INDIRECT EVIDENCE THAT WE LIVE IN A COMPUTER SIMULATION

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Abstract

We examine the possibility that we exist as part of a giant computer simulation. It is quantum field theory that is being simulated. We provide a simple model describing how this might work. If such is the case, a number of agreeable consequences follow. For one thing, it is possible to dispense with renormalization. For another, Haag's theorem is circumvented.

Key Words: Quantum Field Theory, Simulation Hypothesis, Haag's Theorem.

Perhaps owing to the work of Bostrom (1) an increasing number of physicists have considered seriously the possibility that we live in a computer simulation (2). The simulation hypothesis takes a number of forms. The one we are most interested in goes by the name the 'Virtual World Simulation Hypothesis.' According to this notion we and our reality are being simulated by a "computer" (or some equivalent form of information-processing system) in much the same way that 'Sims' are simulated in the eponymous game. Of course, our simulation is far more complicated than that of the 'Sims' — so much so that we cannot even tell that we are 'Sims.' Bostrom's idea raises a number of provocative questions. For one thing, we must wonder whether a 'Sim' *could* be sentient. Lacking any profound insight into the metaphysics of consciousness, we should probably just admit that we do not know. At least, there is no very obvious reason why it could not be. For another thing, we must wonder about the "computer" and the world it is imagined to exist in. Are these things real or just useful fictions?

We assume that our computer simulates a quantum field theory (QFT) and does this in a simple and efficient way. The best thing would be for it to simulate our reality over a lattice of spacetime points. This is not unlike what our small computers do when they try to numerically solve a hyperbolic PDE. It would work with a lattice of coordinate addresses given by $X^{\alpha\beta\gamma\delta}$ = {x, y, z, t} = $\{\alpha\,\epsilon, \beta\,\epsilon, \gamma\,\epsilon, \delta\,\epsilon\}$ where α, β, γ, δ are integers and ϵ denotes the spacing of the lattice. α, β, and γ range from 0 to L/ϵ where L^3 is the volume of the universe being simulated. δ could be unlimited or the simulation might shut down after a finite number of time-steps. (It is a bit like lattice gauge theory, only *for real*.) It will assign the values of the quantum fields to the lattice points. We will also define space-like hypersurfaces, *S(t)*, which consist of the lattice points having $\delta = t/\epsilon$. Differentiation over our lattice is defined easily:

1) $\partial_x\ f\left[X^{\alpha\beta\gamma\delta}\right] = \left(f\left[X^{(\alpha+1)\,\beta\gamma\delta}\right] - f\left[X^{(\alpha-1)\,\beta\gamma\delta}\right]\right) / 2\,\epsilon$,

2) $\partial_{x,x}\, f\left[X^{\alpha\beta\gamma\delta}\right] = \left(f\left[X^{(\alpha+1)\,\beta\gamma\delta}\right] + f\left[X^{(\alpha-1)\,\beta\gamma\delta}\right] - 2\ f\left[X^{\alpha\beta\gamma\delta}\right]\right) / \epsilon^2$, and so forth.

A brief digression is in order since plane-wave solutions (e.g. $e^{i\,k\,x}$) figure prominently in QFT and would, presumably, do so in its digitized version. Suppose our QFT was real scalar field theory. Our computer would want to solve the Klein-Gordon equation which it can do in plane-wave solutions. Suppose we are dealing with a wave propagating in the x direction. Suppose, further, that the computer imposes periodic boundary conditions at the "walls" of its lattice. Writing $\varphi\,(x_i,\ t_i) = e^{i(-E\,t_i + k\,x_i)}$, we notice an interesting thing. As k approaches and exceeds π/ϵ the solutions, as far as the computer can see, wrap back on and duplicate themselves. There is, in effect, a *maximum momentum* available to the wave. The computer obtains a dispersion relation that reads:

3) $\mathrm{Cos}(\epsilon E) = \mathrm{Cos}(\epsilon k) - \epsilon^2 m^2/2$.

When k ($m \neq 0$) exceeds ArcCos($\frac{\epsilon^2 m^2}{2}$ - 1)/ϵ ($\approx \pi/\epsilon$) the energy becomes complex which is, obviously, unacceptable. This limits k even a little more. The plane-wave solutions available to the computer are both discreet and finite in number. If k is angled relative to the axes of the lattice the wave will propagate a little differently. Savage (3) has suggested that this anisotropic propagation could furnish an experimental test of the simulation hypothesis. By looking at observable consequences that are *not* observed he puts a rough limit of $3X10^{-26}$m on how big ϵ can be.

The fact that the hypersurfaces *S(t)* obey periodic boundary conditions and, therefore, contain only finitely many points follows from our assumption that the computer, though doubtless enormous, is, ultimately, finite; it can only calculate at finitely many lattice points. We must admit that there is nothing particularly Lorentz invariant about any of this. The fact that we do not notice any momentum cutoffs or anisotropies is a consequence of the extreme smallness of ϵ. The fact that our world appears to be Lorentz invariant follows from the relativistic invariance of the equations the computer solves.

