Abstract. I will explore in brief a simple geometry that could unify quantum physics with general relativity.
1. Clock model as proper time

Clock is basic idea in all physics, but time that clock shows is observer dependent. Goal of physics is to find things that are invariant and don’t depend on frame of reference. Let’s say i have a set of rules that measure distance in space and clock that measures distance in time, they can be both combined into one object that is proper time, it says how much ticks of an observer clock it takes to get from one point of space-time to another. Now i want to make thinking about it as much general as possible, to do it let’s say i only have rules and clock and i want transform rules and clock into distance, there are two way transform i can do, one transforms clock and rules into number second one transforms number into set of rules and clocks. So to capture all information about space-time i need object that does both. Invariant has to be not the one way transformation but both way transformation for all possible frame of reference. Let’s say i have list of rules and clocks that are vectors assign to some point of space-time, in \( N + 1 \) dimensions of space time i need \( N + 1 \) vectors of space-time to match every direction that space-and time can change. So i need a tensor of rank \( N + 1 \) in covariant and contravariant components. I will denote it \( \tau \) and invariant is number that is equal to contraction:

\[
\tau^{\alpha_1 \ldots \alpha_{N+1}}_{\alpha_1 \ldots \alpha_{N+1}} = \tau \tag{1.1}
\]

Without contraction this tensor has \( N + 1 \) independent components, down indexes represent outside space-time, up indexes how space-time is seen by observer. So all reference frame will agree on \( \tau \) but not all of them will agree how space-time look from outside and seen for observer. I can write that tensor as:

\[
\tau^{\beta_1 \ldots \beta_{N+1}}_{\alpha_1 \ldots \alpha_{N+1}} \tag{1.2}
\]

Now let’s say i have two points of space-time (observers), one of them is in strong gravity field one is at weak gravity field. One in weak gravity field has low value of vectors in outside space-time so it will see that object in strong gravity field slows down, that one in strong gravity field will see the object in weak field does speed up, it comes from fact that both observers have to agree on \( \tau \) and in strong gravity field \( \alpha \) indexes change very fast so it’s rules are very long and clock ticks very fast compared to weak gravity field and when it measures very short rules with long rules and clocks it will state that observer in weak gravity field speeds up. Observer in weak gravity field measures long long rules and fast clocks by short rules and slow clocks so it will state object in strong gravity slows down.
2. Field equation

I will examine an equation that could be a solution to creating a field equation of gravity that has a quantum physics built into it. It has a geometrical meaning behind it, first I will use two objects to create it, rotation matrix in tensor form for each space dimension and proper time tensor that takes $N+1$ vectors where $N+1$ is the number of space and turns it into a number or it takes a number and turns it into a vector. We live as far as we know in four-dimensional space-time but in this model I will explore four-dimensional space-time. I can formally write the field equation as:

$$R^\gamma_1...^\gamma_4(\phi)\tau^\gamma_1...^\gamma_4 = \partial^\beta_1...\partial^\beta_5\tau^\beta_1...^\beta_4$$

(2.1)

Where $R$ is rotation tensor and $\tau$ is proper time tensor. Let's first examine the equation's geometrical meaning without rotation so equation reduces to:

$$\tau^\alpha_1...^\alpha_4 = \partial^\beta_1...\partial^\beta_4\tau^\beta_1...^\beta_4$$

(2.2)

Each index runs from zero to three, on the left side I will have a tensor that takes four vectors $\beta_1\beta_2\beta_3\beta_4\beta_5$ and turns them into proper time. It has $4^4 = 256$ components, generally for $N+1$ dimensions of space-time I have $(N+1)^{N+1}$ components. On the right side of the equation I have a proper time tensor that takes four vectors $\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5$ and turns them into proper time then it takes proper time and turns it into four vectors $\beta_1\beta_2\beta_3\beta_4\beta_5$. Now I take how those five vectors change with respect to their coordinates and I'm left with how four vectors $\alpha_1\alpha_2\alpha_3\alpha_4\alpha_5$ change to proper time. Now it generates static space-time where objects always follow determined trajectories. It's a classical field. But if I add rotation tensor that comes from rotation matrix of four dimensions space and assume that field can be rotated by some angle in any direction now it becomes a quantum field, object can move in any direction and it's all equal state of system. Now a vector can point in any direction in space-time. Rotation can be in both directions positive and negative angle that should be good enough to explain spin states.
3. Physical law from field equation

