Criticism of the relativity theory
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Abstract: In Section 2 the article derives the equation of motion for a test mass falling in a gravitational field. It is shown that there is no moving mass. Section 3 adds a corollary: Einstein's proof of $E = mc^2$ is also incorrect. In Section 4 a simple contradiction invalidating the Special Relativity Theory (SRT) is given. In Sections 5, 6 and 7 SRT is shown to be complete nonsense. Section 8 proves that the General Relativity Theory (GRT) is also nonsense and very bad nonsense. The section also gives a (possibly the first) correct exact solution to GRT, i.e., one where the speed of light is constant, not some Schwarzschild nonsense. The end of the article raises the question why these theories are claimed to be correct, but that question could not be solved in this article.

1. Introduction

The Special Relativity Theory claims that a mass $m_0$ moving with velocity $v$ has the moving mass $m = m_0 / \sqrt{1 - \beta^2}$ where $\beta = v/c$, and that this moving mass is the mass measured in the frame of reference where this mass is moving. This equation is derived by requiring Lorentz invariance for the equation of movement $F = ma$ in the form $F = (d/dt)m(t)(ds/dt)$. Therefore it is proven in a situation of an accelerated frame of reference, not in a frame of reference with constant velocity. From $m = m_0 / \sqrt{1 - \beta^2}$ follows that if an elementary particle with the mass $m_0$ is accelerated to a velocity $v$ close to the speed of light $c$, the mass that needs to be accelerated is the moving mass $m$. As $m$ tends to infinity when $v$ approaches $c$, the energy required becomes infinite. This is given as the reason why a massive particle cannot be speeded to $c$.

One may wonder why $F$ should transfer to itself in the Lorentz transform as force is not a conserved quantity. If e.g., $E = mv^2/2$ or $W = Fs = mas$ is required to be Lorentz invariant then $m$ is not changing. Yet $F = (d/dt)m(t)(ds/dt)$ is the equation from which Einstein got the mass transformation formula and this formula is the basis of his proof of Olinto De Pretto's formula $E = mc^2$. In Einstein's proof the mass is growing in the frame where the mass is moving. If this mass transformation formula is incorrect and mass is not growing in this way in this frame of reference, then Einstein's proof of $E = mc^2$ fails. Mass of course can grow in the moving frame (the rest frame of the test mass), since we can define a transform so that parameters have whatever transformation rules and that mass does not need have any physical meaning, but the claim is that the mass is really growing. It is shown that the natural interpretation is that the mass is not growing and therefore Einstein's proof of $E = mc^2$ is wrong.

2. A test mass falling in the gravitational field of a point mass

Consider a mass $m$ falling freely in a gravitational field created by a point mass $M$ in Newton's gravitation theory. The movement of the test mass $m$ is in the radial direction: the
test mass falls from the initial place \( r_0 \) towards the origin set to the position of the mass \( M \).

The equation of the motion is

\[
F = G m M / r^2 = m a = m \frac{d^2 (r_0 - r)}{dt^2}.
\]

Writing this with the Newtonian gravitational potential field \( \varphi = -G M / r \) we get

\[
m \frac{d}{dr} \left( \frac{GM}{r} \right) = m \frac{d}{dr} \varphi = -m \frac{d^2 r}{dt^2}.
\]

The equation of motion of the test mass \( m \) is

\[
m \frac{d}{dr} \varphi = -m \frac{d^2 r}{dt^2}, \quad \text{simplifying to} \quad \frac{d}{dr} \varphi = -\frac{d^2 r}{dt^2},
\]

that is, the mass cancels out. This is the equivalence principle in the Newtonian gravitation theory: \( m \) in the equation \( F = m a \) (the inertial mass) and \( m \) in the equation \( F = G m M / r^2 \) (the gravitational mass) is the same mass. Einstein wanted this equivalence principle to hold also in a relativistic theory of gravitation. Then it gets some new content: the principle implies that local time also transforms the same way for the inertial mass and gravitational mass. As in a gravitational field a clock slows down (as is verified by the Pound-Rebka experiment), then also in accelerating motion a clock slows down. Let us accept this equivalence principle.

Inserting the Newtonian potential \( \varphi = -G M / r \), we can solve the equation of motion. The solution in the rest frame of the mass \( M \) is

\[
r = \left( GM \frac{9}{2} \right)^{\frac{1}{3}} t^{\frac{2}{3}}
\]

implying that the mass \( m \) is in the place \( r_0 - r = r_0 - \left( GM \frac{9}{2} \right)^{\frac{1}{3}} t^{\frac{2}{3}} \) at the time \( t \).

The solution is easily checked:

\[
- \frac{d^2 r}{dt^2} = -\left( GM \frac{9}{2} \right)^{\frac{1}{3}} \frac{2}{3} \left( -\frac{1}{3} \right) t^{-\frac{4}{3}} = \left( GM \frac{9}{2} \right)^{\frac{1}{3}} \left( \frac{2}{9} \right)^{\frac{2}{3}} t^{-\frac{4}{3}} = \left( GM \frac{9}{2} \right)^{\frac{1}{3}} \left( \frac{2}{9} \right)^{\frac{2}{3}} t^{-\frac{4}{3}}
\]

\[
\frac{d}{dr} \varphi = GM \frac{1}{r^2} = GM \left( GM \frac{9}{2} \right)^{\frac{2}{3}} t^{-\frac{4}{3}} = \left( GM \frac{9}{2} \right)^{\frac{1}{3}} \left( \frac{2}{9} \right)^{\frac{2}{3}} t^{-\frac{4}{3}}.
\]

If you want the solution to be in a familiar form, then you must change the parameters. The solution can be expressed with the radius \( R \) of the Earth and \( g = 9.81 m/s^2 \) as

\[
r = R - \frac{1}{2} g \tau^2 + \frac{2g}{9T} \tau^3 + \cdots \quad \text{where} \quad \tau = T \mp t \quad \text{and} \quad T = R^2 \left( GM \frac{9}{2} \right)^{\frac{1}{2}}, \quad g = \frac{GM}{R^2}.
\]

If \( T \) is large, the trajectory of the falling mass (mass falls to a well from the surface of the Earth) is exactly what we usually expect it to be.

In the Newtonian gravitation theory the test mass \( m \) does not change size. In Einstein's relativity theory a mass becomes larger if it is moving with a velocity close to the speed of light. Einstein wrote the Newtonian equation of motion \( F = ma \) in the form
This form allows the mass \( m \) to increase as a function of the time. We find this formula e.g. as the equation before the equation (51) in Einstein's *The Meaning of Relativity* (1922), the lectures he gave in Princeton. In Einstein's proof of \( E = mc^2 \) the mass \( m \) to increases in the rest frame of the mass \( M \). Let us investigate if this is possible: we assume that the test mass \( m \) can change in the rest frame of \( M \). As the mass is moving in the radial direction towards the origin, the equation (2.5) has the form:

\[
F = -\frac{d}{dt}(m(t)\frac{dr(t)}{dt}).
\]  

(2.8)

As \( m \) is a function of time in (2.8), the equivalence principle requires that \( m(r) = m(r(t)) = m(t) \) is a function of \( r \). Consequently, we have to write the gravitational force as

\[
F = \frac{d}{dr}(m(r)\phi(r)).
\]  

(2.9)

The equation of motion (2.3) gets the form

\[
\frac{d}{dr}(m(r)\phi) = -\frac{d}{dt}m(t)\frac{dr}{dt}.
\]  

(2.10)

This yields

\[
\frac{dm(r)}{dr} \phi + m(r)\frac{d}{dr}\phi = -\frac{dm(t)}{dt}\frac{dr}{dt} - m(t)\frac{d^2r}{dt^2}.
\]  

(2.11)

As \( m(r) = m(r(t)) = m(t) \) we can write

\[
m \left( \frac{d\phi}{dr} + \frac{d^2r}{dt^2} \right) = -\frac{dm(t)}{dt}\frac{dr}{dt} - \frac{dm(r)}{dr}\phi
\]  

(2.12)

and since in this case \( r = r(t) \), we have

\[
m \left( \frac{d\phi}{dr} + \frac{d^2r}{dt^2} \right) = -\frac{dr}{dt} \frac{dm(r)}{dr} \frac{dr}{dr} - \frac{dm(r)}{dr}\phi
\]  

(2.13)

which is simplified to

\[
m \left( \frac{d\phi}{dr} + \frac{d^2r}{dt^2} \right) = -\frac{dm}{dr} \left( \left( \frac{dr}{dt} \right)^2 + \phi \right)
\]  

(2.14)

and finally to the form

\[
m^{-1} \frac{dm}{dr} = -\left( \frac{d\phi}{dr} + \frac{d^2r}{dt^2} \right) \left( \left( \frac{dr}{dt} \right)^2 + \phi \right)^{-1}
\]  

(2.15)

Let us denote

\[
f = \left( \frac{dr}{dt} \right)^2 + \phi
\]  

(2.16)

then
\[
\frac{df}{dr} = \frac{d}{dr} \left( \left( \frac{dr}{dt} \right)^2 + \varphi \right) = \frac{d\varphi}{dr} + 2 \frac{d^2 r}{dt^2} \cdot \tag{2.17}
\]

We can write the equation of motion (2.10) as
\[
m^{-1} \frac{dm}{dr} = \left( \frac{d\varphi}{dr} + 2 \frac{d^2 r}{dt^2} \right) \left( \left( \frac{dr}{dt} \right)^2 + \varphi \right)^{-1} + \frac{d^2 r}{dt^2} \left( \left( \frac{dr}{dt} \right)^2 + \varphi \right)^{-1} \cdot \tag{2.18}
\]
and if \( dm/\,dr \) is not zero we can write (2.18) in the form
\[
\frac{d}{dr} \log(mf) = \frac{d^2 r}{dt^2} f^{-1} \cdot \tag{2.19}
\]

Let \(|dr/dt| \) and \(|d^2 r/\,dt^2| \) be so small that the mass \( m \) does not move with speed close to the speed of light and the increase of the mass \( m \) can be ignored. This does not imply that \(|dr/dt| \) and \(|d^2 r/\,dt^2| \) are very small. They are not infinitesimally small, they are only small compared to \( c \). We say that they are sufficiently small. According to Einstein's relativity theory, the increase of mass \( m \) must be very small, thus
\[
m \left( \frac{d\varphi}{dr} + \frac{d^2 r}{dt^2} \right) = -\frac{dm}{dr} \left( \left( \frac{dr}{dt} \right)^2 + \varphi \right) \approx 0 \tag{2.20}
\]
and the solution must be very close to the solution in the Newtonian gravitation theory
\[
r \approx \left( \frac{GM}{2} \right)^{\frac{3}{2}} t^{\frac{3}{2}} \cdot \tag{2.21}
\]

However, if \(|dr/dt| \) and \(|d^2 r/\,dt^2| \) are sufficiently small, the value of the mass \( m \) cannot have any effect to the right side in the equation (2.19) because mass \( m \) cancels out in the Newtonian gravitation theory: in Newtonian mechanisms all masses fall in a gravitation field with the same speed. Integrating (2.19) with respect to \( r \)
\[
\log(mf) = \int \frac{d^2 r}{dt^2} f^{-1} dr \cdot \tag{2.22}
\]
We see that \( f \) does not depend on \( m \). We also see that the total energy is
\[
E = E_k + E_p = \frac{1}{2} m \left( \frac{dr}{dt} \right)^2 + m\varphi = mf - \frac{1}{2} m \left( \frac{dr}{dt} \right)^2 \cdot \tag{2.23}
\]
If the total energy is constant, \( mf \) depends linearly on \( m \), so \( \log(mf) \) depends on mass. The right side (2.18) does not depend on mass. This is not because there is no parameter \( m \) in the right side. It is because Galileo showed that all masses fall in the same way: the equation of \( r \) in the field \( \varphi \) does not depend on the mass \( m \) of the test particle. This mass cancels out in (2.3). In order to make it clear that the right side of (2.19) does depend on \( m \), compose the mass \( m \) from \( N \) small parts \( \Delta m \), \( m = N\Delta m \) and let \( N \) depend on \( r \), so that \( m(r) = N(r)\Delta m \). Every small mass \( \Delta m \) falls in the same way, so they all give the same
function in the right side of (2.19). The dependence of \( m(r) \) on \( r \) means that \( (d/dr)\log(m) \) is not zero and it depends on \( m(r) \).

