limit of free positions left within the section ( $\sqrt{c}$ ) following ( $c$ ) is the same, as within the last section just below ( $c$ ), **as stated in the lemma and concluding the proof.**

## 3. Consequences of the double density of occupation

Based on the lemmas 2.3 and 2.4, the following law may be formulated:

**Prime distance law:**

**Taking the arbitrary integer ($m$, $4 \le m$), than the series of multiples of all primes up to ($P_{(R(m))}$) are not able to occupy all integer positions within the interval ($m^2 .. (m+1)^2$): there remain at least one position free, representing a prime number.**

**Proof:** Within the interval of the length ($(m+1)^2 - m^2$) there is one section of the length ($m+1$):

$$\frac{(m+1)^2 - m^2}{m+1} = \frac{2 \cdot m + 1}{m+1} = 1 + \frac{m}{m+1} > 1$$

With lemma 2.4 within this section there is a position free from coverage by the union of the series of multiples of the first ($P_{(R(m+1))} \le m+1$) primes, which are covering positions up to ($(m+1)^2$) as stated in the prime distance law and **concluding the proof.**

With lemma 2.4 within each of the sections of the length ($\sqrt{n}$) up to ($n + \sqrt{n}$) there is a position free from the double coverage of the union of the series of multiples of the first ($R(n)$) primes. This is valid for the larger distance of the length ($2 \cdot \sqrt{n}$) above ($n$) too. This has far reaching consequences: The set of the series of multiples of all primes up to ($P_{(R(c))}$), normed with the size of this last prime, represents - as limit - a continuum, since for any prime within the set, the following limit is valid:

$$\frac{P_{(p+1)} - P_{(p)}}{P_{(p+1)}} \le \frac{\left[P_{(p)} + 2 \cdot \sqrt{P_{(p)}}\right] - P_{(p)}}{P_{(p+1)}} = \frac{2 \cdot \sqrt{P_{(p)}}}{P_{(p+1)}} < \frac{2 \cdot \sqrt{P_{(p+1)}}}{P_{(p+1)}} \; ; \; \lim_{c \to \infty} \frac{2}{\sqrt{P_{(p+1)}}} = 0 \qquad (3.1)$$

## 4: Density of occupation of subsets of all integers by the union of the series of multiples of primes

There is an other interesting consequence of the double density of occupation by the reflected series of multiples of the primes: It is the proof of the infinity of the number of diads with any given even number as distance between their components.

**Definition of subsets of all integers:**

With the infinite number of integers ( $m \in Z$, $Z = 1, 2 .. \infty$ ) all integers are part of one of the following infinite subsets:

$$N_{sub(m,a)} = \{ a + m \cdot P_{(1)} \cdot P_{(2)} \; ; \; a = -1, 0, 1, 2, 3, 4 \} \qquad (4.1)$$

All integers in the subsets for ( $a = 0, 2, 3, 4$ ) are divisible either by ( $P_{(1)}$ ), and/or by ( $P_{(2)}$ ). Therefore all primes ( $P_{(n)}$, { $n = 3, 4, 5, ...$ }) are members of one of the infinite subsets ( $a = -1$ ) or ( $a = 1$ ), having the distance ( $\frac{N}{2} = 1$ ) to the members of the set ( $a = 0$ ).

Members of the subset ( $a = 0$ ) with one of their neighboring positions equal to a prime compose the infinite subsets of these subsets:

$$N_{sub(m,-1,n1)} = \{ a + m \cdot P_{(1)} \cdot P_{(2)} = P_{(n1)} \; ; \; a = -1 \} \; ; \; N_{sub(m,1,n2)} = \{ a + m \cdot P_{(1)} \cdot P_{(2)} = P_{(n2)} \; ; \; a = 1 \} \qquad (4.2)$$

