Planck's Formula Violates the Uncertainty Principle

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In this article, the author will try to address the basic aspects of blackbody radiation, after a brief historical review. In this study, it is shown that any potential (which does not depend on time) can be approximated in the first order with a harmonic oscillator potential.

Using statistical physics and the assumptions of quantum mechanics, it is shown that Planck's formula (as the father of quantum mechanics) violates the uncertainty principle (as his son).

I. Introduction

The general thermodynamic consideration allowed Kirchhoff, Boltzmann and Wien to derive rigorously a series of important laws controlling the emission of heated bodies.

However, these general considerations were insufficient for deriving a particular law of energy distribution in the ideal black-body radiation spectrum. It was W.Wien who advanced in this direction more than the others. In 1893 he spread the notions of temperature and entropy to thermal radiation and showed, that the maximum radiation in the black-body spectrum displaces to the side of shorter wavelengths with increasing temperature; and at a given frequency the radiation intensity can depend on temperature only, as the parameter appeared in the $(\frac{\nu}{T})$ ratio. In other words, the spectral intensity should depend on some function $f(\frac{\nu}{T})$. The particular form of this function has remained unknown.

In 1896, proceeding from classical concepts, Wien derived the law of energy distribution in the black-body spectrum. However, as was soon made clear, the formula of Wien's radiating law was correct only in the case of short waves. Nevertheless, these two laws of Wien have played a considerable part in the development of quantum theory (the Nobel Prize, 1911).

J.Rayleigh (19000 and J.Jeans (1905) derived the spectral distribution of thermal radiation on the Basis of the assumption that the classical idea on the uniform distribution of energy is valid. However, the temperature and frequency dependencies obtained basically differed from Wien's relationships.

According to the results of fairly accurate measurements, carried out before that time, and to some theoretical investigations, Wien's expression for spectral energy distribution was Invalid at high temperatures and long wavelengths. This circumstance forced Planck to turn to consideration of harmonic oscillators, which have been taken ass the sources and absorbers of radiation energy. Using some further assumptions on the mean energy of oscillators, Planck derived Wien's and the Rayleigh-Jeans laws of radiation. Finally, Planck obtained the empirical equation, which very soon was reliably confirmed experimentally on this basis, first of all, of the Wien-Lummer black-body model. Searching for the theory modifications which would allow this empirical equation to be derived, Planck arrived at the assumptions constituting the quantum theory basis (the Nobel Prize, 1918).

II. Planck's formula

Planck begins his work with the important assumption that black-body radiation is the product of oscillation the electrical oscillators in the cavity crater. This hypothesis was very novel. Because at that time, the atom's hypothesis was not accepted by all.

Equation of Planck's oscillator is

$$\ddot{x} - \omega^2 x = 0 \quad (1)$$

were ω are constant is related to the oscillator characteristicsis.

Planck by quanta hypothesis

$$E_n = nh\nu \quad (2)$$

And using Boltzmann's expression of the second law of thermodynamics, which has never used any of it. Receive to her popular equation

$$u = \frac{8\pi h\nu^2}{c^3}\frac{h\nu}{e^{\frac{h\nu}{k_bT}} - 1} \quad (3)$$

were u is energy density of oscillators, h is Planck's constant, ν frequency, c light's speed,k_bBoltzmann constant and T is temperature of cavity.

III. Statistical approach

From statistical physics we know that

$$\langle E\rangle = \frac{\sum E_n e^{-\frac{E(n)}{k_bT}}}{\sum e^{e^{-\frac{E(n)}{k_bT}}}} \quad (4)$$

$$E_n = nh\nu \quad (5)$$

$$\langle E\rangle = \frac{h\nu}{e^{\frac{h\nu}{k_bT}} - 1} \quad (6)$$

This result tell us that oscillator's number be in form

$$\langle n \rangle = \frac{1}{e^{\frac{h\nu}{k_b T}} - 1} \quad (8)$$

And its energy be in form

$$\langle E \rangle = \langle n \rangle h\nu \quad (9)$$

This mean that we don't know a single oscillator's energy. We have just energy of oscillator's collection. This prove exactly Einstein point of view where "We just talk about averages".

IV. Viral Theorem

To prove that Planck's oscillator is inconsistent with uncertainty, we must first prove a case.
The theorem shows that for the coordinate oscillator the following relation is established.

$$\langle T \rangle = \langle V \rangle$$

This suggests that the mean value of the potential energy and kinetic energy, which is said to be quantum mechanical in the expected value, is equal to the coordinate oscillator
To prove this, we define the function G as follows

$$G \equiv \sum px \quad (10)$$

That p is the momentum and x is the location Depending on the time from G, it can be written

$$\frac{dG}{dt} = \sum (p\dot{x} + \dot{p}x) \quad (11)$$

Now we compute the average value of $\frac{dG}{dt}$ in the time interval τ as follows

$$\langle \frac{dG}{dt} \rangle = \frac{1}{\tau} \int_0^\tau \frac{dG}{dt}\, dt = \frac{G(\tau) - G(0)}{\tau} \quad (12)$$