We assume that \( dm/ dr \neq 0 \). One way to remove the dependency of the left side of (2.19) from mass is to set \( f = 0 \) exactly (and not approximately) if \( |dr/ dt| \) and \( |d^2r/ dt^2| \) are sufficiently small. This is because even a small nonzero value of \( f \) lets a large value of \( m \) influence the right side. We notice that

\[
f' = \left(\frac{dr}{dt}\right)^2 + \varphi = 0
\]

(2.24)

has a solution that is similar to the previous exact solution, but not the same

\[
r = \left(GM\right)^{\frac{1}{3}}t^{\frac{2}{3}} \quad \text{for} \quad \varphi = -GM/r.
\]

(2.25)

Inserting this solution we get

\[
\left(\frac{d\varphi}{dr} + \frac{d^2r}{dt^2}\right) = \left(GM\right)^{\frac{1}{3}}\frac{4}{3}t^{\frac{4}{3}} - \left(GM\right)^{\frac{1}{3}}\frac{2}{3} t^{\frac{4}{3}} = \frac{7}{9} \left(GM\right)^{\frac{1}{3}}t^{\frac{4}{3}}
\]

(2.26)

Thus (2.20)

\[
m\left(\frac{d\varphi}{dr} + \frac{d^2r}{dt^2}\right) \approx 0
\]

(2.27)

is not satisfied. This way of solving the problem in (2.19) is not possible.

Let us now assume that \( dm/ dr \neq 0 \) and \( f \neq 0 \). There is still one way to try to satisfy (2.19). If

\[
m = \exp\left(\int_{a}^{r} h(r) dr\right) = e^{H(r)} e^{-H(a)}
\]

(2.28)

for some smooth function \( h(r) = dh(r)/ dr \), then

\[
\frac{d}{dr} \log(m) = h(r) \quad \text{and we can set} \quad m_0 = e^{-H(a)}.
\]

(2.29)

The \( H(r) \) can depend only on the trajectory that is the same for all masses. Explicitly, we can demand that

\[
m = m_0 \left(1 - \frac{1}{c^2} \left(\frac{dr}{dt}\right)^2\right)^{-\alpha}.
\]

(2.30)

For \( \alpha = 1/2 \) we have Einstein's formula for moving mass. Let us assume this is the case, then

\[
h(r) = \frac{d}{dr} H(r) = -\alpha \frac{d}{dr} \log\left(1 - \frac{1}{c^2} \left(\frac{dr}{dt}\right)^2\right) = \alpha \frac{1}{c^2-\left(\frac{dr}{dt}\right)^2} \frac{1}{c^2-\left(\frac{dr}{dt}\right)^2} \frac{d}{dr} \left(\frac{dr}{dt}\right)^2 = \alpha \left(c^2-\left(\frac{dr}{dt}\right)^2\right)^{-1} \frac{d^2r}{dt^2}.
\]

(2.31)

From (2.15)
\[ h(r) = m^{-1} \frac{dm}{dr} = -\left( \frac{d\varphi}{dr} + \frac{d^2 r}{dt^2} \right) \left( \frac{dr}{dt} + \varphi \right)^{-1} \] (2.32)

we get an equation
\[ 2\alpha \frac{d^2 r}{dt^2} \left( \frac{dr}{dt} + \varphi \right) = -\left( \frac{d\varphi}{dr} + \frac{d^2 r}{dt^2} \right) \left( c^2 - \left( \frac{dr}{dt} \right)^2 \right). \] (2.33)

We make a small calculation
\[ 0 = 2\alpha \frac{d^2 r}{dt^2} \varphi + (2\alpha - 1) \frac{d^2 r}{dt^2} \left( \frac{dr}{dt} \right)^2 + \frac{d\varphi}{dr} \left( c^2 - \left( \frac{dr}{dt} \right)^2 \right) + c^2 \frac{d^2 r}{dt^2} \] (2.34)
\[ 0 = 2\alpha \frac{d^2 r}{dt^2} \varphi + (2\alpha - 1) \frac{d^2 r}{dt^2} \left( \frac{dr}{dt} \right)^2 + \frac{dr}{dt} \frac{d\varphi}{dr} \left( c^2 - \left( \frac{dr}{dt} \right)^2 \right) + \frac{1}{2} c^2 \frac{d}{dt} \left( \frac{dr}{dt} \right)^2 \] (2.35)

Let us insert \( \alpha = 1/2 \). Then the equation is easily solved:
\[ c^2 \frac{d\varphi}{dt} + \frac{1}{2} c^2 \frac{d}{dt} \left( \frac{dr}{dt} \right)^2 = \frac{d\varphi}{dt} \left( \frac{dr}{dt} \right)^2 - \frac{1}{2} \varphi \frac{d}{dt} \left( \frac{dt}{dt} \right)^2 \] (2.36)
gives
\[ \frac{d\varphi}{dt} \left( c^2 - \left( \frac{dr}{dt} \right)^2 \right) = -\frac{1}{2} \frac{d}{dt} \left( \frac{dt}{dt} \right)^2 \left( c^2 + \varphi \right) \] (2.37)
and
\[ \frac{d\varphi}{dt} \left( c^2 + \varphi \right)^{-1} = -\frac{1}{2} \frac{d}{dt} \left( \frac{dt}{dt} \right)^2 \left( c^2 - \left( \frac{dr}{dt} \right)^2 \right)^{-1}, \] (2.38)

which can be integrated
\[ \log(c^2 + \varphi) = \frac{1}{2} \log \left( c^2 - \left( \frac{dr}{dt} \right)^2 \right) + \log B \] (2.39)
where \( B \) is an integration constant. Thus,
\[ c^2 + \varphi = B \sqrt{c^2 - \left( \frac{dr}{dt} \right)^2}. \] (2.40)

We must set \( B = c \) in order to cancel the leading term in
\[ c^4 + 2c^2 \varphi + \varphi^2 = B^2 c^2 - B^2 \left( \frac{dr}{dt} \right)^2. \] (2.41)
The final equation of movement is
\[ 2\phi + \frac{1}{c^2} \phi^2 = -\left(\frac{dr}{dt}\right)^2. \] (2.42)

This equation looks rather strange, but we can put it to a more familiar form by derivating it with respect to time:
\[ 2\frac{d\phi}{dt}\left(1 + \frac{1}{c^2}\phi\right) = -2\frac{d^2r}{dt^2}\left(\frac{dr}{dt}\right) \] (2.43)
and writing is as
\[ \frac{dt}{dr} \frac{d\phi}{dt}\left(1 + \frac{1}{c^2}\phi\right) = -\frac{d^2r}{dt^2}\left(1 + \frac{1}{c^2}\phi\right) = -\frac{d^2r}{dt^2}. \] (2.44)

From this form it is clear that the classical limit is (2.3). Notice that this formula does not have mass. The rest mass of the test mass is in \( m_0 = \exp(-H(a)) \). What (2.44) is claiming is that all sizes of test masses fall according to (2.44). The situation is spherically symmetric. We notice that the equation (2.44) can be written as
\[ \frac{d\Psi}{dr} = -\frac{d^2r}{dt^2} \] (2.45)
and be interpreted as an equation of two forces
\[ F_{\text{field}} = m\nabla \Psi = F_{\text{acceleration}} = ma = m\frac{d^2(r_0 - r)}{dt^2} \] (2.46)
where the mass \( m \) stays constant. This understanding is possible only if \( \alpha = 1/2 \) in (2.35), which seems to mean that \( \alpha = 1/2 \) is the correct value. Equation (2.46) agrees with the basic concepts of \( F = ma \) and \( F = m\nabla \Psi \). Thus, if the gravitation field created by the mass \( M \) is not \( \phi = -GM/r \) but
\[ \psi = -\frac{GM}{r} + \frac{1}{2c^2}\left(\frac{GM}{r}\right)^2. \] (2.47)
We have a normal Newtonian equation of motion and a field that stops the test mass from reaching the speed of light: in small values of \( r \) the second term in \( \Psi \) gives a negative force and slows down the test mass. This is correct: if a test mass is accelerated to speeds close to \( c \), it cannot increase its speed above \( c \), assuming as we do that the speed of light is the maximal speed. A falling test mass loses potential energy, but cannot gain equally much kinetic energy. If there is an energy difference, then it must go somewhere. If it does not go into building a moving mass, is goes into some other form of energy. However, in (2.46) the test mass does not lose much potential energy when it is falling: the energy stays in the form of potential energy. Equation (2.47) also implies that there is a radius
\[ r = \frac{GM}{2c^2} \] (2.48)
where the field \( \Psi \) is zero. This is one understanding of (2.44) where the mass \( m \) does not grow. The other way of understanding (2.44) is that the equation of motion is (2.10) and the mass grows as in (2.30). Equation (2.44) is a direct consequence of (2.10) and (2.30) with
\( \alpha = 1/2 \). This is Einstein's understanding. It can easily be shown wrong. We can solve (2.42) exactly

\[
\frac{dr}{dt} = \pm \left( -2\varphi - \frac{1}{c^2} \varphi^2 \right)^{1/2} = \left( \frac{2GM}{r} \right)^{1/2} \left( \frac{GM}{2c^2r} \right)^{1/2} \left( \frac{2c^2r}{GM} - 1 \right)^{1/2} = \frac{cr}{GM} \left( \frac{2c^2r}{GM} - 1 \right)^{1/2}
\]

(2.49)

where we selected + from \( \pm \) because both \( r \) and \( t \) are positive and inserted \( \varphi = -GM/r \).