The section of the above sets define an infinite subset of the subset ( $a = 0$ ), with primes at both neighboring positions having the distance between them equal to ( $N = 2$ ), meaning they are **twin primes**:

$$N_{tw(m,n1,n2)} = \{ N_{sub(m,-1,n1)} \cap N_{sub(m,1,n2)} \} \qquad (4.3)$$

The distance of the center point of the diads to the origin is named in the following the **distance of the diads**. The distance between the primes composing the diads is named in the following the **distance of the components**. The distance of the components may be any even number. The only condition is, that the half-distance of the components has to be smaller, then the distance of the diad ( $N/2 < c$ ). With (1.9) and (1.0) the number of diads and their low limit - given by the diads-number-formula are - the following:

$$\pi_{diad\_appr}(c) = \sum_{j=1}^{ceil(\sqrt{c})} \frac{\delta_2 \cdot \sqrt{c}}{\ln(j \cdot \sqrt{c}) \cdot \ln(2 \cdot c - j \cdot \sqrt{c})} \;;\; \pi_{diad\_appr\_low\_}(c) = \frac{\delta_2}{4} \cdot R(c)^2 \qquad (4.4)$$

In the following it is listed, which values may take the distances of the components ( $N$ ) of the diads to any given distances of the diads, belonging to each of the subsets ( $a = 0, 1, 2, 3, 4, 5$ ).

In case of the distance of the diads ( $c$ ) member of the set ( $a = 0$ ), the following distances of the components are possible:

$$\frac{N}{2} = 1,\ 7,\ 13,\ 19, \dots \;;\; N(o,m) = (m-1) \cdot 6 + 1 \;;\; \frac{N}{2} < c(n,0) = n \cdot 6 \;;\; m = 1, 2, 3, 4, \dots \;;\; n = 1, 2, 3, 4, \dots \qquad (4.5)$$

$$\frac{N}{2} = 5,\ 11,\ 17, \dots \;;\; N(0,m) = (m-1) \cdot 6 + 5 \;;\; \frac{N}{2} < c(n,0) = (n+1) \cdot 6 \;;\; m = 1, 2, 3, 4, \dots \;;\; n = 1, 2, 3, 4, \qquad (4.6)$$

In case of the distance of the diads member of the set ( $a = 1$ ), the following distances of the components are possible:

$$\frac{N}{2} = 6,\ 12,\ 18, \dots \;;\; N(1,m) = m \cdot 6 \;;\; \frac{N}{2} < c(n,1) = 1 + (n+1) \cdot 6 \;;\; m = 1, 2, 3, 4, \dots \;;\; n = 1, 2, 3, 4, \dots \qquad (4.7)$$

In case of the distance of the diads member of the set ( $a = 2$ ), the following distances of the components are possible:

$$\frac{N}{2} = 3,\ 9,\ 15, \dots \;;\; N(2,m) = m \cdot 6 + 3 \;;\; \frac{N}{2} < c(n,2) = 2 + (n+1) \cdot 6 \;;\; m = 1, 2, 3, 4, \dots \;;\; n = 1, 2, 3, 4, \dots \qquad (4.8)$$

In case of the distance of the diads member of the set ( $a = 3$ ), the following distances of the components are possible:

$$\frac{N}{2} = 4,\ 10,\ 16, \dots \;;\; N(3,m) = m \cdot 6 + 4 \;;\; \frac{N}{2} < c(n,3) = 3 + (n+1) \cdot 6 \;;\; m = 1, 2, 3, 4, \dots \;;\; n = 1, 2, 3, 4 \dots \qquad (4.9)$$

$$\frac{N}{2} = 2,\ 8,\ 14, \dots \;;\; N(3,m) = m \cdot 6 + 2 \;;\; \frac{N}{2} < c(n,3) = 3 + (n+1) \cdot 6 \;;\; m = 1, 2, 3, 4, \dots \;;\; n = 1, 2, 3, 4 \dots \qquad (4.10)$$

In case of the distance of the diads member of the set ( $a = 4$ ), the set of the possible distances correspond to the distances in case of ( $a = 2$ ).

Similarly, in case of the distance of the diads member of the set ( $a = 5$ ), the set of the possible distances correspond to the distances in case of ( $a = 1$ ).

The union of all half distances of the components ( $\frac{N}{2}$ ) listed in (4.5) through (4.10), as well as the distances of the diads ( $c$ ) include all integers.