Let τ be equal to the oscillator period, or even if it is very large

$$G(\tau) = G(0) \quad (13)$$

So we will conclude that

$$\langle \frac{dG}{dt} \rangle = 0 \quad (14)$$

And according to (2), the following relation results

$$\langle \sum p\dot{x} \rangle = -\langle \sum \dot{p}x \rangle \quad (15)$$

Use the word to the right

$$F = \dot{p} \quad (16)$$

It can be rewritten like this

$$\langle \sum \dot{p}x \rangle = \langle Fx \rangle \quad (17)$$

And in the case of the coordinate oscillator, we know that F is as follows

$$F = -m\omega^2 x \quad (18)$$

Where m is a mass and ω is a fixed component of the intrinsic properties of the oscillator
So (8) can be rewritten as follows

$$\langle Fx \rangle = -\langle m\omega^2 x^2 \rangle \quad (19)$$

Planck considers the oscillator's potential energy $V = \frac{1}{2} m\omega^2 x^2$with respect to this subject.

$$\langle Fx \rangle = -2\langle V \rangle \quad (20)$$

For the side (6) we can write with respect to $T = \frac{1}{2} m\dot{x}^2$

$$\langle \sum p\dot{x} \rangle = 2\langle T \rangle \quad (21)$$

So finally, using (20) and (21) it follows that

$$\langle T \rangle = \langle V \rangle \quad (22)$$

We will use this to show that Planck oscillator violates the following relationship

$$\Delta p \Delta x > \frac{\hbar}{2} \quad (23)$$

Where Δp and Δx are uncertainties in the measurement of location and momentum observations
But to do this, we have to find a way to calculate these values.

V. Expecting value for the observable

if we want calculate the probability of observe “a_j” in measurement of “A” (a observable) when the system in “β” state we should write

$$\hat{A}|a_j\rangle = a_j|a_j\rangle \quad (24)$$

$$\mathcal{P}_\beta^{a_j} = |\langle a_j|\beta\rangle|^2 \quad (25)$$

Expecting value of “A” equal to

$$\overline{A_\beta} = \frac{\sum a_j \mathcal{P}_\beta^{a_j}}{\sum \mathcal{P}_\beta^{a_j}} \quad (26)$$

After simple calculate we arrive to uncertainty of an observable in the measurement to form

$$\Delta\hat{A} = \left[\left\langle B\left|\left(\hat{A} - \langle\hat{A}\rangle\right)^2\right|B\right\rangle\right]^{\frac{1}{2}} \quad (27)$$

$$\Delta\hat{A} = \left[\langle\hat{A}^2\rangle - \langle\hat{A}\rangle^2\right]^{\frac{1}{2}} \quad (28)$$

VI. Trouble beginning

We should calculate expecting value for momentum and position of Planck oscillator

We have

$$\langle E \rangle = \langle T + V \rangle \quad (29)$$

With due attention to (21) and (9) we have

$$\langle V \rangle = \langle T \rangle = \frac{\langle n \rangle h\nu}{2} \quad (30)$$

we know that

$$\langle T \rangle = \frac{\langle p^2 \rangle}{2m} \quad (31)$$

If we combine (29) and (28) we have

$$\langle p^2 \rangle = \frac{\langle n \rangle h\nu}{2} \quad (32)$$

Similarly, for V we have

$$\langle V \rangle = \frac{K}{2} \langle x^2 \rangle \quad (33)$$

$$\langle x^2 \rangle = \frac{nh\nu}{K} \quad (34)$$

We know from symmetry that

$$\langle p \rangle = \langle x \rangle = 0 \quad (35)$$

Know we could calculate Δx and Δp

If we put $K = m\omega^2$

$$\Delta p = \sqrt{\langle p^2 \rangle - \langle p \rangle^2} = \sqrt{\langle n \rangle \hbar m\omega} \quad (36)$$

$$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \sqrt{\frac{\langle n \rangle \hbar}{m\omega}} \quad (37)$$

Now if we calculate $\Delta x \Delta p$ we have

$$\Delta p \Delta x = \langle n \rangle \hbar \quad (38)$$

This equation for ground state equal to zero that it's disagree with uncertainty principle.

This is very bad situation. Statistical physics and Bose-Einstein statistics together predict this result.

Although Planck distribution work very well in all field of physics but (38) show us that this distribution has a fundamental problem.

The source of this problem is that (3) doesn't predict zero-point-energy (Z-P-E). although Planck tried to incorporate (Z-P-E) into this formula in 1912, but he didn't succeed.

Z-P-E is the product of quantum mechanics and "uncertainty principle" so if the Planck distribution doesn't predict Z-P-E, then there will also underscore uncertainty principle.

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