Writing

\[
t = \left( \frac{2}{9GM} \right)^{1/3} r^2 + g(r)
\]

(2.50)

for some smooth function \( g(r) \) we get from (2.49)

\[
g'(r) = \frac{cr}{GM} \left( \frac{2c^2r}{GM} - 1 \right)^{1/2} - \frac{\sqrt{r}}{\sqrt{2GM}}
\]

(2.51)

which is integrated to

\[
g(r) = r^{3/2} \sqrt{\frac{2}{GM}} \left\{ 1 - \frac{GM}{2c^2r} - \frac{2}{3} \left( 1 - \frac{GM}{2c^2r} \right)^{3/2} - \frac{1}{3} \right\}.
\]

(2.52)

The integration constant is zero because if \( c \to \infty \), then \( g(r) = 0 \). Thus, (2.50) and (2.52) give the exact trajectory of the test mass \( m \) in Einstein's understanding of (2.10) and (2.30).

We can see from (2.49) that if

\[
\begin{align*}
 & r = \frac{GM}{c^2}, \quad \text{then} \quad \frac{dr}{dt} = c \quad \text{and} \quad F = m \nabla \varphi = \frac{d}{dr} (m(r) \varphi(r)) = 0 \\
 & r = \frac{2GM}{3c^2}, \quad \text{then} \quad \frac{dr}{dt} = \sqrt{\frac{3}{2}} c \\
 & r = \frac{GM}{2c^2}, \quad \text{then} \quad \frac{dr}{dt} = 0 \quad \text{and} \quad \psi = 0.
\end{align*}
\]

(2.53)

Firstly, according to (2.43), the mass \( m \) does reach the speed \( c \) at one point. The inertial mass does grow to infinity at this point if (2.30) is assumed to describe the physical reality, but the gravitational mass in the left side of (2.10) also grows to infinity at the same point: the attraction force also becomes infinite. Secondly, after this point the mass slows down and its velocity goes to zero at the radius (2.48). In Einstein's understanding the gravitational attraction force in (2.10) always increases when \( r \) decreases, so there is no reason why the mass should start slowing down. We conclude that Einstein's understanding is not possible. The natural way to understand what happens in (2.45)-(2.47): the gravitational force induced by the field \( \varphi \) as two components and \( F = m \nabla \psi \), not \( F = m \nabla \varphi \). The reason for this is that the field geometry does not allow a test mass to exceed the speed of light.

The field \( \Psi \) can be continued as zero, or some other function, inside the radius (2.48). Therefore there need not be any singularity in the gravitational field, which is good as singularities should not appear in physical systems. The field \( \Psi \) is a spherically symmetric scalar field. Such a field cannot be obtained from the field equation of the General Relativity Theory, but it comes naturally from Nordström's first gravitation theory where the field equation is

\[
\Box \varphi = 4\pi G \rho \quad \text{when} \quad \eta = (+,-,-,-).
\]

(2.54)
with
\[ F = m \nabla \Psi \quad \text{where} \quad \Psi = \phi + \frac{1}{2c^2} \phi^2. \]

(2.55)

Applied to electromagnetism, the Maxwell equations (the field equation) is not changed, but the Coulomb force is changed. We can now look at Einstein's proof of \( E = mc^2 \) as it is often presented in modern times.

Gunnar Nordström collaborated with Einstein and Einstein got him confused with the proper time (and later also with the stress tensor). Thus, Nordström wrote the equation of motion of the test mass with the proper time of the Special Relativity Theory (SRT). We will show that SRT is wrong and the proper time should not be used. The time in the equation of motion is most naturally understood as Euclidean time, though it might be the gravitational local time. The correct equation of motion is simply the Newtonian formula:
\[ F = ma = m \frac{d^2s}{dt^2}. \]

(2.56)

### 3. The error in Einstein's proof of \( E=mc^2 \)

The proof is very simple. From \( m_0 = m \sqrt{1 - \beta^2} \) we get by squaring
\[ m_0^2 c^2 = m^2 c^2 - m^2 v^2. \]

(3.1)

Assuming that this equation holds when \( v \) is not constant, we can differentiate
\[ 0 = 2mc^2 dm - 2mv^2 dm - 2m^2 v dv \]

(3.2)

and obtain
\[ c^2 dm = v^2 dm + mv dv. \]

(3.3)

Inserting the equation of motion
\[ F = \frac{d}{dt} (mv) = \frac{dm}{dt} v + m \frac{dv}{dt} \]

(3.4)

to
\[ dW_K = F ds = v \frac{dm}{dt} ds + m \frac{dv}{dt} ds = v \frac{ds}{dt} dm + m \frac{ds}{dt} dv = v^2 dm + mv dv = c^2 dm \]

(3.5)

we get
\[ E = \int_{m_0}^m dW_K = \int_{m_0}^m c^2 dm = mc^2 - m_0 c^2. \]

(3.6)

Section 2 shows in (2.53) that \( m \) does not grow. Since \( m_0 = m \sqrt{1 - \beta^2} \) is not correct, this proof fails. Notice that this proof works in the frame of reference where the mass is moving. It is not in the rest frame of the mass. In that frame \( v = 0 \).

The equation \( E = mc^2 \) is not any deep result and it has nothing to do with mass growing when the velocity is increasing. This equation was first published by Olinto de Pretto and should be called De Pretto's equation. In a discrete model it is trivial to derive it. Consider mass \( m \) being originally at rest and then speeded in the time \( \Delta t \) to the velocity \( c \). Thus, the velocity difference is \( \Delta v = c \) and the acceleration is \( a = \Delta v / \Delta t = c (1 / \Delta t) \). The force needed for giving the mass \( m \) this acceleration is \( F = ma = mc (1 / \Delta t) \). In the time \( \Delta t \) the object
moves a distance $\Delta s$. The force acts for this distance $\Delta s$, thus the work is $W = F\Delta s$ and we get

$$W = F\Delta s = mc \frac{\Delta s}{\Delta t}. \quad (3.7)$$

The term $\Delta s/\Delta t$ is a velocity. In a continuous space-time this velocity would be the average velocity where the velocity increases linearly from zero to $c$. Thus, we would get the usual formula for the kinetic energy for mass $m$ moving with speed $c$

$$W = mc \frac{\Delta s}{\Delta t} = mc \frac{1}{2}c = \frac{1}{2}mc^2. \quad (3.8)$$

In a discrete model this is different. The mass accelerates in one discrete time unit $\Delta t$ and the space unit is $\Delta s = c\Delta t$. The velocity in a space unit can be either zero or $c$ and nothing between. Then we do get

$$E = W = mc \frac{\Delta s}{\Delta t} = mc^2. \quad (3.9)$$

Speeding a mass to a velocity $c$ in (3.9) must be understood in the sense that what gets this velocity $c$ must be massless. The baryon number must be conserved in any nuclear reaction where mass changes to energy. If the sum of mass before and after the reaction does not match, then the missing mass has turned to energy.

A discrete space-time model also explains why the maximum speed is $c$: it is the lattice speed of the space. A discrete model with a lattice speed is usually discarded as such a model is not Lorentz invariant. It will be seen in Sections 4 and 5 that the Lorentz transform is incorrect and there is no reason to demand a model to be Lorentz invariant. The correct demand is that the geometry is conformal. In Section 4 we show that the Special Relativity Theory is wrong by a simple counterexample so that there remains no question that the derivation of $m = m_0/\sqrt{1 - \beta^2}$ by requiring Lorentz invariance of $F = (d/dt)m(t)(ds/dt)$ could be considered as a valid argument.

### 4. A simple counterexample to the Special Relativity Theory

Chapter 4 in the book *The River of Time* (1997) by Igor Novikov outlines the evidence for the Special Relativity Theory and on page 83 of 265 pages in my translation of the book he explains the situation where an observer is in moving frame (M-frame) and another observer is in a rest frame (R-frame). The observer in the rest frame calculates that the observer in the moving frame ages slower, while the observer in the moving frame calculates that the observer in the rest frame ages slower. Novikov explains this in the way that both are correct and there is no contradiction in it: an observer in each frame simply has his own view and the views can differ. He compares it to a situation that if the M-frame observer is bouncing a ball, it goes up and down, while the observer in the R-frame sees the ball bouncing and advancing at the same time.

The comparison is incorrect. There really is a contradiction. Let the M-frame be a very fast train and the R-frame the railway platform. We put $n$ lamps to the end of the train hanging them outside so that they can be seen to the platform. We also put $n$ lamps to the midpoint of the platform so that they can be seen to an observer standing in a doorway of the train. The number $n$ is assumed large and the observers in the train and on the platform very good in
counting lamps though the train moves with the speed close to the speed of light. (We can do this type of experiments with muons, though not necessarily this one.)

Let all lamps be burning at the starting time \(T=0\). Then the lamps go off in each frame separately according to a negative exponential process. The number of lamps that have turned off in the \(R\)-frame at the time \(T\) is thus:

\[
n_{RR}(T) = n\left(1 - e^{-\lambda_{RR}T}\right).
\]

We choose

\[
\lambda_{RR} = \frac{2\ln 2}{T_R},
\]

where \(T_R\) is the measurement time in the \(R\)-frame. In the time \(T_R\) the number of lamps in the \(R\)-frame that have turned off is

\[
n_{RR} = n_{RR}(T_R) = n\left(1 - e^{-\lambda_{RR}T_R}\right) = n(1 - e^{-2\ln 2}) = \frac{3}{4} n.
\]

Let us assume, like Novakov does, that it is only about a different view and both frames are exactly the same. After all, for the Special Relativity Theory there is no preferred frame of reference. Therefore the number of lamps that have turned off in the \(M\)-frame in the time \(T'\), the local time in the \(M\)-frame (we have different local times in the two frames), is

\[
n_{MM}(T) = n\left(1 - e^{-\lambda_{MM}T'}\right)
\]

where we choose

\[
\lambda_{MM} = \frac{2\ln 2}{T_M}.
\]

\(T_M\) is the measurement period in the \(M\)-frame. In the \(M\)-frame's time the measurement period is \(T_M\) the number of lamps that have turned off in the \(M\)-frame is

\[
n_{MM} = n_{MM}(T_M) = n\left(1 - e^{-\lambda_{MM}T_M}\right) = n(1 - e^{-2\ln 2}) = \frac{3}{4} n.
\]

Clearly, this is a symmetric situation.

Let the train move with the speed \(v = c\frac{\sqrt{3}}{2}\). Then \(\sqrt{1 - (v/c)^2} = 1/2\).

The observer in the \(R\)-frame calculates from the Special Relativity Theory that the time in the \(M\)-frame is twice as long as in the \(R\)-frame, thus he sees that the lamps in the \(M\)-frame turn off with the process

\[
n_{MR}(T) = n\left(1 - e^{-\lambda_{MR}T}\right)
\]

where

\[
\lambda_{MR} = \frac{\ln 2}{T_R}
\]

and \(n_{MR}(T)\) is the number of lamps that the observer in the \(R\)-frame saw turn off in the \(M\)-frame in the measurement time \(T_R\). This number is

\[
n_{MR} = n_{MR}(T_R) = n\left(1 - e^{-\lambda_{MR}T_R}\right) = n(1 - e^{-\ln 2}) = \frac{1}{2} n.
\]
Here ≜ stands for defined by. As \( n_{RR} > n_{MR} \) the observer in the R-frame concludes that the time goes slower in the M-frame.