Therefore the set of distances between primes composing diads ( $N$ ) includes all even numbers and grows to infinity with the distance of the center point of the diads ( $c$ ). Differently expressed: The number of diads - at any distance, with the corresponding even number as distance of their components - is unlimited. With (4.4) the **low limit of the number of diads - including twin primes - is given by the diads-number-formula and grows to infinity.**

$$\pi_{diad\_appr}(n,a) = \sum_{j=1}^{ceil(\sqrt{c(n,a)})} \frac{\delta_2 \cdot \sqrt{c(n,a)}}{\ln(j \cdot \sqrt{c(n,a)}) \cdot \ln(2 \cdot c(n,a) - j \cdot \sqrt{c(n,a)})} \;;\; \pi_{diad\_appr\_low\_}(n,a) = \frac{\delta_2}{4} \cdot R(c(n,a))^2 \qquad (4.11)$$

As an example for the diads the twin primes are evaluated as a special case of (4.5), with ( $\frac{N}{2} = 1$ ).

For ( $c(n) = n \cdot 6$ ) the density of occupation of twins is:

$$D_{tw\_freq}(c,j) = \frac{\delta_2}{\ln(j \cdot \sqrt{c} - 1) \cdot \ln(j \cdot \sqrt{c} + 1)} \qquad (4.12)$$

The approximating function of the total number of twins up to ( $c$ ), with density in case of the double occupation by the union of the series of multiples of the primes and with the factor of correction gives the **twin-number-formula** as well:

$$TW_{appr\_}(c) = \delta_2\cdot\sqrt{c}\cdot\sum_{j=1}^{floor(\sqrt{c})} D_{tw\_free}(c,j) = \delta_2\cdot\sqrt{c}\cdot\sum_{j=1}^{floor(\sqrt{c})}\left(\frac{1}{\ln(j\cdot\sqrt{c})^2}\right) > TW(c) := \frac{\delta_2\cdot c}{\ln(c)^2} \qquad (4.13)$$

The evaluation and the comparison with the effective number twins are carried out in Annex 4.

## Annexes

All above formula are checked in the following annexes with numeric results. The annexes together compose executable files in MATHCAD from PTC. The syntax corresponds to the MATHCAD syntax.

## A0: General data, vectors and functions

The set of primes and the listed known formula below is used for this checking. Some vectors of the results with time consuming evaluation are evaluated once and the results are saved reading later.

The set of primes is read from a file: ( $P := READPRN("Primes\_large.prn")$ ). The number of primes in the set and their numbering are: ( $Np := rows(P) - 1 = 5003713$ , $n := 1,2..Np$ ).

The number of primes up to ( $c$ ) is approximated with the prime-number-formula. At ( $c$ ) only the multiples of the primes up to ( $\sqrt{c}$ ) are covering free integral positions. The numbers of these primes are:

$$P_{(S(c))} < c < P_{(S(c)+1)} \quad ; \quad P_{(R(c))} < \sqrt{c} < P_{(R(c)+1)} \qquad (A0.1)$$

$$\pi_c(c) > S(c) := floor\left(\frac{c}{\ln(c)}\right) \quad ; \quad \pi_c(\sqrt{c}) > R(c) := floor\left(\frac{\sqrt{c}}{\ln(\sqrt{c})}\right) \approx \frac{\sqrt{c}}{\ln(\sqrt{c})}$$

For the evaluation of the number of free positions up to the distance ( $c$ ) the routine ( $n_{next}(c, n_{last})$ ) resulting the index ( $n$ ) of the prime next to any integer is needed ( $P_{(n)} \le c < P_{(n+1)}$ ). The evaluation starts either at the last evaluated index ( $n_{last}$ ), or at the index resulting from the prime-number-formula. This, in order to shorten some of the evaluation processes.

Further functions are the formula evaluating the index of the next smaller prime and of the next smaller twin prime to any distance as well as the effective number of primes up to any distance:

(A0.2)

```
n_next_P_(c, n_last) := | if c ≠ 0
                        |   | n ← S(c)  if n_last = 1
                        |   | n ← n_last  otherwise
                        |   | while P(n) ≤ c
                        |   |   n ← n + 1
                        |   | Res ← n − 1
                        | Res ← 0  otherwise

n_next_TW_(c, TW) := | if c > 0
                     |   | nn ← floor(δ2·c / ln(c)^2)
                     |   | while TW(nn) ≤ c
                     |   |   nn ← nn + 1
                     | Res ← 0  otherwise
                     | Res ← nn − 1
```