So what does the computer do and how does it do it? It is programmed with the 'Theory of Everything' which we assume to be an interacting QFT (or something very like it). We ask only that the theory, in its continuum limit, conserve 4-momentum and be relativistically invariant. We imagine that the computer works in the Dirac Interaction Picture. We assume the theory has a Hamiltonian that can be separated into a free-field (linear) part, H_0, and a part, H_I, that describes the interactions. Suppose it begins its work at $t = 0$. It has to be supplied with knowledge of the quantum fields on *S(0)* and *S(*ϵ*)*. To make things easy let us suppose these are given as $\frac{1}{\sqrt{L^3}} \sum_k (u_k a_k + u_k{}^* a_k^\dagger)$ (as would be the case if the QFT were real scalar field theory) or something similar otherwise. Let u_k and $u_k{}^*$ be the above-mentioned plane-wave solutions. The computer now has the information it needs to calculate the quantum fields at later times.

a_k and $a_k^\dagger$ represent annihilation and creation operators. The former are associated with the positive-frequency waves and the latter with their conjugates. The a_k s always annihilate |0> and $[a_k, a_{k'}^\dagger]$ ($\{a_k, a_{k'}^\dagger\}$) = $\delta_{kk'}$. These operators require something to operate on. The computer, in addition to simulating the quantum fields, also simulates a state vector in the Fock space defined by the operators. This vector encompasses all the particles in the universe. The computer is provided with | Ψ (*0*) > and, at each time-step, it evolves this state vector according to $i\, \partial_t\, |\Psi(t)> = H_I\, |\Psi(t)>$. Since the number of basis vectors in this Fock space looks denumerably infinite we might worry that an infinitely large computer would be necessary. This is not the case, however. Since the simulated universe is spatially finite it can contain only a finite number of particles. Say there are 10^{60} of them. The computer would have no use for a basis vector describing 10^{70} particles. Also, the momentum cuts off at π/ϵ. The computer could get by with only a finite number of basis vectors.

We could even imagine the computer being more ambitious and trying to simulate a curved spacetime. Perhaps it attaches new functions — a metric tensor $g_{\mu\nu}$ — to the lattice points and evolves them using a digitized version of Einstein's equation. Equations like 1) and 2) would have to be adjusted to ensure their covariance. We will not explore this possibility further here.

QFTs are, generally, plagued by divergences. We assume the computer disposes of the zero point energy by something like normal ordering. Interacting QFTs also tend to suffer from UV divergences taken to indicate a problem with the theory at very small distances. But, for our computer, there are no distances smaller than ϵ nor momenta greater than π/ϵ — there exists a kind of automatic high-momentum cutoff. This would render finite the results of calculations that yield infinities in the continuum limit. The bare masses, charges, and so forth employed by the computer would differ from the physical quantities we observe but still be finite. A more devastating threat to interacting QFTs comes from Haag's theorem (4) which states that, given a set of operators satisfying $[a_k, a_{k'}^{\dagger}] = \delta_{kk'}$, which also possess a vacuum state from which we can construct a Fock space, we can assemble infinitely many other such sets of operators, satisfying these commutation relations, for which no vacuum state constructible from our original operators can exist. These sets of operators will not be unitarily equivalent to our original choice and the physics deriving from them must, therefore, yield ambiguous results. Haag's argument centers on the infinitely many degrees of freedom assumed in QFTs. After assuring us of the mathematical soundness of theories where the degrees of freedom are finite, he goes on to state: *"If we pass now to the limit $N \to \infty$ one new feature appears. A possible basis vector results from any distribution of integer numbers ν_k over the infinitely many oscillators. The 'number' of these possibilities is no longer countable. It is given by $\aleph_0^{\aleph_0} = \aleph_1$."* He concludes: *"The point is, however, that for infinite N, (14) is no longer a consequence of (12). In other words, there will be different irreducible representations of (12)."* (By *(12)* Haag means our commutation relations. By *(14)* he means $a_k|\psi_0> = 0$, defining the existence of a unique vacuum state.) But, in this theory, the number of "oscillators" is finite; Haag's argument cannot go forward. It has been appreciated since the work of Reed and Simon (5) that Haag's theorem fails if we live in a large, periodic, box. But we do not live in a box. Or do we? If the computer is simulating us as suggested we live in a box (3-torus) resulting from the finite nature of the lattice employed for our simulation. It is a mathematical box (not a physical one). But the consequences are the same. Similar observations have been made by Sheikholeslami-Sabzevari and Rahmati (6) in connection with lattice gauge theory. But they do not seem to take the simulation hypothesis seriously.

QFT is somewhat reminiscent of the bumblebee that cannot fly but does so anyway. In spite of Haag's theorem, and the rather ugly and arbitrary-looking process of renormalization, it provides a very good description of Nature. We speculate that this may be due to the fact that the bee is not flying in the kind of "air" we think it is. If we actually are being simulated on a giant computer an interesting question arises — does this computer exist as a real thing in a higher-order reality we have no knowledge of or is this just simply the way our universe works? As a philosophical matter we would say that, if there are conscious observers present in the higher-order reality, then it is the first way. If not, the "computer" is just a useful example — a heuristic device for understanding an unfamiliar concept (7).

References and Footnote.

1) Bostrom, N. *Philosophical Quarterly*, (2003), Vol. **53**, No.211, 243.

2) For a general review see Wikipedia: Simulation Hypothesis.

3) Beane, S. R., Davoudi, Z., Savage, M. J. arXiv:1210.1847 [hep-ph].

4) Haag, R. *Mathematisk-fysiske Meddelelser*, **29**, 12 (1955). (See also Hall, D: Wightman, A. S. *Mathematisk-fysiske Meddelelser*, **31**, 1 (1957).)

5) Reed, M. C., Simon, B. *Scattering Theory*. Methods of Mathematical Physics. **III**. New York, NY. Academic Press (1979).

6) Sheikholeslami-Sabzevari, B., Rahmati, H. arXiv:1011.2278 [hep-lat].

7) It is interesting to observe that, if $\epsilon = 3X10^{-26}$ m and the universe is about 26 billion light years on a side, the lattice would require about $6X10^{155}$ points. Modern supercomputers can, generally, work with lattices of about 1024^3 points (Harting, J., Venturoli, M., Coveney, P. V. arXiv:0312038v2 [comp-ph]). Assuming Moore's Law — that computational power doubles every two years — we should be able build our hypothetical computer in roughly 1000 years.