Likewise, the observer in the M-frame calculates from the Special Relativity Theory that the time in the R-frame is twice as long as in the M-frame, thus he sees that the lamps in the R-frame turn off with the process

\[
n_{RM}(T) = n(1 - e^{-\lambda_{RM} T})
\]

(4.10)

where

\[
\lambda_{RM} = \frac{\ln 2}{T_M}
\]

(4.11)

and \( n_{MR}(T') \) is the number of lamps that the observer in the M-frame saw turn off in the R-frame in the measurement time \( T_M \). This number is

\[
n_{RM} = n_{RM}(T_M) = n(1 - e^{-\lambda_{RM} T_M}) = n(1 - e^{-n^2}) = \frac{1}{2} n
\]

(4.12)

As \( n_{MM} > n_{RM} \) the observer in the M-frame concludes that the time goes slower in the R-frame. Very clearly, we still have a completely symmetric situation. Now we go to the problem. The observer in the R-frame can see if lamps turn off in the M-frame. He need not see all in the measurement time, but we assume that the moving frame is close to the R-frame the distance being \( d \). The time for the light from the lamps requires the time

\[
T_I = \frac{2d}{c}
\]

(4.13)

for a roundtrip. Thus, the observer in the R-frame can see lamps that turn off in the time period if the length \( T_R - T_I \). From this we can conclude that he can see more lamps turn off in the M-frame than in that frame turns off in half of the measurement period \( T_M \). In half of the measurement period in the M-frame turn off

\[
n_{MM}(\frac{1}{2}T_M) = n\left(1 - e^{-\lambda_{MM} \frac{T_M}{2}}\right) = n(1 - e^{-n^2}) = \frac{1}{2} n
\]

(4.14)

lamps. (Logically, in this situation the observer in the R-frame would see all or almost all \((3/4)n\) lamps turn off in the M-frame in the measurement period, but we do not need more than strictly more than \( n/2 \) to get a contradiction.)

It follows that the number \( n_{MR} \) of lamps that the observer in the R-frame saw turn off in the M-frame in the measurement time satisfies the inequality:

\[
n_{MR} > \frac{1}{2} n = \frac{2}{3} n = \frac{2}{3} n_{MM}
\]

(4.15)

Likewise, the number \( n_{RM} \) of lamps that the observer in the M-frame saw turn off in the R-frame in the measurement time satisfies the inequality:

\[
n_{RM} > \frac{1}{2} n = \frac{2}{3} n = \frac{2}{3} n_{RR}.
\]

(4.16)

Thus

\[
n_{MM} = \frac{3}{4} n = \frac{3}{2} n_{RM} > \frac{3}{2} n_{RR} = n_{RR}
\]

(4.17)
and
\[
\frac{n}{k} = \frac{3}{4} n = \frac{3}{2} n_{MR} > \frac{3}{2} \frac{2}{3} n_{MM} = n_{MM}.
\] (4.18)

We have a contradiction:
\[
n_{MM} > n_{RR} > n_{MM}.
\] (4.19)

This contradiction comes because the observers can count most of the lamps that turn off in the other frame. The delay of light from the train to the platform must be sufficiently small, so this is not a counterexample for two stars or planets having a different local time determined by their gravitational fields. As this example does not need events to happen at the same time, this contradiction cannot be explained away by saying that events happening at the same time or in a given order need not do so in another frame of reference. Here we count lamps that turn off in a longer period of time. This simple counterexample shows that SRT is wrong, but a question may remain that how it could be wrong as it is derived correctly. The following three sections show that it is not derived correctly. Section 5 shows that the Michelson-Morley experiment could not measure anything; it is flawed. Section 6 shows that there is a coordinate transform that can set the roundtrip speed of light to \( c \) in a moving frame, but it is not the Lorentz transform, and even the transform that sets the roundtrip speed to \( c \) cannot set the speed of one-way trips to \( c \). There is no coordinate transform that can do it. Section 7 briefly looks at the way the mass transformation formula is obtained from the Lorentz transform. As this transform does not do what it should, there is no reason to demand Lorentz invariance from any equations.

### 5. The error in Michelson-Morley experiment

Let us look at the Michelson-Morley experiment in a simplified version. A square box with each side having the length \( L \) is moving with a constant speed \( v \) with respect to a rest frame of reference \( R \). From the midpoint of one of the sides of the square light is emitted. It shines on the opposite side, reflects from it and returns to the starting point. In the frame \( R \) this roundtrip time is \( T = T_1 + T_2 \) where \( T_1 \) is the time for the light beam to reach the mirror and \( T_2 \) is the time for the light beam to travel from the mirror back to the starting point. Initially, let the velocity \( \vec{v} \) be in the same direction as the light. We select the frame \( R \) such that light travels with a constant speed \( c \) to each direction in \( R \).

\[
\begin{align*}
T_1 & \quad T_2 \\
\text{\quad s}_1 = cT_1 = L + vT & \quad \nu T_1 \\
\nu(T_1 + T_2) & \quad s_2 = cT_2
\end{align*}
\]

Figure 1. The scenario in a simplified Michelson-Morley experiment.
The distance traveled by light in the frame R in the time $T_1$ is $s_1 = cT_1 = L + vT_1$ because the box moves forward distance $vT_1$ in the time $T_1$. The returning light beam hits the side of the box at time $T_1 + T_2$ and the box has moved forward distance $v(T_1 + T_2)$. Thus,

$$s_1 = L + vT_1 = v(T_1 + T_2) + cT_2$$

(yielding $T_2 = \frac{c-v}{c+v} T_1$). From $cT_1 = L + vT_1$ we get $T_1 = \frac{L}{c-v}$ and therefore

$$T = T_1 + T_2 = \left(1 + \frac{c-v}{c+v}\right) \frac{L}{c-v} = \frac{2L}{c} \frac{1}{1-\beta^2}$$

where $\beta = \frac{v}{c}$. (5.2)

The speed of light in the fixed frame R is

$$\frac{cT_1 + cT_2}{T_1 + T_2} = c$$

(5.3)

but we expect to measure the speed of light in the moving frame R' as

$$c' = \frac{2L}{T} = c(1 - \beta^2)$$

(5.4)

but Michelson and Morley said that they measured the speed as $c' = c$.

We can generalize the above outlined arrangement to a case when $\vec{v}$ is not in the same direction as the light beam. If the angle that $\vec{v}$ makes with the direction of the light beam during the time $T_1$ is $\pi/2 - \alpha$, then in time $T_1$ the box moves towards the positive x-direction distance $vT_1 \sin \alpha$ and to the negative y-axis distance $vT_1 \cos \alpha$. From the theorem of Pythagoras we get the equation:

$$s_1^2 = (cT_1)^2 = (L + vT_1 \sin \alpha)^2 + (vT_1 \cos \alpha)^2$$. This second order polynomial equation of $T_1$ is easily solved:

$$T_1 = \frac{L}{c^2 - v^2} \left(\sin \alpha \pm \sqrt{c^2 - v^2 \cos^2 \alpha}\right)$$. (5.5)

For $T_2$ we get the equation

$$s_2^2 = (cT_2)^2 = (L - vT_2 \sin \alpha)^2 + (vT_2 \cos \alpha)^2$$. which yields

$$T_2 = \frac{L}{c^2 - v^2} \left(-\sin \alpha \pm \sqrt{c^2 - v^2 \cos^2 \alpha}\right)$$. (5.6)

Thus,

$$T = T_1 + T_2 = \frac{2L}{c^2 - v^2} \sqrt{c^2 - v^2 \cos^2 \alpha} = \frac{2L}{c} \frac{1}{1-\beta^2} \frac{1}{\sqrt{1-\beta^2 \cos^2 \alpha}}$$

were we had to take the + alternative from ± as the time cannot be negative. The measured velocity of light in the moving frame R' is

$$c' = \frac{2L}{T} = c \left(1 - \beta^2\right) \frac{1}{\sqrt{1-\beta^2 \cos^2 \alpha}}$$. (5.7)

Clearly $c'$ is not $c$, yet Michelson and Morley claimed to have measured that $c' = c$.

Unfortunately, the way Michelson and Morley measured $c'$ was flawed. Michelson and Morley split a beam of light into two beams. Each beam makes a roundtrip that has the length $2L$. These roundtrips are not the same and if the ether hypothesis holds, the roundtrip
time is different in the two paths because the speed of light is different in each path. Michelson and Morley thought that there should be an interference picture when these two light beams are added at the end of the roundtrip. The error in this logic should be obvious: the two beams have the same frequency and they are in the same phase at any chosen time. There cannot be any interference picture when the beams are combined. In order to see this, let the roundtrip times on the two paths be denoted by $T_i$, $i = 1, 2$. The frequency is the same on both paths and only the wavelength is different on each path, thus a frequency component $f$ has the same oscillation time $T_f = 1/f$ on both paths. In order to interfere the two beams must be taken to the same place (the end of the roundtrip) at the same time $T_f$. In order to be at the end of the roundtrip at the time $T_f$, the beam $i$ must have left the splitter at the time $T_F = T_f$. Before the beam left the splitter it had made $f(T_f - T_i)$ wavelengths on the frequency $f$. On the roundtrip the beam $i$ made $T_i / T_f$ oscillations. Thus, at the time $T_F$ both beams made in total $(T_F - T_i) / T_f + T_i / T_f = T_f / T_f$ oscillations in the frequency $f$. The situation is the same for every frequency and we notice that both beams are in exactly the same phase when the researchers try to make them interfere. Naturally there is no interference picture. For some reason this obvious logical error was not noticed. The result of the Michelson-Morley experiment was unexpected and Einstein proceeded to solve the problem how the speed of light can be the same in all moving frames by defining new coordinates for the moving frame $R'$.

That the Michelson-Morley experiment was flawed does not mean that their result was wrong. The result was correct, but for a different reason. The "ether" where light undulates is the gravitational field. The gravitational field of the Earth follows the Earth. Thus, the speed of light is the same to all directions on the Earth. Even if the experiment had been made correctly, the result had been the same, but this result cannot be interpolated to a situation where a bus is moving on the Earth. The bus creates a very weak gravitational field and the field inside the bus is essentially the gravitational field of the Earth. We should not see equal speed of light to all directions in the rest frame of a moving bus (or a muon in a laboratory).