$S_{eff}(c) := n_{next\_P\_}(c, 1) \qquad R_{eff}(c) := n_{next\_P\_}(\sqrt{c}, 1)$

In the following all functions, which are evaluated for illustration, are evaluated at sparse distances, equal to multiples of the square root of the largest distance considered ( $c_{sp_{(k)}} = k\cdot\sqrt{P_{(Np)}}$ ), resp. at the next smaller prime ( $P_{\left[n_{sp_{(k)}}\right]} < c_{sp_{(k)}} < P_{\left[n_{sp_{(k+1)}}\right]}$ ):

$$\Delta c_{sp} := \sqrt{P_{(Np)}} \; ; \; k_{limit} := floor\left[\frac{P_{(Np)}}{\Delta c_{sp}}\right] - 1 = 9277 \; ; \; k := 1,2..k_{limit} \; ; \; c_{sp_{(k)}} := k\cdot\Delta c_{sp} \qquad (A0.3)$$

$$\pi(c) = S_{eff}(c) \; ; \; n_{sp_{(k)}} := S_{eff}\left[c_{sp_{(k)}}\right] \; ; \; kk_{limit} := floor\left(\frac{k_{limit}}{2}\right) - 1 \; ; \; kk := 1,2..kk_{limit} \; ; \; kk_{limit} = 4637$$

The values are evaluated once at sparse distances and written to a file. They are read from this file: ( $WRITEPRN("index\_sparse\_primes.prn") := n_{sp}$ ) ; ( $n_{sp} := READPRN("index\_sparse\_primes.prn")$ ).

## A1: Evaluation of the constant of the density of diads

The constant in (1.4) is evaluated the following way: The factor ( $\delta_{2(c)}$) expressed from (1.4), the relation below quickly converges to a constant value as limit, resulting the factor of correction. Its graphical representation for the sparse values ( $n_{sp_{(k)}}$) is shown below:

$$\delta_2 = \lim_{S \to \infty} \left[ \frac{1 - \frac{1}{P_{(1)}}}{\left[1 - \frac{1}{P_{(1)}}\right]^2} \cdot \frac{\prod_{n=2}^{S} \left[1 - \frac{2}{P_{(n)}}\right]}{\left[\prod_{n=2}^{S} \left[1 - \frac{1}{P_{(n)}}\right]\right]^2} \right] = 2 \cdot \delta = 2 \cdot \lim_{S \to \infty} \left[ \prod_{n=2}^{S} \frac{1 - \frac{2}{P_{(n)}}}{\left[1 - \frac{1}{P_{(n)}}\right]^2} \right] = \delta_2 = 1.3203348166$$

$$nn := 2, 3 .. Np \qquad \delta_1 := 1 \qquad \delta_{nn} := \delta_{nn-1} \cdot \frac{\left(P_{nn}\right)^2 - 2 \cdot P_{nn}}{\left(P_{nn}\right)^2 - 2 \cdot P_{nn} + 1} \qquad \delta_{Np} = 0.6601618 \qquad \text{(A1.1)}$$

$$\delta_2 := 2 \cdot \delta_{Np} \qquad \delta_2 = 1.3203236325$$

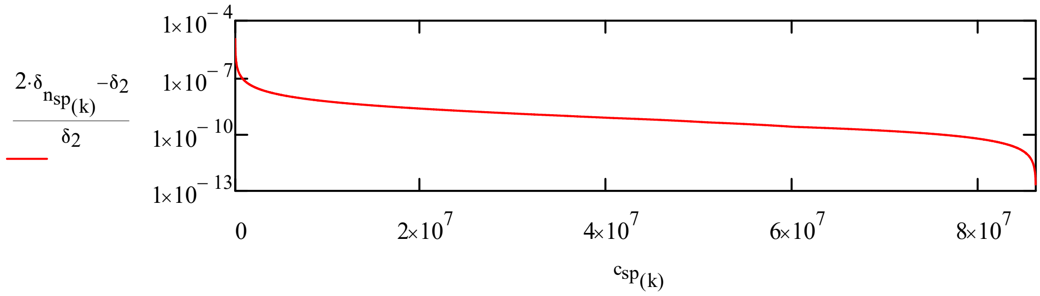



**Figure A1.1: Evolution of the factor of correction ( $\delta_2$ ), approaching the final constant value**

## A2: Evaluation of the effective number of central triads and of their approximation

The approximate values of the number of triads (1.9) is evaluated once for sparse distances ( $P_{\left[n_{sp(kk)}\right]} \le c_{sp(kk)}$), with the center of the triads being equal to the next smaller prime to the sparse distances - and the results are written to a file. They are read from this file :