Field equation states very simple yet complex mathematical law. Change in how outside space-time is seen, is equal to its gravity field (geometry). Geometry of a field is given by \( \mathcal{N} + 1 \) independent components. Solving that equation must give those independent components. In first section i explained what that tensor means but what is physical law that comes out of field equation? First i will integrate both sides to get rid of how it changes:

\[
\int \tau_{\alpha_1...\alpha_4} dx^{\beta_1}...dx^{\beta_4} = \tau^{\beta_1...\beta_5}_{\alpha_1...\alpha_4}
\]

(3.1)

Now it just state that if i take five vectors that represent space-time geometry (gravity) and change it into number (proper time-distance in space-time) and then change that number into five vectors set of all transformation that do change them are equal to hyper-volume of how that five vectors change to number. in direction of how that number changes to vectors. So i take a hyper-volume of some part of space time and integrate it over observer seen world directions and i get all possible transformations. The larger the volume in some part of space-time the bigger is proper time transformation. So it needs a curved space-time in order to work. So how much ruler and clocks i can put in some region of space-time is equal to its invariant proper time but invariant proper time is a contraction so i discard components that are not independent-independent components are physical objects.

\[
\tau = \int \tau_{\alpha_1...\alpha_4} dx^{\alpha_1}...dx^{\alpha_4} = \tau^{\alpha_1...\alpha_4}_{\alpha_1...\alpha_4}
\]

(3.2)

This approach leads to static space-time in sense that it’s geometry does not change globally. What does change is how observers see it from any point of it. But that leads to clear conflict with quantum physics, here if I know starting point i know ending point. Proper time only depends on position in space-time, so does as it follows from it a trajectory that goes in that space-time. In next chapter will explain how to change it, but for now i need to summarize what physical law states for any dimensions of space-time: If i have a given \( \mathcal{N} + 1 \) dimensions space-time it’s proper time tensor is equal to how much proper time i can put in that \( \mathcal{N} + 1 \) hype-volume of that space-time.
I add rotation operator of angle $\phi$ to make field equation quantum. It states that any rotation by any constant angle $\phi$ of proper time is still solution to field equation. It means I have infinite number of possible directions object can move- but after measurement it just changes to one path. I can write wave function as integral of invariant proper time in angle ranging from some staring angle $\phi_0$ to some end angle $\phi_1$, divided by integral of all possible angles:

$$\Psi (\phi_0, \phi_1) = \frac{\int_{\phi_0}^{\phi_1} \int R_{\alpha_1...\alpha_4}^{\gamma_1...\gamma_4} (\phi) \tau_{\gamma_1...\gamma_4} dx^{\alpha_1}...dx^{\alpha_4} d\phi}{\int \int R_{\alpha_1...\alpha_4}^{\gamma_1...\gamma_4} (\phi) \tau_{\gamma_1...\gamma_4} dx^{\alpha_1}...dx^{\alpha_4} d\phi} = \frac{\int_{\phi_0}^{\phi_1} \tau d\phi}{\int \tau d\phi} \quad (4.1)$$

Probability is a scalar object and it depends on invariant proper time value, so bigger the invariant proper time value the bigger the probability, more angles its taken into integral more probability. So state before measurement is equal to all possible rotation angles integral after measurement it changes to some region of rotation angles. From integral comes fact that if I take integral from angle to same angle I will get zero- it means that there is uncertainty in how small change in angle can be, I can’t rid of rotation operator and find direct path I can find only small amount of paths that are possible. It means object trajectory has always some randomness to it. So one path is not one rotation angle. Planck scale is fundamental in quantum physics, to make this equation work I need to assume that it’s fundamental and I can’t have lower values that one Planck length and time. It means that invariant proper time can’t have value lower than Planck time to power of dimensions if I use only clocks as measuring tool (I just divide all space directions by speed of light) or length to the power of dimensions (I multiply time direction by speed of light) :

$$\tau_R = \int \tau_{\alpha_1...\alpha_4} dx^{\alpha_1}...dx^{\alpha_4} = \tau_{\alpha_1...\alpha_4}^{\alpha_1...\alpha_4} \geq t_P^{N+1} \quad (4.2)$$

$$\tau_C = \int \tau_{\alpha_1...\alpha_4} dx^{\alpha_1}...dx^{\alpha_4} = \tau_{\alpha_1...\alpha_4}^{\alpha_1...\alpha_4} \geq t_P^{N+1} \quad (4.3)$$

Where subscripts means ruler for $R$ and clock for $C$. Minimal angle that can be written as special case where $n=1$: $\phi_1 - \phi_0 = 2\pi \sigma nl_P$, so it means that spin is minimal angle that object be rotated and thus follow that rotation can have only values of $\phi = 2\pi \sigma nl_P$ where $n$ is natural number. Where spin can take values $\sigma = 0, \pm \frac{1}{2}, \pm 1..., \pm \sigma$. 