6. A transform to make the speed of light equal to $c$ in $R'$

Let us find a transform that gives the speed of light as $c$ in $R'$ by assuming that the time is not the same in $R$ and in $R'$. If the time in $R'$ is not $T$ but

$$ T' = T \sqrt{1 - \beta^2} = \gamma^{-1}T, \text{ where } \gamma = (1 - \beta^2)^{-\frac{1}{2}}, $$

(6.1)

then for $\cos \alpha = 1$ we have a correct speed:

$$ \frac{2L}{T} = c \sqrt{1 - \beta^2} = c\gamma^{-1} \text{ if } \cos \alpha = 1. $$

(6.2)

Thus

$$ c' = \frac{2L}{T'} = \frac{2L}{\gamma^{-1}T} = \frac{1}{\gamma}\gamma^{-1}c = c \text{ if } \cos \alpha = 1. $$

(6.3)

This works, but in order to get the transform work for other values of $\cos \alpha$ we must change also the roundtrip distance $2L$. That is, there is nothing else we can change: the time cannot depend on $\cos \alpha$ if the time is a scalar. If the roundtrip distance is not $2L$ but $2L'$ then
\[ c' = \frac{2L'}{T'} = \frac{2L'}{T \sqrt{1 - \beta^2}} = c \frac{L'}{L} \frac{\sqrt{1 - \beta^2}}{\sqrt{1 - \beta^2 \cos^2 \alpha}}. \]  

(6.4)

If we define
\[ L' = L \frac{\sqrt{1 - \beta^2 \cos^2 \alpha}}{\sqrt{1 - \beta^2}} = \gamma L \frac{1 - \beta^2 \cos^2 \alpha}{\sqrt{1 - \beta^2}} \]  

(6.5)

then \( c' = c \). Especially, if \( \vec{v} \) is in the direction of the light beam, then \( \cos \alpha = 0 \) and in \( R' \) the measurement rod \( L' \) is longer:
\[ L' = \gamma L, \]  

(6.6)

The distance \( x' \) in \( R' \) corresponding to the distance \( x \) in \( R \) is then of the form
\[ x' = \gamma (x - f_1(t)) \]  

(6.7)

There can be any function \( f_1(t) \) included because \( L' = x'_b - x'_a \) for the beginning and ending points \( x'_a, x'_b \) of the side length \( L' \) measured at the same time instant. Thus
\[ L' = x'_b - x'_a = \gamma (x_b - f_1(t_b)) - \gamma (x_a - f_1(t_a)) = \gamma (x_b - x_a) - \gamma (f_1(t_b) - f_1(t_a)) \]  

(6.8)

Here we noticed that \( L = x_b - x_a \) and when \( L \) is measured, the beginning and ending points \( x_a, x_b \) must be taken at the same time. Thus \( t_b = t_a \). The function \( f_1(t) \) can naturally contain constants \( v \) and \( c \).

In the same way we can construct a time transform:
\[ t' = \gamma^{-1} (t - f_2(x)) \]  

(6.9)

The function \( f_2(x) \) of \( x \) can be chosen freely because the time \( T' = t'_b - t'_a \) is a difference of two times measured in the same point \( x \). The derivation is quite similar to the one in (6.8).

We can complete the transform into a coordinate transform \((x, t, y, z) \rightarrow (x', t', y', z')\) of a 4-space. Let us select \( f_1(t) = vt \) and \( f_2(x) = vx / c^2 \), as these functions can be freely chosen. Then
\[ x' = \gamma (x - vt) \quad x = \gamma (x' + v\gamma t') \]  

(6.10)

\[ t' = \gamma^{-1} \left( t - \frac{vx}{c^2} \right) \quad t = \gamma \left( \gamma^2 t' + \frac{vx'}{c^2} \right) \]

\[ y' = y \quad z' = z \quad \text{where} \quad \beta = \frac{v}{c}, \quad \gamma = (1 - \beta^2)^{-\frac{1}{2}} \]

This is not the Lorentz transform, but we can change it to the Lorentz transform by defining another transform
\[ t'' = \gamma^2 t', \quad x'' = x', \quad y'' = y', \quad z'' = z'. \]  

(6.11)

Then
\[ x'' = \gamma (x - vt) \]  

(6.12)

\[ t'' = \gamma \left( t - \frac{vx}{c^2} \right) \]

\[ y'' = y, \quad z'' = z. \]
This transform has the inverse
\[
x = \gamma(x + \gamma v t)
\]
\[
t = \gamma(t + \frac{\gamma v x}{c^2})
\]
\[
y = y', \quad z = z', \quad \beta = \frac{v^2}{c^2}, \quad \gamma = \gamma^{-1}(1 - \beta^2)^{-1} = (1 - \beta^2)^{\frac{1}{2}}.
\]

The transform (6.12) is known as the Lorentz transform, but it is not the transform that gives the roundtrip speed of light in Figure 1 as \(c\) in the frame \(R'\).

Let us consider a ball that is making the roundtrip of length \(2L\) in the box in Figure 1 just like the light beam, only that the ball has the constant speed \(w\) that is much smaller than \(c\). Let \(v\) also be small and let the ball and the box move in the same direction in the beginning. The situation for the ball is exactly the same as for a light beam, thus the roundtrip time in \(R\) is as in (5.2) by changing \(c\) to \(w\).

\[
T = \frac{2L}{w} \left(1 - \frac{v^2}{w^2}\right)^{-1}.
\]

In order to get the speed of the ball in the frame \(R'\) we simply need to change the coordinates as in (6.10) to
\[
T' = \gamma^{-1}T \quad \text{and} \quad L' = \gamma L.
\]

The result is what we expect it to be
\[
w' = \frac{2L'}{T'} = \frac{2L}{T} \gamma^2 = w \left(1 - \frac{v^2}{w^2}\right)^{\frac{1}{2}}.
\]

If \(\beta\) is very small, the speed \(w'\) close approximates the value it has in Newtonian mechanics (i.e., (5.4) with \(c\) changed to \(w\)), while if \(w = c\) then \(w' = c\). This is exactly as we want it to be: in the \((x',t')\)-coordinates light travels with the speed \(c\) in both frames \(R\) and \(R'\) in Figure 1. If we instead of (6.10) use the Lorentz transform (6.12), then
\[
T'' = \gamma T \quad \text{and} \quad L'' = \gamma L
\]

and
\[
w'' = \frac{2L''}{T''} = \frac{2L}{T} \gamma w = \left(1 - \frac{v^2}{w^2}\right).
\]

According to (6.18), if \(w = c\), we still measure the roundtrip speed in Figure 1 in the moving frame \(R'\) according to the Newtonian formula (5.4). The Lorentz transform (6.12) did not do the trick we wanted it to do. It is not the transform that gives the speed of light as \(c\) in the moving frame in Figure 1. The transform (6.10) is the correct transform, because in that transform time stops in \(R'\) when \(v\) approaches \(c\). In the Lorentz transform (6.12) time goes to infinity (a clock runs very fast) when \(v\) approaches \(c\).

Interestingly, if we do not look at the roundtrip speed but the speed only on the first part of the trip in Figure 1, then the transforms are as follows. The time \(T_i\) in (5.1)-(5.2)
\[
T_i = \frac{L}{w - v}
\]

is a time period and it transforms as in (6.10) as \(T_i' = \gamma^{-1}T_i\) and in (6.12) as \(T_i'' = \gamma T_i\). Thus
\[ w' = \frac{L'}{T'_1} = \frac{xL}{y'^{-1}T'_1} = \gamma^2 (w-v) \] (6.20)

\[ w'' = \frac{L''}{T''_1} = \frac{xL}{yT'_1} = w-v. \]

Let \( w = c \) in (6.20):

\[ c' = \gamma^2 (c-w) = \frac{c-w}{1 - \frac{v^2}{c^2}} = \frac{c-w}{c^2 - v^2} = c \frac{c}{c+v} \] (6.21)

\[ c'' = c - v \]

We notice that the speed of light in the moving frame \( R' \) in the first part of the roundtrip in Figure 1 is not measured as \( c \) in either of the transforms (6.10) or (6.12). The transform (6.10) can make the roundtrip measured speed to \( c \) in \( R' \), the transform (6.12) cannot even do that, but neither can achieve what Einstein wanted the transform to do.

### 7. Invariant equations of motion

Maxwell equations are invariant in the Lorentz transform (but only if E and B are required to transform in a special way to make the equations Lorentz invariant) and Einstein proceeded to require that all equations of motion must be invariant in the Lorentz transform. Later the equations were required to be covariant, but this concept only applies to tensor equations. There is no difference between these concepts in ordinary partial differential equations. What is meant is that if \((x', t')\) of the moving frame of reference is inserted to the equations of \((x, t)\) for the rest frame of reference, then the equation of motion for \((x'', t'')\) has the same form as for \((x, t)\).

The basic equation of movement in Newtonian mechanics is \( F = ma \). Let the frame \( R' \) move with a constant speed \( \vec{v} \) and let the mass \( m \) move in the same direction as \( \vec{v} \). Initially the velocity of the mass in the frame \( R \) seems to be

\[ w = \frac{dx}{dt} \] (7.1)

but this is not so for the following reason. In the Lorentz transform (6.12) \( x \) and \( t \) are independent coordinates in \( R \). In \( R' \) the coordinates \( x'' \) and \( t'' \) are independent. Thus

\[ x = \gamma (x'' + vt'') \quad \text{gives} \quad \frac{dx}{dt''} = \gamma v, \] (7.2)

\[ t = \gamma \left( t'' + \frac{vx''}{c^2} \right) \quad \text{yields} \quad \frac{dt}{dt''} = \gamma \]

and

\[ x'' = \gamma (x - vt) \quad \text{leads to the velocity in the frame} \ R' \]

\[ w'' = \frac{dx''}{dt''} = \gamma \left( \frac{dx}{dt''} - v \frac{dt}{dt''} \right) = \gamma (\gamma v - v) = 0. \] (7.3)

This is not correct: the mass is not in rest in \( R' \). The velocity of the mass is not obtained by derivation from (6.12). The velocity is actually
\[ w = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}, \quad (7.4) \]

and both \( \Delta x \) and \( \Delta t \) are intervals. Therefore they transfer in (6.12) as

\[ \Delta x'' = \gamma \Delta x \quad \text{and} \quad \Delta t'' = \gamma \Delta t. \quad (7.5) \]

We get from (6.12)

\[ w'' = \lim_{\Delta t'' \to 0} \frac{\Delta x''}{\Delta t''} = \lim_{\gamma \Delta t \to 0} \frac{\gamma \Delta x}{\gamma \Delta t} = w. \quad (7.6) \]

In the transform (6.10) we have

\[ \Delta x'' = \gamma \Delta x \quad \text{and} \quad \Delta t'' = \gamma^{-1} \Delta t. \quad (7.7) \]

We get from (6.10)

\[ w'' = \lim_{\Delta t'' \to 0} \frac{\Delta x''}{\Delta t''} = \lim_{\gamma \Delta t \to 0} \frac{\gamma \Delta x}{\gamma^{-1} \Delta t} = \gamma^2 w. \quad (7.8) \]

Notice that \( \beta = v/c \) is constant as the velocity \( \overline{v} \) is constant. Only the mass \( m \) can accelerate in this case, not the frame \( R' \). In \( R \) the equation of motion is

\[ F = \frac{d}{dt} \left( m \frac{dx}{dt} \right). \quad (7.9) \]

