Conservation of Energy and Particle Moving Towards a Mass

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Abstract

We consider a zero rest mass classical particle moving from infinity towards a point mass along a fixed line containing the mass. We show gravitation with only constants $c$ and $G$ with dimension does not satisfy conservation of energy.

1 Introduction

We restrict to gravitation that has only constants $c$ and $G$ with dimension. Units are chosen so that $c = G = 1$. Let $x, y, z$ be coordinates of space and consider a point mass $A$ on the $x$ axis. Let $\gamma$ be a zero rest mass particle moving along the $x$ axis from infinity towards $A$. Here $\gamma$ being considered as a classical particle. When $\gamma$ is at infinity let $A$ be at rest at the origin and have total energy $M$. Let $E$ be the energy of $\gamma$ at infinity.

2 Energy gain function

As $\gamma$ moves towards $A$ it gains energy from $A$. Let the function $W(M, E, h, R)$ be the amount of energy $\gamma$ gains on moving from an $x$ value of $R + h$, with $R > 0$ and $h > 0$, to an $x$ value of $R$. For small $E/M$ and $M/R$ the amount of energy $\gamma$ gains on moving from infinity to $R$ is approximately $ME/R$.

Since $c$ and $G$ are the only constants with dimension there is then a dimensionless function $F$ of the dimensionless variables $M/R, E/R$, and $h/R$ such that we can write

$$ W(M, E, h, R) = \frac{MEh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) $$

We will assume $W(M, E, h, R)$ is an increasing function of $E$.

3 Bound on energy gain

By conservation of energy $\gamma$ cannot gain more than an amount $M$ of energy so

$$ W(M, E, h, R) \leq M $$

As a consequence of this bound there is then a dimensionless function $B(M/R, h/R)$ such that

$$ \sup_{E} \left\{ \frac{MEh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \right\} = \frac{Mh}{R} B\left(\frac{M}{R}, \frac{h}{R}\right) \leq M $$

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For small $E/M, M/R, h/R$ since $W(M, E, h, R)$ is approximately $MEh/R^2$ and by the assumption that $W(M, E, h, R)$ is an increasing function of $E$ we have $B(M/R, h/R) > 0$ for small $M/R$ and $h/R$. Consequently we can define

$$b = \inf_{R > R_0} \left\{ B\left(\frac{M}{R}, \frac{h}{R}\right) \right\}$$

(4)

where $R_0$ is chosen so that $M/R_0$ and $h/R_0$ are small. We have $b \geq 0$.

4 $b = 0$

The amount of energy $\gamma$ gains on moving from $R + (N + 1)h$ to $R$ is the amount of energy $\gamma$ gains on moving from $R + (N + 1)h$ to $R + Nh$ plus the amount of energy $\gamma$ gains on moving from $R + Nh$ to $R + (N - 1)h$ and so on. For a $\gamma$ having large $E$ this is approximately

$$\sum_{n=0}^{N} \frac{Mh}{R + (N - n)h} B\left(\frac{M}{R + (N - n)h}, \frac{h}{R + (N - n)h}\right) \geq \sum_{n=0}^{N} \frac{Mhb}{R + (N - n)h}$$

(5)

where $R > R_0$. It follows by section (3) the energy $\gamma$ gains on moving from $R + (N + 1)h$ to $R$ becomes closer and closer to the left hand side of (5) as $E$ becomes larger and larger. If $b > 0$ the right hand sum of (5) becomes unbounded as $N \to \infty$. Consequently for some $N$ the left hand sum would become larger than $M$ hence the energy $\gamma$ gains, for large $E$, would be larger than $M$ violating conservation of energy. We must have $b = 0$.

5 Contradiction

Since $B(M/R, h/R) > 0$ for $R > R_0$ and $b = 0$ it follows there must be a sequence $\{R_k\}$ where $R_k \to \infty$ as $k \to \infty$ such that $B(M/R_k, h/R_k) \to 0$ as $k \to \infty$. Define the function

$$C(M, h, R) = RB\left(\frac{M}{R}, \frac{h}{R}\right)$$

(6)

We have $C(M_k, h_k, R) \to 0$ as $k \to \infty$ where $M_k = MR/R_k$ and $h_k = hR/R_k$. By (3) and (6)

$$\frac{MEh}{R^2} F\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \leq \frac{Mh}{R} B\left(\frac{M}{R}, \frac{h}{R}\right) = \frac{Mhb}{R^2} C(M, h, R)$$

(7)

hence

$$EF\left(\frac{M}{R}, \frac{E}{R}, \frac{h}{R}\right) \leq C(M, h, R)$$

(8)

Substitute $M_k$ for $M$ and $h_k$ for $h$ in this inequality and let $k \to \infty$ gives since $M_k, h_k$, and $C(M_k, h_k, R)$ go to zero and $E > 0$ that

$$F\left(0, \frac{E}{R}, 0\right) \leq 0$$

(9)

As stated before for small $E/M, M/R, h/R$ that $W(M, E, h, R)$ is approximately $MEh/R^2$. Comparing this with (1) we have $F(0, E/R, 0)$ for small $E/R$ is approximately one contradicting (9).

6 Conclusion

Assuming that the energy gain of $\gamma$ on moving from $R + h$ to $R$ increases as the energy $\gamma$ has at infinity increases it was shown that a gravitation with only constants $c$ and $G$ with dimension does not satisfy conservation of energy. Other conservation of energy arguments are presented in [1] and [2].
References
