An Interpretation of the Fine Structure Constant Formula Found by Hans de Vries

Reinhard Kronberger

Abstract
The formula found by Hans de Vries for the fine structure constant is very elegant and accurate but there exists no explanation for it. In this paper, I try to give an interpretation. It is also shown why we have an electromagnetic field and why we have the value for the fine structure constant.

The Hans de Vries formula:

\[ \alpha = \frac{1}{\pi} \left( 1 + \frac{\alpha}{(2\pi)^2} \right)^{\frac{1}{2}} \]

where \( \Gamma = 1 + \frac{\alpha}{(2\pi)^2} \left( 1 + \frac{\alpha}{(2\pi)^2} \right) \left( 1 + \frac{\alpha}{(2\pi)^2} \right) \left( 1 + \ldots \right) \)

Someone can proof that the HdV formula is identical to

\[ \alpha = \sum_{n=0}^{\infty} \frac{\alpha^n}{(2\pi)^n} \left( \frac{2\pi}{\alpha} \right)^n e^{-\frac{x^2}{2}} \]

then

\[ \sqrt{\alpha} = \sum_{n=0}^{\infty} \frac{\alpha^n}{(2\pi)^n} \left( \frac{2\pi}{\alpha} \right)^n e^{-\frac{x^2}{2}} = \left( 1 + \frac{\alpha}{(2\pi)^2} + \frac{\alpha}{(2\pi)^4} \right) \left( 1 + \frac{\alpha}{(2\pi)^2} \right) \left( 1 + \frac{\alpha}{(2\pi)^2} \right) \left( 1 + \ldots \right) e^{-\frac{x^2}{2}} \]

The challenge now is to interpret this formula.
The factor \( e^{-\frac{x^2}{2}} \) looks like the expectation value of the wrapped normal distribution which is

\[ \langle z \rangle = e^{\mu - \frac{\sigma^2}{2}} = e^{\frac{\pi^2}{2}} \]

for \( \mu = 0 \) and \( \sigma = \frac{\pi}{\sqrt{2}} \)

see https://en.wikipedia.org/wiki/Wrapped_normal_distribution

And the factor

\[ (1 + \frac{\alpha}{(2\pi)^2} + \frac{\alpha}{(2\pi)^4} + \frac{\alpha}{(2\pi)^6} + \frac{\alpha}{(2\pi)^8} + \ldots) \]

looks like the series of conditional probabilities.

clearly (details see https://en.wikipedia.org/wiki/Conditional_probability)

\[ \sqrt{\alpha} = \sum_{n=1}^{\infty} P(A_1 \cap \ldots \cap A_n) = \left( 1 + \frac{\alpha}{(2\pi)^2} + \frac{\alpha}{(2\pi)^4} + \frac{\alpha}{(2\pi)^6} + \frac{\alpha}{(2\pi)^8} + \ldots \right) e^{-\frac{x^2}{2}} \]

with

\[ P(A_1 \cap \ldots \cap A_n) = P(A_1) \cdot P(A_2 | A_1) \cdot P(A_3 | A_1 \cap A_2) \ldots P(A_n | A_1 \cap \ldots \cap A_{n-1}) = \frac{\alpha^{n-1}}{(2\pi)^{\frac{n(n-2)}{2}}} e^{-\frac{x^2}{2}} \]

\[ \frac{\alpha}{(2\pi)^2} \cdot \frac{\alpha}{(2\pi)^4} \cdot \frac{\alpha}{(2\pi)^6} \cdot \frac{\alpha}{(2\pi)^8} \cdot \ldots \]

the denominator of \( \frac{\alpha}{(2\pi)^2} \) looks like the i-dimensional 'volume' of a torus therefore

the factors \( \frac{1}{(2\pi)^2} \) can be seen as normalization factors.

Now if we understand what is \( A_1, A_2, A_3, \ldots \) then we understand the HdV formula.
And furthermore we understand why we have an electrical charge.

Normally a n-dimensional torus is defined as \( T^n := S^1 \times \ldots \times S^1 = (S^1)^n \)
But in our formula we have two denominators which have the dimension of a point and a line.
Therefore we define the torus as

\[ \tilde{T}^n := \{0\} \times [0,1] \times S^1 \times \ldots \times S^1 = \{0\} \times [0,1] \times (S^1)^{n-2} \]
The infinite torus $\tilde{T}^\infty$ then can be seen as infinite ladder.

$$\tilde{T}^\infty = \begin{cases} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{cases} \begin{array}{l} \text{level } n \\ \text{level } n-1 \\ \text{level } 3 \\ \text{level } 2 \\ \text{level } 1 \end{array} \bigg\{ \begin{array}{l} \tilde{T}^3 \\ \tilde{T}^2 \\ \tilde{T}^1 \end{array} \bigg\} \bigg\{ \tilde{T}^{n-1} \bigg\} \tilde{T}^n$$

With this geometrical picture we can explain our probability sum. \(P(\text{absorbing or emitting a photon}) = P(\pm \gamma) = \sqrt{\alpha}\) is given by the different levels of the \(\tilde{T}^\infty\).

A photon is emitted when we climb down from one level to the prior level.
or is absorbed when we climb up on the torusladder one step from one level to the next.

Our events \(A_1, A_2, \ldots \) are then

\(A_1\) absorbing a photon by climbing up to level 1 from vacuum or emitting a photon by climbing down from level 1 to vacuum.

\(A_2\) absorbing a photon by climbing up to level 2 from level 1 or emitting a photon by climbing down from level 2 to level 1, and so on.

We call the factor \(e^{-\frac{z^2}{4}}\) the Basic-Generator of the electromagnetic field (short BG).

Explaination and visualisation of the Basic Generator BG.

\[ \text{BG} = E(X_0) = \text{expectation value of the wrapped normal distribution which is} \]
\[ <z> = e^{\mu - \frac{z^2}{2}} = e^{-\frac{z^2}{4}} \text{ for } \mu = 0 \text{ and } \sigma = \frac{\pi}{\sqrt{2}} \]

see https://en.wikipedia.org/wiki/Wrapped_normal_distribution

We write \(E(X_0) = e^{-\frac{z^2}{4}}\) \(X_0 = \{x \mid x = e^{i\theta}, 0 \leq \theta < 2\pi\}\)

The factor \(\frac{\pi}{\sqrt{2}}\) comes from a projection of a distribution with standard deviation \(\sigma = \pi\).

The projective normal distribution with \(\sigma = \frac{\pi}{\sqrt{2}}\) then will be wrapped.
Last but not least the value for the Fine-structure Constant by the Hans de Vries formula.

I have called the sum on \( n = 100 \) and calculated the result by iteration.

\[
\alpha = \left[ \sum_{n=0}^{\infty} \frac{\alpha^n}{(2\pi)^4} \right]^2 e^{-\frac{x^2}{2}}
\]

\[\alpha \approx 0.0072973525686 \approx \frac{1}{137.035999096}\]

Value for \( \alpha \) by Wikipedia

\[\alpha = 0.0072973525693(11)\]

The calculated value by the H\( dV \) formula fits very good to the empirical measurements.