In \( R' \) following the transform (6.12) we get

\[ F'' = \frac{d}{dt''} \left( m'' \frac{dx''}{dt''} \right) = \frac{1}{\sqrt{1 - \beta^2}} \frac{d}{dt} \left( m' \frac{dx}{dt} \right). \quad (7.10) \]

In the frame \( R' \) taking the transform (6.10) we have

\[ F' = \frac{d}{dt'} \left( m' \frac{dx}{dt'} \right) = \frac{1}{\gamma^{-1}} \frac{d}{dt} \left( m' \gamma^2 \frac{dx}{dt} \right) = \frac{1}{(1 - \beta^2) \sqrt{1 - \beta^2}} \frac{d}{dt} \left( m' \frac{dx}{dt} \right). \quad (7.11) \]

Assuming that the force \( F'' \) equals force \( F' \), we notice that there is a solution for (7.10) where \( m' \) and \( m'' \) do not depend on the time \( t \). For

\[ F = m \frac{d^2 x}{dt^2} \quad (7.12) \]

\[ F = F'' = \frac{m''}{\sqrt{1 - \beta^2}} \frac{d^2 x}{dt^2}. \]

The equations (7.9) and (7.10) are identical (the equation is invariant in the Lorentz transform) if

\[ m = \frac{m''}{\sqrt{1 - \beta^2}}. \quad (7.13) \]

This is enough for the equation of motion to be invariant, but Einstein went further and decided that the mass changes in the frame \( R \) as
and in a frame that is accelerating with the mass he defined $m'' = m_0$. Notice this very odd choice. In the cases of $T''$ and $L''$ the properties in $R$ are kept as they were and the modified versions are in $R'$, but the mass changes in $R$. This is essential for Einstein's proof of $E=mc^2$ as was seen in Section 3.

If we use the transform (6.10), which gives the speed of light in $R'$ in the roundtrip in Figure 1 as $c$, then the time independent value of $m'$ that makes (7.9) and (7.10) identical if $F' = F$ is

$$m = \frac{m'}{(1-\beta^2)\sqrt{1-\beta^2}}.$$

This mass change formula would ruin Einstein's proof of $E=mc^2$, but notice that (6.12) does not give gives the speed of light in $R'$ in the roundtrip in Figure 1 as $c$, and neither (6.10) nor (6.12) give the speed of light in $R'$ as $c$ for the first part of the roundtrip in Figure 1. Einstein's proof of $E=mc^2$ fails if the mass change is in $R'$.

The question is: what is the sense with requiring invariance of $F = ma$ under the Lorentz transform? The force is not a conserved property. The Lorentz transform does not make the speed of light in the roundtrip in the Michelson-Morley experiment equal to $c$. The transform that makes the roundtrip speed of light equal to $c$ cannot make the speed of light in a one-way trip equal $c$. The Michelson-Morley experiment is in flawed and should not be considered at all. Finally, by Section 2 the mass does not change in the frame where the gravitational field is at rest, so there is no sense to write $F = (d/dt)\rho(t)(ds/dt)$.

The parts of the relativity theory that we so far looked at (the mass transformation formula, Einstein's proof of $E=mc^2$, and the Lorentz transform in SRT) all seem to be incorrect. In case someone still has any hope that some part of the relativity theory might be correct, let us continue to the General Relativity Theory. It is fundamentally flawed as the next section shows.

8. A simple calculation shows the General Relativity Theory incorrect

Einstein proposed four experiments for verifying his General Relativity Theory (GR). He forgot the most obvious test: does the General Relativity Theory predict correctly the gravitational field on the Earth or close to the Earth on a satellite orbit. It does not and this is the fatal error in this theory. We can make this test on paper because the experimental part has been done long ago: the gravitational field on the Earth appears to be the Newtonian gravitation potential:

$$\phi = \phi(r) = -\frac{G\rho}{r},$$

which is the solution to the Newtonian field equation

$$\Delta \phi = -4\pi G\rho.$$

We will assume that the mass is a spherical mass with a finite radius. In the empty space outside this radius $\rho$ is constant and we have

$$m = \frac{m_0}{\sqrt{1-\beta^2}}$$

(7.14)
\[ \frac{\partial^2 \phi}{\partial x^2} = G \rho \frac{1}{r^5} (r^2 - 3x^2) . \]  
(8.3)

By symmetry
\[ \Delta \phi = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi = G \rho \frac{1}{r^5} (3r^2 - 3x^2 - 3y^2 - 3z^2) = 0. \]  
(8.4)

Thus, in the empty space outside the mass the solution fulfills the equation
\[ \Delta \phi = 0 . \]  
(8.5)

In the same way, if we solve the field equation of GR
\[ R_{ab} - \frac{1}{2} R = k_g T - \lambda g_{ab} \]  
(8.6)

in the empty 4-space outside a spherical mass, the equation reduces to
\[ R_{ab} = \frac{1}{2} R - \lambda g_{ab} . \]  
(8.7)

The space around us is three dimensional and we can give it a metric. Typically we give the space a Cartesian metric and get the Euclidean three-dimensional space. However, Einstein wanted a different metric. He wanted that light rays move on geodesics of the metric and that at any point and at any time light has the constant speed \( c \). Einstein had more design criteria: e.g. he wanted that the theory is Lorentz covariant and that causality is preserved, but at the moment we focus only on these two demands: 1) light follows geodesics and 2) in the metric \( g_{ab} \) light has the constant speed \( c \) in any space element into any direction.

The Newtonian gravitational potential (8.1) yields in orthogonal local coordinates the metric
\[ ds^2 = c^{-2} g_{00} \phi^2 - g_{11} \phi^2 - g_{22} \phi^2 - g_{33} \phi^2 \]  
(8.8)

For easier notations we set \( c = 1 \) by rescaling seconds and meters. For any scalar potential field \( \varphi \), the metric \( g_{ab} \) corresponding to the field \( \varphi \) in Cartesian coordinates \((t, x, y, z)\) where \( x^0 = t \), \( x^1 = x \), \( x^2 = y \), \( x^3 = z \) and the signs are (+,-,-,-), is given by
\[ g_{00} = \varphi^2 , \quad g_{11} = -\varphi^2 , \quad g_{22} = -\varphi^2 , \quad g_{33} = -\varphi^2 , \quad \text{and} \quad g_{ab} = 0 \text{ if } a \neq b . \]  
(8.9)

In spherical coordinates \((t, r, \theta, \phi)\), where \( x^0 = t \), \( x^1 = r \), \( x^2 = \theta \), \( x^3 = \phi \) and the signs are (+,-,-,-), the metric is given by
\[ g_{00} = \varphi^2 , \quad g_{11} = -\varphi^2 , \quad g_{22} = -\varphi^2 r^2 , \quad g_{33} = -\varphi^2 r^2 \sin^2 \theta , \quad \text{and} \quad g_{ab} = 0 \text{ if } a \neq b . \]  
(8.10)

For any orthogonal metric (i.e., \( g_{ab} = 0 \) if \( a \neq b \)) holds
\[ g^{aa} = \frac{1}{g_{aa}} \]  
(8.11)

and the Christoffel symbols satisfy (the notation \( g_{ab,c} = \partial_c g_{ab} \))
\[ \Gamma^a_{aa} = \frac{1}{2} g^{aa} g_{aa,a} , \]  
(8.12)
\[ \Gamma^a_{ba} = \frac{1}{2} g^{aa} g_{ab}, \text{ if } a \neq b \]
\[ \Gamma^a_{bb} = -\frac{1}{2} g^{aa} g_{bb}, \text{ if } a \neq b \]
\[ \Gamma^a_{bc} = 0, \text{ if } a \neq b, a \neq c \text{ and } c \neq b. \]

In order to get the Einstein equations we calculate the Christoffel symbols for the metric in both coordinate systems. Then we calculate the Ricci entries
\[ R_{bd} = R^a_{bad} = \Gamma^a_{ba,d} - \Gamma^a_{bd} \Gamma^d_{ae} - \Gamma^e_{bd} \Gamma^a_{ae}. \] (8.13)

All ways to do the calculation are tedious. One way to calculate is the following. In an orthogonal metric
\[ R_{ji} = \sum_{i,j} \left[ \frac{1}{4} g^{ii} g_{jj} \left( g^{jj} g_{jj,j} - g^{ij} g_{ij,j} \right) - \frac{1}{2} \frac{\partial}{\partial j} \left( g^{ii} g_{ij,j} \right) \right], \quad j = 0, 1, 2, 3 \] (8.14)

where we have avoided implicit summation to make it clearer what this calculation implies.

In an orthogonal metric the off-diagonal Ricci entries have the equation
\[ R_{ji} = \frac{1}{4} \sum_{k=0}^{4} g^{kk} g_{kk,j} \left( g^{ii} g_{ii,j} - g^{ij} g_{ij,j} \right) + \frac{1}{2} \frac{\partial}{\partial j} \left( g^{ii} g_{ii,j} \right) \]
\[ + \frac{1}{4} g^{ij} g_{jj} \sum_{k=0}^{4} g^{lk} g_{kk,j}. \]

The off-diagonal Ricci entries are zero both in Cartesian and in spherical coordinates. After a fairly long calculation the result is as follows.

For Cartesian coordinates the nonzero Ricci entries are:
\[ R_{00} = -\frac{1}{c^2} \psi^{-1} \square \psi + 3 \psi^{-2} \left( \frac{\partial \psi}{\partial t} \right)^2 + \frac{1}{c^2} \psi^{-2} \sum_{i=1}^{3} \left( \frac{\partial \psi}{\partial x_i} \right)^2 - 2 \psi^{-1} \frac{\partial^2 \psi}{\partial t^2} \]
\[ R_{ii} = \psi^{-1} \square \psi + c^2 \psi^{-2} \left( \frac{\partial \psi}{\partial t} \right)^2 - \psi^{-2} \sum_{i=1}^{3} \left( \frac{\partial \psi}{\partial x_i} \right)^2 - 2 \psi^{-1} \frac{\partial^2 \psi}{\partial t^2} + 4 \psi^{-2} \left( \frac{\partial \psi}{\partial x_i} \right)^2, \quad i = 1, 2, 3 \] (8.16)

where we wrote \( x^i \) as \( x \), in order not to confuse an index with a power. The box \( \square \) is the D'Alembertian and the signs (i.e., \( \eta^{ab} \)) are (+, - , - , - )
\[ \square = c^2 \frac{\partial^2}{\partial t^2} - \nabla_i \nabla^i = - \frac{\partial^2}{\partial t^2} - \nabla_i \nabla^i. \] (8.17)

The Ricci scalar
\[ R = g^{aa} R_{aa} \] (8.18)
for the metric given by a scalar field is
\[ R = c^2 \psi^2 R_{00} - \psi^2 R_{11} - \psi^2 R_{22} - \psi^2 R_{33} = -6\psi^{-3} \Box \psi. \tag{8.19} \]