$$\tau_{c\_appr\_sp(kk)} := \tau_{c\_appr}\left[P_{\left[n_{sp(kk)}\right]}\right] \qquad \text{WRITEPRN}(\text{"triades\_appr\_sp.prn"}) := \tau_{c\_appr\_sp} \qquad \text{(A2.1)}$$

$$\tau_{c\_appr\_sp} := \text{READPRN}(\text{"triades\_appr\_sp.prn "})$$

The total number of triads around the center point at ( $c$ ) and within the last section before ( $2 \cdot c$ ) is given by (1.8). It is evaluated once for sparse distances ( $P_{\left[n_{sp(kk)}\right]} \le c_{sp(kk)}$) - with the center of the triads being equal to the next smaller prime to the sparse distances - and the results are written to a file. They are read from this file:

$$\tau_{c\_last\_appr\_sp(kk)} := \tau_{c\_last\_appr}\left[P_{\left[n_{sp(kk)}\right]}\right] \qquad \text{WRITEPRN}(\text{"triades\_last\_appr\_sp.prn"}) := \tau_{c\_last\_appr\_sp}$$

$$\tau_{c\_last\_appr\_sp} := \text{READPRN}(\text{"triades\_last\_appr\_sp.prn "}) \qquad \text{(A2.2)}$$

The routine evaluating the effective number of triads at ( $c$ ) checks for each prime ( $P_{(2)} < P_{(kk)} < c$ ), if the integer at de distance ( $d = 2 \cdot c - P_{(kk)}$) were a prime too. If it is a prime, than the sum of the diads around the central point is risen by one. The checking is made by control of the equality of the prime next to the distance ( $d$ ) to the distance itself. Similarly the diads within the last section are evaluated. For the evaluation of the index of the next smaller prime to a given distance the routine (0.2) is used. The results are written to files and are read from these files:

$$\tau_{c\_eff\_}(c) := \begin{array}{|l} S \leftarrow 0 \\ k_{last} \leftarrow 1 \\ n \leftarrow n_{next\_P\_}(c, k_{last}) - 1 \\ \text{while } n \geq 1 \\ \quad \begin{array}{|l} d \leftarrow 2 \cdot c - P_{(n)} \\ k \leftarrow n_{next\_P\_}(d, k_{last}) \\ S \leftarrow S + 1 \ \text{ if } d = P_{(k)} \\ k_{last} \leftarrow k \\ n \leftarrow n - 1 \end{array} \\ S \leftarrow \frac{S}{\ln(c)} \end{array} \qquad \tau_{c\_last\_eff\_sp\_}(c) := \begin{array}{|l} S \leftarrow 0 \\ k_{last} \leftarrow 1 \\ n \leftarrow n_{next\_P\_}(\sqrt{c}, k_{last}) - 1 \\ \text{while } n \geq 1 \\ \quad \begin{array}{|l} d \leftarrow 2 \cdot c - P_{(n)} \\ k \leftarrow n_{next\_P\_}(d, k_{last}) \\ S \leftarrow S + 1 \ \text{ if } d = P_{(k)} \\ k_{last} \leftarrow k \\ n \leftarrow n - 1 \end{array} \\ S \leftarrow \frac{S}{\ln(c)} \end{array} \qquad \text{(A2.2)}$$

$$\tau_{c\_eff\_sp(kk)} := \tau_{c\_eff\_}\left[P_{\left[n_{sp(kk)}\right]}\right] \qquad \tau_{c\_last\_eff\_sp(kk)} := \tau_{c\_last\_eff\_sp\_}\left[P_{\left[n_{sp(kk)}\right]}\right]$$

$$\text{WRITEPRN}(\text{"triades\_c\_eff\_sp\_.prn"}) := \tau_{c\_eff\_sp} \qquad \text{WRITEPRN}(\text{"triades\_last\_eff\_sp.prn"}) := \tau_{c\_last\_eff\_sp}$$

$$\tau_{c\_eff\_sp} := \text{READPRN}(\text{"triades\_c\_eff\_sp\_.prn"}) \qquad \tau_{c\_last\_eff\_sp} := \text{READPRN}(\text{"triades\_last\_eff\_sp.prn"})$$

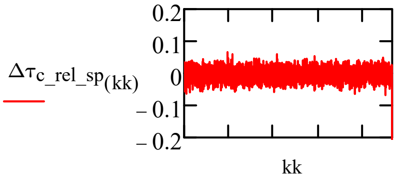
## A3: Evaluation of the properties of the dispersion of triads and of diads

The dispersion of both, of the free positions left by both, by the straight and by the reflected series of multiples of primes are proportional to the number of primes ( $R(c)$), the series of multiples of primes, which are covering free positions at ( $c$ ). Therefore this is the case by the dispersion of the number of triads around its best estimate approximation as well. In case of the number of diads within the last section the width of the dispersion is proportional to the number of primes up to the square root of the distance.

The relative dispersion with reference to ( $R(c)$) respectively ( $R(\sqrt{c})$ ) are evaluated and shown in the figure below. The dispersion is equal in both cases, how it was expected:

$$\Delta\tau_{c\_sp(kk)} := \tau_{c\_eff\_sp(kk)} - \tau_{c\_appr\_sp(kk)} \qquad \Delta\tau_{c\_rel\_sp(kk)} := \frac{\Delta\tau_{c\_sp(kk)}}{R_{eff}\left[P_{\left[n_{sp(kk)}\right]}\right]} \qquad \text{(A3.1)}$$

$$\Delta\tau_{c\_last\_sp(kk)} := \tau_{c\_last\_eff\_sp(kk)} - \tau_{c\_last\_appr\_sp(kk)} \qquad \Delta\tau_{c\_last\_rel\_sp(kk)} := \frac{\Delta\tau_{c\_last\_sp(kk)}}{R_{eff}\left[\sqrt{P_{\left[n_{sp(kk)}\right]}}\right]}$$



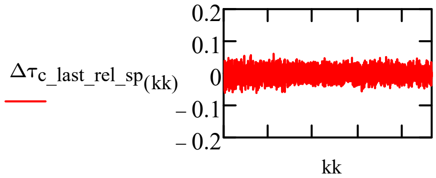




**Figure A3.1: The relative dispersion of the number of triads and of triads in the last section, around their approximation**

The width of the dispersion of the triads is therefore estimated as ( $\Delta B(c) = K_{\Delta B} \cdot R(c) \;\; ; \;\; K_{\Delta B} \geq 0.1$ ) (A3.2)

.
The standard deviation SD from of the dispersion (A3.1) - relative to ( $R(c)$) - has to be constant for large distances. It is evaluated and illustrated below:

$$SD_{\Delta\tau\_rel}(c) = \sqrt{\frac{1}{c} \cdot \sum_{c} \left[\frac{\Delta\tau_c(c)}{R_{eff}\left[P_{\left[n_{sp(kk)}\right]}\right]}\right]^2} \;\; ; \;\; SD_{\Delta\tau\_rel(kk)} := \sqrt{\frac{1}{kk} \cdot \sum_{j=1}^{kk} \left[\Delta\tau_{c\_rel\_sp(j)}\right]^2} \qquad \text{(A3.3)}$$

$$SD_{\Delta\tau\_last\_rel}(c) = \sqrt{\frac{1}{c} \cdot \sum_{c} \left[\frac{\Delta\tau_{c\_last}(c)}{R_{eff}\left[\sqrt{P_{\left[n_{sp(kk)}\right]}}\right]}\right]^2} \;\; ; \;\; SD_{\Delta\tau\_last\_rel\_sp(kk)} := \sqrt{\frac{1}{kk} \cdot \sum_{j=1}^{kk} \left[\Delta\tau_{c\_last\_rel\_sp(j)}\right]^2}$$

The factor of proportionality for the standard deviation of the triads and of the triads in the last section are constants and about equal:

$$F_{SD\_\Delta\tau} := SD_{\Delta\tau\_rel}(kk_{limit}-1) = 0.01805 \qquad \text{(A3.4)}$$

$$F_{SD\_\Delta\tau\_last\_rel} := SD_{\Delta\tau\_last\_rel\_sp}(kk_{limit}-1) = 0.0176596$$

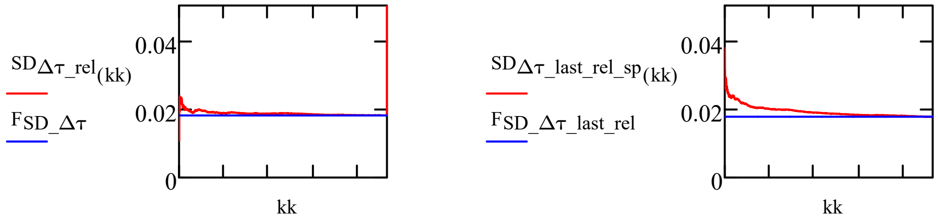



**Figure A3.2: Standard deviation of the dispersion of the number of triads and of the number of triads in the last section, around their approximating functions**

The difference between the best estimate approximation of the number triads and the value resulting from the triads-number-formula defined in (1.10) is proportional to the best estimate value, divided by the logarithm of the distance ( $\ln(c)$). The factor of proportionality is evaluated below and illustrated in the figure below.

$$\Delta\tau_{c\_TNF\_sp(kk)} := \tau_{c\_appr\_sp(kk)} - T_{c\_TNF}\left[c_{sp(kk)}\right] \qquad \Delta\tau_{c\_TNF\_rel\_sp(kk)} := \Delta\tau_{c\_TNF\_sp(kk)} \cdot \frac{\ln\left[c_{sp(kk)}\right]}{\tau_{c\_appr\_sp(kk)}}$$

$$\Delta\tau_{c\_TNF}(c) = F_{\Delta\tau.c\_TNF} \cdot \frac{\tau_{c\_appr}(c)}{\ln(c)} > F_{\Delta\tau.c\_TNF} \cdot \frac{T_{c\_TNF}(c)}{\ln(c)} = F_{\Delta\tau.c\_TNF} \cdot \frac{\delta_2 \cdot c}{\ln(c)^4} \qquad \text{(A3.5)}$$

The factor of proportionality is: $F_{\Delta\tau.c\_TNF} := \Delta\tau_{c\_TNF\_rel\_sp}(kk_{limit}-1) = 0.6980952$

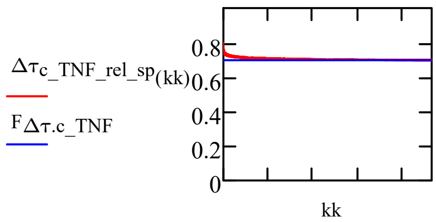



**Figure A3.3: Factor of proportionality of the difference between the best estimate number of triads and the value of the triads-number-formula, to the best estimate number of triads, divided by the logarithm of the distance**

## A4: The effective number of twin primes and its dispersion

The evaluation procedure of the best estimate number of twin primes corresponding to (4.12) is given below. The routine returns the value of the best estimate value:

(A4.1)

$$TW_{appr\_}(c) := \left| \begin{array}{l} sqr \leftarrow \sqrt{c} \\ S \leftarrow 0 \\ j \leftarrow 1 \\ \text{while } j < sqr \\ \quad \left| \begin{array}{l} S \leftarrow S + \dfrac{1}{\ln(j\cdot\sqrt{c}-1)\cdot\ln(j\cdot\sqrt{c}+1)} \\ j \leftarrow j+1 \end{array} \right. \\ S \leftarrow S \cdot \delta_2 \cdot sqr \end{array} \right.$$

The approximate number of twins at sparse distances are evaluated once and the results are written to a file and they are read from it. The vector of the effective number of twin primes is evaluated elsewhere and the results are written to a file. They are read from this file:

$$TW_{appr\_sp(k)} := TW_{appr\_}\left[c_{sp(k)}\right] \qquad WRITEPRN(\text{"tw\_appr\_sp.prn"}) := TW_{appr\_sp} \qquad \text{(A4.2)}$$

$$TW_{appr\_sp} := READPRN(\text{"tw\_appr\_sp.prn"}) \qquad TW := READPRN(\text{"number\_TW\_eff\_sp.prn"})$$

In order to evaluate the variables at sparse distances, first the indexes of the next smaller twins to the sparse distances are evaluated and written to a file for later reading:

$$TW\_sp_{(k)} := n_{next\_TW}\left[c_{sp_{(k)}}, TW\right] \qquad (A4.3)$$

$$WRITEPRN("k\_TW\_eff\_sp.prn") := TW\_sp \qquad TW\_sp := READPRN("k\_TW\_eff\_sp.prn")$$

The difference between the effective number of twins and their approximating function is:

$$limit := length(TW_{appr\_sp}) - 1 \qquad k := 1 .. limit - 1 \qquad (A4.4)$$

$$\Delta TW(c) = TW(c) - TW_{appr}(c) \qquad \Delta TW_{sp_{(k)}} := TW\_sp_{(k)} - TW_{appr\_sp_{(k)}}$$

Because of the larger initial deviation of the relative dispersion around the approximating function at small distances the dispersion is evaluated starting from the distance ( $k_{start} := 5415;\ c_{sp}(k_{start}) = 5.024 \times 10^7$ ):

$$\Delta TW_{rel}(c) = \frac{\Delta TW(c)}{R(c)} \qquad \Delta TW_{rel\_sp_{(k)}} := \frac{\Delta TW_{sp_{(k)}}}{R_{eff}\left[c_{sp_{(k)}}\right]} \qquad (A4.5)$$

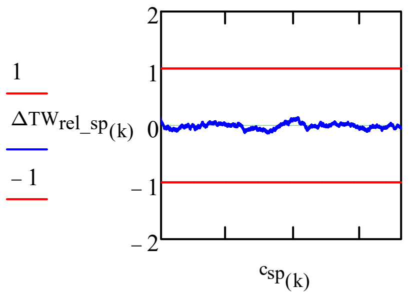



**Figure A4.1: Dispersion of the effective number of twins around its approximating function and its limiting boundaries at sparse distances**

The standard deviation of the relative dispersion of the number of twin primes around its approximating function is:

$$k := k_{start} + 1 .. k_{limit} - 1$$

$$SD\Delta TW\_rel(c) = \sqrt{\frac{1}{c} \cdot \sum_{c} \left(\frac{\Delta TW(c)}{R_{eff}(c)}\right)^2}\ ; \quad SD\Delta TW\_rel_{(k)} := \sqrt{\frac{1}{k - k_{start}} \cdot \sum_{j = k_{start}}^{k} \left[\Delta TW_{rel\_sp_{(j)}}\right]^2} \qquad (A4.6)$$

The factor of proportionality for the standard deviation of the twin primes are approaching a constant value:

$$F_{SD\_\Delta TW} := SD\Delta TW\_rel(k_{limit-1}) = 0.0511355 \qquad (A4.7)$$

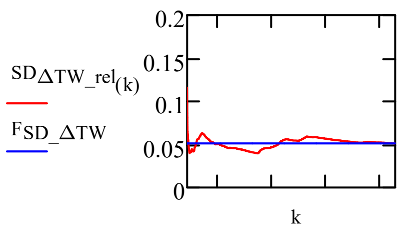



**Figure A4.2: Standard deviation of the relative dispersion of the effective number of twins around its approximating function**

The standard deviation of the dispersion of the number of twins is therefore about 5% of the value of the approximating function.

## Conjectures:

Reflecting the union of the series of multiples of the primes additionally over the ails of triads define five equidistant primes, pentides. Continuing this way, septides and so on may be defined. The number of all these multiple equidistant primes is unlimited.
The primes are defined on the non divisibility condition by all smaller integers. Similar infinite subsets of the integers may be defined on similar conditions. If the density of the instances of such subsets is decreasing with the inverse of the logarithm of the distance, or powers of the inverse of the logarithm, than congruent relative dispersion may be defined and the subsets may be regarded as continuums.

## References:

[1]. Chen J. R., On the representation of a large even integer as the sum of a prime and the product of at most two primes, Science in China, 16 (1973), No.2, pp. 111-128..