For spherical coordinates the nonzero Ricci entries are (when \( c = 1 \))
\[
R_{00} = \frac{1}{2} \psi^2 \left\{ \frac{\partial^2 \psi^2}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 \psi^2}{\partial \theta^2} + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi^2}{\partial \phi^2} - 3 \frac{\partial^2 \psi^2}{\partial t^2} \right\} + \frac{1}{2} \psi^{-2} \left\{ \frac{2}{r} \frac{\partial \psi^2}{\partial r} + \frac{1}{r^2} \cot \theta \frac{\partial \psi^2}{\partial \theta} \right\} + \frac{3}{2} \psi^{-4} \left( \frac{\partial \psi^2}{\partial t} \right)^2.
\]
\[
R_{11} = \frac{1}{2} \psi^2 \left\{ -3 \frac{\partial^2 \psi^2}{\partial r^2} - \frac{1}{r^2} \frac{\partial^2 \psi^2}{\partial \theta^2} - \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 \psi^2}{\partial \phi^2} + \frac{\partial^2 \psi^2}{\partial t^2} \right\} + \frac{1}{2} \psi^{-2} \left\{ -2 \frac{\partial \psi^2}{\partial r} - \frac{1}{r^2} \cot \theta \frac{\partial \psi^2}{\partial \theta} \right\} + \frac{3}{2} \psi^{-4} \left( \frac{\partial \psi^2}{\partial \theta} \right)^2.
\]
\[
R_{22} = \frac{1}{2} \psi^2 \left\{ -r^2 \frac{\partial^2 \psi^2}{\partial r^2} - 3 \frac{\partial^2 \psi^2}{\partial \theta^2} - \frac{1}{\sin^2 \theta} \frac{\partial^2 \psi^2}{\partial \phi^2} + r^2 \frac{\partial^2 \psi^2}{\partial t^2} \right\} + \frac{1}{2} \psi^{-2} \left\{ -4r \frac{\partial \psi^2}{\partial r} - \cot \theta \frac{\partial \psi^2}{\partial \theta} \right\} + \frac{3}{2} \psi^{-4} \left( \frac{\partial \psi^2}{\partial \theta} \right)^2 + 1.
\]
\[
R_{33} = \frac{1}{2} \psi^2 \left\{ -r^2 \sin^2 \theta \frac{\partial^2 \psi^2}{\partial r^2} - \sin^2 \theta \frac{\partial^2 \psi^2}{\partial \theta^2} - 3 \frac{\partial^2 \psi^2}{\partial \phi^2} + r^2 \sin^2 \theta \frac{\partial^2 \psi^2}{\partial t^2} \right\} + \frac{1}{2} \psi^{-2} \left\{ -4r \sin^2 \theta \frac{\partial \psi^2}{\partial r} + 3 \sin \theta \cos \theta \frac{\partial \psi^2}{\partial \theta} \right\} + \frac{3}{2} \psi^{-4} \left( \frac{\partial \psi^2}{\partial \theta} \right)^2 - \sin^2 \theta.
\]

The Ricci scalar in spherical coordinates gives the same equation also in spherical coordinates
\[ R = g^{ab} R_{ab} = \psi^2 R_{00} - \psi^2 R_{11} - \psi^2 R_{22} - \psi^2 R_{33} = -6\psi^{-3} \Box \psi \tag{8.21} \]
when the D'Alembertian is expressed in spherical coordinates and the signs are set to (+,−,−,−).

Ricci entries in Cartesian coordinates are easier to work with, and in a small area in the empty space outside the mass we certainly can use Cartesian coordinates. Equation (8.16) is also better for us because \( c \) is not set to one.

The Newtonian gravitational potential (8.1) is a solution to the case of empty space with a point mass at the origin. The Ricci tensor entries for the metric are obtained from the potential field \( \phi \) in (8.1) in Cartesian coordinates from (8.16). \( \partial_i \phi = 0 \) and \( \Box \phi = -\Delta \phi = 0 \), thus
\[
R_{00} = \frac{1}{c^2} \sum_{j=1}^{3} \left( \frac{\partial \phi}{\partial x^j} \right)^2 = \frac{1}{c^2} \left( \frac{\partial \phi}{\partial x} \right)^2 + \left( \frac{\partial \phi}{\partial y} \right)^2 + \left( \frac{\partial \phi}{\partial z} \right)^2 = \frac{1}{c^2} \frac{1}{r^2}.
\]
\[
R_{ii} = \frac{1}{r^2} + 2 \frac{x^2}{r^4} \text{ for } i=1,2,3.
\]

We notice that these entries are not zero, but the Ricci scalar is zero. In the classical limit \( R_{00} = 0 \) but \( R_{ii} \neq 0 \) for \( i=1,2,3 \). Especially \( R_{ii} - R_{00} \) is not zero and does not tend to any small number in the classical limit \( c \to \infty \). For any solution of the Einstein equations in this case we have from (8.7)
\[
R_{00} - R_{ii} = -\lambda g_{00} + \lambda g_{ii} = \lambda (1 - c^{-2}) \psi^2. \tag{8.23}
\]
Measurements show that the cosmological constant is very small in our solar system. We can conclude that no solution of the Einstein equations can give a good approximation to the Newtonian gravitation potential in our solar system. This is a fatal error and it nullifies the experimental verification of GRT by bending of light close to the sun (the Eddington measurement of 1919), by the correction to the percession of the perihelion of Mercury and by the Pound-Rebka experiment. GRT cannot give a correct result in any of these experiments since GRT does not yield a solution that can be used in these experiments in our solar system.

It may see that there cannot be a more serious error in a theory of gravitation than a total failure to explain gravitation in our solar system, but GRT manages to surpass this level: there is a bigger error which ruins the theory in all solar systems. This is because if the speed of light is constant, then the metric must necessarily be induced by a scalar field. Let us explain why this is so.

We assume that light travels along geodesics of the gravitational field. From the place \((x, y, z)\) and the time \(t\) light can move an infinitesimally small distance to any direction. We can consider the 3-space and the time as a four-dimensional Euclidian space. In the Euclidean 4-space the infinitesimal movements would be \((dx, dy, dz, dt)\), but we want to give this 4-space a different metric. Therefore let the infinitesimals at the place \((x, y, z)\) and the time \(t\) be \((Adx, Bdy, Cdz, Ddt)\) where \(A, B, C, D\) are functions of the place \((x, y, z)\) and the time \(t\). The 3-dimensional sphere of the metric at the place \((x, y, z)\) and the time \(t\) is

\[
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\]

where \(CdzBdyAdx\) and as in the Euclidean metric the infinitesimals in each direction are equally large (the Euclidean infinitesimal ball is round), we have the conformal metric

\[
A^2dx^2 + A^2dy^2 + A^2dz^2 \quad \text{and} \quad D = \frac{1}{c}A. \quad (8.26)
\]

Writing \(\varphi = \eta\varphi\), where \(\varphi < 0\) is a scalar gravitational field (\(\varphi\) is a special case of \(\varphi\)), the infinitesimal sphere of the 4-dimensional space in the geometry for gravitation is \((-\varphi dx, -\varphi dy, -\varphi dz, -c^{-1}\varphi dt)\) and the line element is

\[
2222222
\]

In the Minkowski space there is a pseudometric where \(\eta_{\alpha\beta} = (+, -, -, -)\) the line element is

\[
ds^2 = -\psi^2 dx^2 - \psi^2 dy^2 - \psi^2 dz^2 + c^2\psi^2 dt^2. \quad (8.27)
\]

Identifying \(g_{ii}\) for a general metric in orthogonal local coordinates

\[
ds^2 = -g_{11}dx_1^2 - g_{22}dx_2^2 - g_{33}dx_3^2 + g_{00}dx_0^2. \quad (8.29)
\]

we see that \(g_{00} = c^2\psi^2\) and \(g_{ii} = \psi^2\), \(i = 1, 2, 3\). There is no other metric that can yield the velocity of light as a scalar constant. It the speed of light is not a scalar constant, then it
becomes a vector function \( c_i(x_0, x_1, x_2, x_3), \ i = 1,2,3 \). No formula from SRT can handle such vector function speed of light. It is also not supported by measurements. It is necessary to have the speed of light as constant, else there is no Einsteinian relativity theory.

In case it seems strange that the metric must necessarily be induced by a scalar field because GRT does not mention this condition, consider the following argument. A photon is in the origin of the infinitesimal ball at the place \((x, y, z)\) and the time \(t\). This photon reaches the surface of the ball in the same time \(Ddt\) regardless of into what direction it moves. The length in the infinitesimal ball must be the same to each direction if the speed of the photon is the same to each direction. Thus, the speed of light is not

\[
\begin{align*}
\sqrt{A^2 dx^2 + B^2 dy^2 + C^2 dz^2} &= \frac{Dx}{Ddt} + \frac{Dy}{Ddt} + \frac{Dz}{Ddt} = \sqrt{3} Adx \\
\end{align*}
\]

In (8.30) the photon travels to a direction that is in such an angle that all displacements in \(x, y\) and \(z\) directions are equal. It does not reach the surface of the infinitesimal ball at the point \(dx, dy, dz\) as is implied in (8.30). It reaches it already as the point \(dx/\sqrt{3}, dy/\sqrt{3}, dz/\sqrt{3}\).

We notice that the metric that gives equal speed \(c\) of light to each direction in any place \((x, y, z)\) and any time \(t\) is necessarily created by a scalar field \(\phi\) and the geometry is conformal: the space can expand and contract in any place, but the infinitesimal ball, if taken as a ball of a 4-dimensional manifold, is always perfectly round. This condition implies that angles are preserved. We know this condition from electro-magnetism: Maxwell's equations imply that the power lines of the EM field map to the grid in Euclidean geometry in a conformal way.

Solutions of Einstein's field equations for GRT, such as the Schwarzschild solution, do not have a round infinitesimal ball. Therefore the speed of light is different in different places \((x, y, z)\) and times \(t\). The geometry of the Schwarzschild solution is hyperbolic. It is possible that the global geometry of the universe is hyperbolic and there exists such a region in the universe where the Schwarzschild solution is realized, but it is against the explicit requirements that Einstein imposed when creating the General Relativity Theory. So far we have not found evidence that the speed of light is not locally constant in any place in the universe. It can be useful to study such a possibility, and it is not excluded, but this possibility is outside the General Relativity Theory. If such solutions are studied, in the context of black holes or other ways, then one must remember that light does not have a constant speed. This lack of the constant speed of light in the Schwarzschild solution is the reason why GRT fails the Shapiro delay test when the test is calculated from the Schwarzschild solution. Nordström's gravitation theory passes the Shapiro delay test. Einstein proposed the Shapiro delay test as a test of the General Relativity Theory. This test fails always if the infinitesimal ball is not round. Therefore the General Relativity Theory passes the test only if it has a solution where the infinitesimal ball is round. Let us calculate if there is such a solution. The only way to have a round infinitesimal ball is that the metric corresponds to a scalar field \(\phi\).

The way to solve the Einstein equations is to first solve the case of an empty space with a point mass at the origin and then sum such fields in order to get any mass distribution. As the metric is induced by a scalar field, the case with a point mass in the origin gives \(\Box \phi = 0\). Therefore also holds \(R = -6\phi^{-3} \Box \phi = 0\). The Einstein equations reduce to (8.23). We can insert the forms of the Ricci tensor entries from (8.16) and first remove \(\lambda\)

\[
R_u - c^2 R_{00} = -\lambda \phi^2 + \lambda \phi^2 = 0.
\]

From (8.16) we get
\[ 0 = R_{00} - c^2 R_{ii}. \]  

(8.32)

\[ R_{ii} - c^2 R_{00} = -2 \psi^{-2} \sum_{j=1}^{3} \left( \frac{\partial}{\partial x_j} \psi \right)^2 - 2c^2 \psi^{-2} \left( \partial_0 \psi \right)^2 + 2c^2 \psi^{-1} \partial_0^2 \psi + 4 \psi^{-2} \left( \partial_i \psi \right)^2 - 2 \psi^{-1} \partial_i^2 \psi \]

Thus, for every \( i \in \{1, 2, 3\} \) holds

\[ 2 \psi^{-2} \sum_{j=1}^{3} \left( \frac{\partial}{\partial x_j} \psi \right)^2 - 2c^2 \psi^{-2} \left( \partial_0 \psi \right)^2 = 4 \psi^{-2} \left( \partial_i \psi \right)^2 - 2 \psi^{-1} \partial_i^2 \psi \]  

(8.33)

From the right side of (8.33) for \( i, j \in \{1, 2, 3\} \) we get the equation

\[ \frac{\partial}{\partial x_i} \left( \frac{\partial \psi}{\partial x_i} \psi^{-2} \right) = \frac{\partial}{\partial x_j} \left( \frac{\partial \psi}{\partial x_j} \psi^{-2} \right). \]  

(8.34)

This equation has a solution of the form (possibly it has other solutions)

\[ \psi = Ae^{c x^2 + b(x + y + z)} \]  

(8.35)

where the parameters can be functions of \( t \). Calculating

\[ 2 \lambda \psi^{-1} = R_{ii} + c^2 R_{00} = 4c^2 \psi^{-2} \left( \partial_0 \psi \right)^2 - 2c^2 \psi^{-1} \partial_0^2 \psi + 4 \psi^{-2} \left( \partial_i \psi \right)^2 - 2 \psi^{-1} \partial_i^2 \psi . \]  

(8.36)

In order to find one exact solution to Einstein's equations that actually has constant speed of light, let \( \lambda = 0 \). Inserting (8.35) into the right side of

\[ -4c^2 \psi^{-2} \left( \partial_0 \psi \right)^2 + 2c^2 \psi^{-1} \partial_0^2 \psi = 4 \psi^{-2} \left( \partial_i \psi \right)^2 - 2 \psi^{-1} \partial_i^2 \psi \]  

(8.37)

yields

\[ -2c^2 \psi^{-2} \left( \partial_0 \psi \right)^2 + c^2 \psi^{-1} \partial_0^2 \psi = \psi \left( 3(2cx + b)^2 + 2c \right) = \psi \left( 3(2cy + b)^2 + 2c \right). \]  

(8.38)

That is, unless \( c = 0 \) the right side of (8.38) depends on any free coordinate \( x_i \). We must set \( c = 0 \) and therefore

\[ 2 \psi^{-1} \left( \partial_0 \psi \right)^2 - \partial_0^2 \psi = -3b^2 c^{-2}. \]  

(8.39)

This has at least the solution \( \psi = \exp \left( i \sqrt{3} bt / c \right) \). We have found one exact solution for the Einstein equations for a point mass in the origin of an empty space

\[ \psi(x, y, z, t) = A \exp \left( b(x + y + z) + i \sqrt{3} bt / c \right). \]  

(8.40)

In order to get a solution for any mass distribution, solutions of (8.40) must be summed. I very much doubt that the solution (8.40) is of any application in theoretical physics, as I doubt that the relativity theory should be.

9. A short comment of the tests that have "proven" the relativity theory

It is by popular science books by experts that the wide audience learns what the scientific truth is. Therefore the blame is on these authors. Let us see how one of them proves that the relativity theory is correct. In Section 4 we looked at one Igor Novikov's (1997) explanations and found it incorrect. He gives in this book in Chapter 4 many other examples how the Special Relativity Theory is "proven". One example, that he calls impressive, is that there are cosmic protons that travel very close to the speed of light and he concludes that their local time is very slow. Unfortunately we cannot know what the local time of a proton is because
protons are stable particles and do not have a half-time. Another example he gives is that muons created by cosmic radiation in high atmosphere are seen to travel longer than their half-time would allow. But half-time is only the time when half of the muons break up. There are muons that last longer. Should one measure the muon half-time, it might show the claim, but that calculation requires knowing the distribution and intensity of cosmic radiation, the distribution of the velocities of muons created by this radiation, and an estimation how many are detected in a given speed range. This calculation has many uncertainties. Novikov also claims that a precise clock, an atomic clock, in an airplane measures the time dilation of the Special Relativity Theory. We may have a doubt: the airplane accelerates and moves in a gravitational field which influences the clock. Besides, atomic clocks are fairly sensitive equipment. In a space orbit, like in a GPS satellite, the time delay that has any significance is caused by the gravitation field, not by the velocity of a satellite. Finally Novikov gives an astronomical example: object SS 433 shows a redshift that is not entirely explainable by the Doppler phenomenon. He does not discuss the other possible reasons, like a gravitational field or expansion of the space, but concludes that the Special Relativity Theory explains this redshift, but many researchers feel that the redshift of astronomical objects is so far not explained in a satisfactory way and even the expansion of the space can be questioned.

In Chapter 5 Novikov discusses the twin paradox and explains it away in the standard way that the astronauts cannot meet again unless one of them has accelerating orbit, and that taking this acceleration into account one obtains exactly the correct result. However, for the twin paradox we do not need to have the astronauts meet. It is exactly the calculation we just made with the R and M frames. There is no need to introduce an accelerating frame in order to derive a real paradox. As real paradoxes do not exist, the Special Relativity Theory is wrong.

The example of muons is often given as evidence of time dilation in the Special Relativity Theory. Muons are unstable and they have a half-time, which can be measured. If muons are accelerated close to the speed of light, then their measured half-time is longer and this is expected to prove that the Special Relativity Theory is correct. However, it does not prove anything of this type. Muons are created in this experiment and for most of their lifetime they are in the accelerator in accelerated motion that slows down their internal clock time. This time dilation is higher if the muons are accelerated to a higher speed.

Time dilation in accelerated motion is not described by the Special Relativity Theory. It happens in the General Relativity Theory and it also happens in Nordström's gravitation theory. It happens in any geometric gravitation theory where light moves along geodesics of the geometry of the gravitational field and the speed of light is constant. Why it happens is easiest to explain by first noticing that a clock slows down in a gravitational field. A clock slows down in a gravitational field because field geometry is larger when the field is stronger. It is the same case as when Canadian hockey players play in Europe where the hockey rink is larger: they skate with the same speed, but the tempo of the game, the relevant clock, is slower. Time dilation in a gravitational field has been verified by the Pound-Rebka experiment. By the equivalence principle a clock also slows down in accelerated motion.

During the whole time of acceleration, the main lifetime of these muons, the clocks of the muons ticked slower. Thus, they were younger when they arrived to the bubble chamber. Therefore they lived longer before decaying: the measured half-time was longer. There is no need to assume that their clocks were ticking slower in the very short time that they were in the bubble chamber traveling with a constant velocity, the only situation that the Special Relativity Theory claims to describe in this whole experiment.

Let us add that this gravitational time delay is nothing mystical and all clocks do not get delayed. Traditional clocks like hourglass, pendulum and water clock do depend on gravitation, but they go slower when the gravitation field is weaker. A sundial does not
depend on the Earth's gravitation field, but it depends on the Sun's gravitational pull as else the position of the Sun would be different. I am not aware if my inherited spring watch from the 1970s is influenced much by gravitation. Aging does seem to be influenced by gravitation: fighter pilots experiencing high G-values are said to age faster, while people living in mountains may have a shorter lifespan due to gravitation related issues, like avalanches and falling down. It is likely that a life-form that developed to some range of gravitation thrives best in that range. However, in relativity theory we assume, with a good reason, that light does travel on geodesics of the gravitation field. If so, some clocks that are related to light or electromagnetism do slow down in a gravitational field. It has been shown that stronger gravitational field causes a redshift in atomic oscillators. Yet, this phenomenon does not justify attempts to build a time machine, which abound in popular science books by cosmology experts.

There are so many false "proofs" of the Relativity Theory. The mass transformation formula is claimed to be proven by the fact that speeding particles in a particle accelerator takes more energy than it should if the mass were constant. I wonder how they can as they are typically speeded up by an electric field and the particle simply changes potential energy to kinetic energy. But "the only possibility" (always there is only one possibility) to explain what happens is that the mass of the particle has grown. Yet, it is not the only possibility. The more likely possibility is that the field does not allow the particle to reach the velocity of light and there is an additional term to the potential field, like in (2.42), that slows down a particle in high speeds.

There is the "proof" of the General Relativity Theory that it gives a very good correction to the precession of Mercury's perihelion. This is already absurd. 19th century astronomers could not calculate precisely the perihelion as it is a many-body problem, and the General Relativity Theory (GRT) certainly does not give a good correction because the field equation of GRT does not give any solution that is close to the Newtonian gravitation potential and the gravitational field of the sun certainly is close to the Newtonian gravitation potential. The equation from which this good correction was calculated is most probably similar to (2.41). Notice that (2.41) is not derived from GRT field equations. It is derived from the mass transformation formula only, but it can also be obtained form assuming that the field is $\Psi$ in (2.42). Amusingly, there is a claim that Nordström's gravitation theory does not give the correction to Mercury's apsidal precession. It is exactly the field equation of Nordström's gravitation theory that can give (2.42), not GRT. There is another false claim against Nordström's gravitation theory: that light does not bend in that theory. Of course light bends in every geometric gravitational theory where light travels along geodesics of the gravitational field, so also in Nordström's gravitation theory, but Einstein mixed up Nordström so badly that he wrote an incorrect equation of motion. Only the field equation is good. This double strategy of claiming false "proofs" of the Special and General Relativity Theory and invalid falsifications of competing theories of gravitation is very common in literature.

Igor Novikov is one of the leading black hole experts. I recently read about a dozen of popular science cosmology and relativity theory books written by leading experts. They all testify that Special and General Relativity Theories have been verified and they are correct. The problem in all these books, and not only popular science books but also university text books, is that nobody seems to have calculated and checked if the Lorentz transform actually can give constant speed of light for the moving frame. It cannot. It can only make the roundtrip delay in the Michelson-Morley experiment have constant speed $c$ in the moving frame. It cannot make one-way trips in the moving frame to have the speed $c$. No coordinate transform can do it. None of these experts also cared to calculate the field equations of GRT and notice that they cannot give any close approximation to the Newtonian gravitational
potential and that if the speed of light is constant to all directions, then the gravitational field must be scalar. But none of these experts checked.

References: