A Formula for the Function $\pi(x)$ to Count the Number of Primes Exactly if $25 \leq x \leq 846$ with Python Code to Test It v. 2.0

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Abstract

This paper shows a very elementary way of counting the number of primes under a given number with total accuracy. Is the function $\pi(x)$ if $25 \leq x \leq 846$.

Keywords— prime, number, pi, composite, formula, function

1 Introduction

The function $\pi(x)$ is very known to every well documented mathematician interested in number theory. Here we present not an approximation of the function but instead an exact solution. The key idea was born in the February 26 of 2021 at morning when I was thinking about how to test the primality of a given number. By some years I was studying the possibilities of the composite numbers of the form $6k + 1$ and $6k - 1$. Because the prime numbers bigger than 3 has the same structure, I tried to figure how to filter primes from composites. I discover that the composite numbers has a very regular structure beginning from $(2a + 1)(2b + 1) = 4ab + 2a + 2b + 1$, the base of every odd composite number. So the numbers with the form $6k + 1$ has the structure $(6m + 1)(6n + 1)$ or $(6m - 1)(6n - 1)$ and the numbers with the form $6k - 1$ has the structure $(6m - 1)(6n + 1)$. These structures are totally predictable if we choose any pair of numbers but the primes are unpredictable. The set of composite numbers are placed with regularity but the prime numbers are the tiny holes that escapes from the structure of the composites, truly randomly placed if we want. The key idea takes advantage of the regularity of the structures of the composite numbers of the form $6k + 1$ and $6k - 1$. If we can count the number of composites of the form
6k + 1 and 6k − 1 under a given number we can know the number of primes under that number only making a very basic math: \( \pi(x) = \text{Numbers}(6k + 1) + \text{Numbers}(6k - 1) - \text{Numbers}((6m + 1)(6n + 1)) - \text{Numbers}((6m - 1)(6n - 1)) - \text{Numbers}((6m - 1)(6n + 1)) + 2 \). The number 2 at the end represents the additional count of primes 2 and 3. So in this paper we develop the formula to count every set of numbers involved in some interval. On April 3 of 2021 we derived the necessary theorems and the first version of \( \pi(x) \). On April 9 of 2021 we derived the partial version with and interval of \( 25 \leq x \leq 538 \). On January 22 of 2022 we derived the proofs of the structures of the composites that has repetitions.

2 Prime Numbers of the Form 6k + 1 and 6k − 1

Because we want to know the quantity of the primes under \( x \), first we note that every prime greater than 3 has the form 6k + 1 or 6k − 1, the next theorem shows that.

**Theorem 2.1.** (Aurelio Baldor, 1985) [1] Every prime number \( N > 3 \) has the form \( N = 6k + 1 \) or \( N = 6k - 1 \)

**Proof.** Let \( N > 3 \) a prime number, we will show that \( N = 6k + 1 \) or \( N = 6k - 1 \). Divide \( N \) between 6, \( q \) is the quotient and \( R \) the residue. We have \( N = 6q + R \), \( R < 6 \). \( R \) can not be zero because \( N \) is not a multiple of 6 (\( N \) is prime!). \( R \) must be 1, 2, 3, 4 or 5. \( R \) can not be 2 because we would have \( N = 6q + 2 \) and the number would be divisible by 2 (\( N \) is prime!). \( R \) can not be 3 because we would have \( N = 6q + 3 \) and the number would be divisible by 3 (\( N \) is prime!). \( R \) can not be 4 because we would have \( N = 6q + 4 \) and the number would be divisible by 2 (\( N \) is prime!). So, if \( R \) can not be 2, 3 or 4, then \( R \) is 1 or 5. We conclude that \( N \) is of the form \( N = 6k + 1 \) or \( N = 6m + 5 = 6k - 1 \).

Quod erat demonstrandum (Q.E.D).

3 Composite Numbers of the Form 6k + 1 and 6k − 1

To calculate the quantity of primes lesser or equal than \( x \), we need to subtract the composites that has the forms 6k + 1 or 6k − 1 and later add 2 (because we need to take in account the primes 2 and 3). Here we show two theorems that present us the composites with that forms.

**Theorem 3.1.** (Danilo Chávez, April 3, 2021) If a composite number \( N \) is of the form \( N = 6k + 1 \), then \( N = (6m + 1)(6n + 1) \) or \( N = (6m - 1)(6n - 1) \).

**Proof.** Every odd composite number \( N \) is of the form \( N = (2a + 1)(2b + 1) = 4ab + 2a + 2b + 1 \). If \( N = 6k + 1 \) we have \( 4ab + 2a + 2b + 1 = 6k + 1 \) then \( 2ab + a + b = 3k \). As this expression is a multiple of 3, if we suppose \( a \) as a multiple of 3, we conclude that \( b \) also is a multiple of 3, therefore \( N = (6m + 1)(6n + 1) \). As \( m \) and \( n \) are integers, it takes negative values, so \( N = (-6p + 1)(-6q + 1) = (6m - 1)(6n - 1) \), therefore, if \( N = 6k + 1 \), it takes the form \( N = (6m + 1)(6n + 1) \) or \( N = (6m - 1)(6n - 1) \).

Quod erat demonstrandum (Q.E.D).

**Theorem 3.2.** (Danilo Chávez, April 3, 2021) If a composite number \( N \) is of the form \( N = 6k - 1 \), then \( N = (6m - 1)(6n + 1) \)
Proof. Every odd composite number $N$ is of the form $N = (2a + 1)(2b + 1) = 4ab + 2a + 2b + 1$. If $N = 6k - 1$ we have $4ab + 2a + 2b + 1 = 6k - 1$ then $2ab + a + b + 1 = 3k$. As this expression is a multiple of 3, if we suppose $a$ as a multiple of 3, we conclude that $b + 1$ also is a multiple of 3, we say $b + 1 = 3s$ therefore $N = (6m + 1)(6n - 1)$. As $m$ and $n$ are integers, it takes negative values, so $N = (-6p + 1)(-6q - 1) = (6m - 1)(6n + 1)$. As $N$ can take the form $N = (6m + 1)(6n - 1)$ and $N = (6c - 1)(6d + 1)$ we can conclude that if $N = 6k - 1$, it has the unique form $N = (6m - 1)(6n + 1)$. Quod erat demonstrandum (Q.E.D).

4 The Number of Primes Under a Given Number, $\pi(x)$, between an interval of the variable

**FUNCTION $\pi(x)$ BETWEEN AN INTERVAL OF THE VARIABLE**
(Danilo Chávez, April 3, 2021)

If $25 \leq x \leq 846$ and

\[
C_{6k+1}(x) = \left\lfloor \frac{x - 1}{6} \right\rfloor
\]

\[
C_{6k-1}(x) = \left\lceil \frac{x + 1}{6} \right\rceil
\]

\[
C_{10}(x, m) = \left\lfloor \frac{x - (6m + 1)}{6(6m + 1)} \right\rfloor - m + 1
\]

\[
C_{11}(x, m) = \left\lceil \frac{x - 7(6m + 1)}{42(6m + 1)} + 1 \right\rfloor - \left\lfloor \frac{m - 1}{7} \right\rfloor
\]

\[
C_{12}(x, s) = \left\lfloor \frac{x - 7(6s + 1)}{42(6s + 1)} \right\rfloor - 7s
\]

\[
C_{13}(x, s) = \left\lfloor \frac{x - 49(6s + 1)}{294(6s + 1)} \right\rfloor - s + 1
\]

\[
C_{14}(x) = \left\lfloor \frac{x - 7}{42} \right\rfloor
\]

\[
C_{20}(x, m) = \left\lfloor \frac{x + (6m - 1)}{6(6m - 1)} \right\rfloor - m + 1
\]

\[
C_{21}(x, m) = \left\lceil \frac{x - 5(6m - 1)}{30(6m - 1)} + 1 \right\rceil - \left\lfloor \frac{m - 1}{5} \right\rfloor
\]

\[
C_{22}(x, s) = \left\lfloor \frac{x + 5(6s + 1)}{30(6s + 1)} \right\rfloor - 5s
\]

\[
C_{23}(x, s) = \left\lfloor \frac{x - 25(6s + 1)}{150(6s + 1)} \right\rfloor - s + 1
\]

\[
C_{24}(x) = \left\lceil \frac{x + 5}{30} \right\rceil
\]
\[ C_{121}(x, m) = \left\lfloor \frac{x + 5(6m + 1)}{30(6m + 1)} \right\rfloor - \left\lfloor \frac{m}{5} \right\rfloor \]
\[ C_{122}(x, s) = \left\lfloor \frac{x - 5(6s - 1)}{30(6s - 1)} \right\rfloor - 5s + 2 \]
\[ C_{123}(x, s) = \left\lfloor \frac{x + 25(6s - 1)}{150(6s - 1)} \right\rfloor - s + 1 \]
\[ C_{30}(x, m) = \left\lfloor \frac{x - (6m - 1)}{6(6m - 1)} \right\rfloor \]
\[ C_{31}(x, m) = \left\lfloor \frac{x + 5(6m - 1)}{30(6m - 1)} \right\rfloor \]
\[ C_{32}(x, m) = \left\lfloor \frac{x - 7(6m - 1)}{42(6m - 1)} \right\rfloor \]
\[ C_{33}(x, m) = \left\lfloor \frac{x + 35(6m - 1)}{210(6m - 1)} \right\rfloor \]
\[ C_{34}(x, m) = \left\lfloor \frac{x - (6m - 1)}{6(6m - 1)} \right\rfloor - 1 \]
\[ C_{35}(x, m) = \left\lfloor \frac{x - (6m - 1)}{6(6m - 1)} \right\rfloor - 1 \]
\[ C_{36}(x, m) = \left\lfloor \frac{x - (6m - 1)}{6(6m - 1)} \right\rfloor - 1 \]
\[ C_{37}(s) = 35(6s + 1) \]

\[ C_{(6m+1)(6n+1)}(x) = \sum_{m \geq 1 \atop C_{10}(x,m) > 0} C_{10}(x,m) - \sum_{m \geq 1 \atop C_{11}(x,m) > 0} C_{11}(x,m) - \sum_{s \geq 0 \atop C_{12}(x,s) > 0} C_{12}(x,s) \]
\[ + \sum_{s \geq 0 \atop C_{13}(x,s) > 0} C_{13}(x,s) + C_{14}(x) \]

\[ C_{(6m-1)(6n-1)}(x) = \sum_{m \geq 1 \atop C_{20}(x,m) > 0} C_{20}(x,m) - \sum_{m \geq 1 \atop C_{21}(x,m) > 0} C_{21}(x,m) - \sum_{s \geq 0 \atop C_{22}(x,s) > 0} C_{22}(x,s) \]
\[ + \sum_{s \geq 0 \atop C_{23}(x,s) > 0} C_{23}(x,s) + C_{24}(x) \]

\[ C_{\text{common}}(x) = \sum_{m \geq 1 \atop C_{121}(x,m) > 0} C_{121}(x,m) + \sum_{s \geq 1 \atop C_{122}(x,s) > 0} C_{122}(x,s) - \sum_{s \geq 1 \atop C_{123}(x,s) > 0} C_{123}(x,s) \]

4
\[ C_{(6m-1)(6n+1)}(x) = \sum_{m \geq 1, C_{30}(x,m) > 0} C_{30}(x,m) - \sum_{m \geq 2, m \equiv \pm 1 \mod(5) \land m \equiv \pm 1 \mod(7), C_{31}(x,m) > 0} C_{31}(x,m) - \sum_{m \geq 2, m \equiv \pm 1 \mod(5), C_{32}(x,m) > 0} C_{32}(x,m) \]

\[ + \sum_{m \geq 2, m \equiv \pm 1 \mod(5) \land m \equiv \pm 1 \mod(7), C_{33}(x,m) > 0} C_{33}(x,m) - \sum_{m \equiv 1 \mod(5), C_{34}(x,m) > 0} C_{34}(x,m) - \sum_{m \equiv -1 \mod(7), C_{35}(x,m) > 0} C_{35}(x,m) \]

\[ + \sum_{m \geq 2, m \equiv 1 \mod(5) \land m \equiv -1 \mod(7), C_{36}(x,m) > 0} C_{36}(x,m) - \sum_{s \geq 1, C_{37}(s) < x} 1 \]

then, the number of primes lesser or equal to some number \( x \) is

\[ \pi(x) = C_{6k+1}(x) + C_{6k-1}(x) - C_{(6m+1)(6n+1)}(x) - C_{(6m-1)(6n-1)}(x) \]

\[ + C_{\text{common}}(x) - C_{(6m-1)(6n+1)}(x) + C_{2} \]

**DEDUCTION OF THE FUNCTION \( \pi(x) \) BETWEEN AN INTERVAL OF \( x \)**

To find the number of primes lesser or equal to some number \( x \geq 25 \), we calculate the total quantity of numbers with the form \( 6k + 1 \) and \( 6k - 1 \) (primes or composites), we subtract the quantity of composite numbers with the form \( (6m + 1)(6n + 1) \), we subtract the quantity of composite numbers with the form \( (6m - 1)(6n - 1) \), we subtract the quantity of composite numbers with the form \( (6m - 1)(6n + 1) \) and finally we add 2 (prime numbers 2 and 3).

Let \( x \geq 25 \) be some integer.

**A) QUANTITY OF NUMBERS WITH THE FORM 6k + 1 AND 6k − 1**

The total quantity of numbers of the form \( 6k + 1 \) lesser or equal to \( x \).

If \( 6k + 1 \leq x \) then

\[ k \leq \frac{x - 1}{6} \]

Now we define

\[ C_{6k+1}(x) = \left\lfloor \frac{x - 1}{6} \right\rfloor \]
The total quantity of numbers with the form $6k - 1$ lesser or equal to $x$.
If $6k - 1 \leq x$ then

$$k \leq \frac{x + 1}{6}$$

Because we want integers we define

$$C_{6k-1}(x) = \left\lfloor \frac{x + 1}{6} \right\rfloor$$

**B) QUANTITY OF COMPOSITE NUMBERS WITH** $(6m+1)(6n+1) = (6p+1)(6q+1)$

![Figure 1: Composite numbers with the form $(6m + 1)(6n + 1) = (6p + 1)(6q + 1)$, the pink numbers are those which are repeated](image)

The next theorem shows the structure of the composites with $(6m+1)(6n+1) = (6p+1)(6q+1)$.

**Theorem 4.1.** (Danilo Chávez, January 22, 2022)

Let be $m, n, p, q, r, t, k$ integers. Let be $1 \leq m, 1 \leq n, 1 \leq p, 1 \leq q, 0 \leq r, 1 \leq t, 1 \leq k$.

When

$$(6m + 1)(6n + 1) = (6p + 1)(6q + 1)$$

If

$$p = 7r + 1$$

with $0 \leq r$, then

$$q = 1, 2, 3, 4...$$
or by symmetry, if

\[ q = 7r + 1 \]

with \( 0 \leq r \), then

\[ p = 1, 2, 3, 4... \]

Proof. Let be \( m, n, p, q, r, t, k \) integers. Let be \( 1 \leq m, 1 \leq n, 1 \leq p, 1 \leq q, 0 \leq r, 1 \leq t, 1 \leq k \). We take the line \( m = 1 \) and we have

\[ 7(6n + 1) = (6p + 1)(6q + 1) \]

or \((6p + 1) = 7t \) or \((6q + 1) = 7t \). By symmetry we can take \((6p + 1) = 7t \) and the same will happen with \((6q + 1) = 7t \) with inverted values.

Let's take \((6p + 1) = 7t \) and we have

\[ p = \frac{7t - 1}{6} \]

We are looking for the values where \( p \) is integer and we have that \( t \) is of the form \( 6r + 1 \) with \( r = 0, 1, 2, 3... \)

\[ p = \frac{7(6r + 1) - 1}{6} = 7r + 1 \]

\( p \) takes the values \( p = 1, 8, 15, 22... \)

Now we are looking for the values of \( q \) given \( p \). We start with the equation

\[ 7(6n + 1) = (6p + 1)(6q + 1) \]

We can see that

\[ q = \frac{7n - p + 1}{6p + 1} \]

As we are looking for the integer values given \( p \), we suppose that for \( k = 1, 2, 3, 4... \)

\[ 7n - p + 1 = k(6p + 1) \]

giving

\[ 7n = 6kp + k + p - 1 \]
Substituting we have
\[
q = \frac{7n - p + 1}{6p + 1} = \frac{6kp + k + p - 1 - p + 1}{6p + 1} = k \left( \frac{6p + 1}{6p + 1} \right) = k
\]
that shows that \( q \) takes all the values \( k = 1, 2, 3, 4..., \) for any value of \( p \), remember that \( p = 7r + 1 \).
We conclude that when
\[
p = 7r + 1
\]
with \( 0 \leq r \)
\[
q = 1, 2, 3, 4...
\]
or by symmetry if
\[
q = 7r + 1
\]
then
\[
p = 1, 2, 3, 4...
\]
Quod erat demonstrandum (Q.E.D).

To find the total quantity of composite numbers with the form \((6m+1)(6n+1) = (6p+1)(6q+1)\), we calculate the total composite numbers with that form lesser or equal to \( x \), we subtract the numbers in the columns (it has repetitions), we subtract the numbers in the rows (it has repetition), we need to add the numbers between intersections (it was subtracted twice) and finally we add the numbers in the first row (it was subtracted before).

**B.0)** To find the total quantity of composite numbers with the form \((6m + 1)(6n + 1) = (6p + 1)(6q + 1)\) lesser or equal to \( x \).

If \((6m + 1)(6n + 1) \leq x\) then
\[
n \leq \frac{x - (6m + 1)}{6(6m + 1)}
\]
Because we want integers we define
\[
C_{10}(x, m) = \left\lfloor \frac{x - (6m + 1)}{6(6m + 1)} \right\rfloor - m + 1
\]
where the last term eliminates the number of composite numbers under the main diagonal. Thus, the total quantity of composite numbers with the form \((6m + 1)(6n + 1)\), lesser or equal to \(x\) is

\[
\sum_{m \geq 1} \sum_{c_{10}(x, m) > 0} C_{10}(x, m)
\]

**B.1)** To find the total quantity of composite numbers with the form \((6m + 1)(6(7t + 1) + 1)\) lesser or equal to \(x\) in the columns.

\[
(6m + 1)(6(7t + 1) + 1) = 7(6m + 1)(6t + 1)
\]

If \(7(6m + 1)(6t + 1) \leq x\) then

\[
t \leq \frac{x - 7(6m + 1)}{42(6m + 1)}
\]

but \(t \geq 0\) and we want to count the number of columns, so

\[
t + 1 \leq \frac{x - 7(6m + 1)}{42(6m + 1)} + 1
\]

Because we want integers we define

\[
C_{11}(x, m) = \left\lfloor \frac{x - 7(6m + 1)}{42(6m + 1)} + 1 \right\rfloor - \left[ \frac{m - 1}{7} \right]
\]

where the last term eliminates the number of extra columns under the main diagonal. Thus, the total quantity of composite numbers with the form \((6m + 1)(6(7t + 1) + 1)\), lesser or equal to \(x\) is

\[
\sum_{m \geq 1} \sum_{c_{11}(x, m) > 0} C_{11}(x, m)
\]
B.2) To find the total quantity of composite numbers with the form \((6(7s + 1) + 1)(6n + 1)\) lesser or equal to \(x\) in the rows.

\[
(6(7s + 1) + 1)(6n + 1) = 7(6s + 1)(6n + 1)
\]

If \(7(6s + 1)(6n + 1) \leq x\) then

\[
n \leq \frac{x - 7(6s + 1)}{42(6s + 1)}
\]

Because we want integers we define

\[
C_{12}(x, s) = \left\lfloor \frac{x - 7(6s + 1)}{42(6s + 1)} \right\rfloor - 7s
\]

where the last term eliminates the number of composite numbers before the columns that touches the main diagonal. Thus, the total quantity of composite numbers with the form \((6(7s + 1) + 1)(6n + 1)\), lesser or equal to \(x\) and above the main diagonal is

\[
\sum_{s \geq 0} C_{12}(x, s)_{C_{12}(x,s) > 0}
\]

B.3) To find the total quantity of composite numbers with the form \((6(7s + 1) + 1)(6(7t + 1) + 1)\) lesser or equal to \(x\) that are intersections between columns and rows.

\[
(6(7s + 1) + 1)(6(7t + 1) + 1) = 49(6s + 1)(6t + 1)
\]

If \(49(6s + 1)(6t + 1) \leq x\) then

\[
t \leq \frac{x - 49(6s + 1)}{294(6s + 1)}
\]

Because we want integers we define

\[
C_{13}(x, s) = \left\lfloor \frac{x - 49(6s + 1)}{294(6s + 1)} \right\rfloor - s + 1
\]
where the last term eliminates the number of composite numbers under the main diagonal. Thus, the total quantity of composite numbers with the form \((6(7s + 1) + 1)(6(7t + 1) + 1)\), lesser or equal to \(x\) and above the main diagonal is

\[
\sum_{s \geq 0}^{C_{13}(x, s) > 0} C_{13}(x, s)
\]

**B.4)** To find the total quantity of composite numbers with the form \(7(6n + 1)\) lesser or equal to \(x\) that are numbers in the first row.

If \(7(6n + 1) \leq x\) then

\[
n \leq \frac{x - 7}{42}
\]

Because we want integers we define

\[
C_{14}(x) = \left\lfloor \frac{x - 7}{42} \right\rfloor
\]

Thus, the total quantity of composite numbers with the form \(7(6n + 1)\), lesser or equal to \(x\) in the first row is

\[
C_{14}(x)
\]

**B. Final** The total quantity of composite numbers with the form \((6m + 1)(6n + 1)\), without repetition is

\[
C_{(6m+1)(6n+1)}(x) = \sum_{m \geq 1}^{C_{10}(x, m) > 0} C_{10}(x, m) - \sum_{m \geq 1}^{C_{11}(x, m) > 0} C_{11}(x, m) - \sum_{s \geq 0}^{C_{12}(x, s) > 0} C_{12}(x, s)
\]

\[
+ \sum_{s \geq 0}^{C_{13}(x, s) > 0} C_{13}(x, s) + C_{14}(x)
\]

**C) Quantity of Composite Numbers With** \((6m-1)(6n-1) = (6p-1)(6q-1)\)

The next theorem shows the structure of the composites with \((6m-1)(6n-1) = (6p-1)(6q-1).\)
Figure 2: Composite numbers with the form \((6m - 1)(6n - 1) = (6p - 1)(6q - 1)\), the pink numbers are those which are repeated.

**Theorem 4.2. (Danilo Chávez, January 22, 2022)**

Let be \(m, n, p, q, r, t, k\) integers. Let be \(1 \leq m, 1 \leq n, 1 \leq p, 1 \leq q, 0 \leq r, 1 \leq t, 1 \leq k\).

When

\[(6m - 1)(6n - 1) = (6p - 1)(6q - 1)\]

If

\[p = 5r + 1\]

with \(0 \leq r\), then

\[q = 1, 2, 3, 4, ...\]

or by symmetry, if

\[q = 5r + 1\]

with \(0 \leq r\), then

\[p = 1, 2, 3, 4, ...\]

**Proof.** Let be \(m, n, p, q, r, t, k\) integers. Let be \(1 \leq m, 1 \leq n, 1 \leq p, 1 \leq q, 0 \leq r, 1 \leq t, 1 \leq k\). We take the line \(m = 1\) and we have

\[5(6n - 1) = (6p - 1)(6q - 1)\]

or \((6p - 1) = 5t\) or \((6q - 1) = 5t\). By symmetry we can take \((6p - 1) = 5t\) and the same will happen with \((6q - 1) = 5t\) with inverted values.
Let’s take $(6p - 1) = 5t$ and we have

\[ p = \frac{5t + 1}{6} \]

We are looking for the values where $p$ is integer and we have that $t$ is of the form $6r + 1$ with $r = 0, 1, 2, 3...$

\[ p = \frac{5(6r + 1) + 1}{6} = 5r + 1 \]

$p$ takes the values $p = 1, 6, 11, 16...$

Now we are looking for the values of $q$ given $p$. We start with the equation

\[ 5(6n - 1) = (6p - 1)(6q - 1) \]

We can see that

\[ q = \frac{5n + p - 1}{6p - 1} \]

As we are looking for the integer values given $p$, we suppose that for $k = 1, 2, 3, 4...$

\[ 5n + p - 1 = k(6p - 1) \]

giving

\[ 5n = 6kp - k - p + 1 \]

Substituting we have

\[ q = \frac{5n + p - 1}{6p - 1} = \frac{6kp - k - p + 1 + p - 1}{6p - 1} = k \left( \frac{6p - 1}{6p - 1} \right) = k \]

that shows that $q$ takes all the values $k = 1, 2, 3, 4...$, for any value of $p$, remember that $p = 5r + 1$.

We conclude that when

\[ p = 5r + 1 \]

with $0 \leq r$

\[ q = 1, 2, 3, 4... \]

or by symmetry if

\[ q = 5r + 1 \]
then

\[ p = 1, 2, 3, 4... \]

Quod erat demonstrandum (Q.E.D).

To find the total quantity of composite numbers with the form \((6m - 1)(6n - 1) = (6p - 1)(6q - 1)\), we calculate the total composite numbers with that form lesser or equal to \(x\), we subtract the numbers in the columns (it has repetitions), we subtract the numbers in the rows (it has repetition), we need to add the numbers between intersections (it was subtracted twice), we add the numbers in the first row (it was subtracted before) and finally we subtract the composite numbers that has repetition between the composite numbers with the form \((6m + 1)(6n + 1)\) and the composite numbers with the form \((6m - 1)(6n - 1)\).

**C.0)** To find the total quantity of composite numbers with the form \((6m - 1)(6n - 1) = (6p - 1)(6q - 1)\) lesser or equal to \(N\).

If \((6m - 1)(6n - 1) \leq x\) then

\[ n \leq \frac{x + (6m - 1)}{6(6m - 1)} \]

Because we want integers we define

\[ C_{20}(x, m) = \left\lfloor \frac{x + (6m - 1)}{6(6m - 1)} \right\rfloor - m + 1 \]

where the last term eliminates the number of composite numbers under the main diagonal. Thus, the total quantity of composite numbers with the form \((6m + 1)(6n + 1)\), lesser or equal to \(x\) is

\[ \sum_{m \geq 1, C_{20}(x, m) > 0} C_{20}(x, m) \]

**C.1)** To find the total quantity of composite numbers with the form \((6m - 1)(6(5t + 1) - 1)\) lesser or equal to \(x\) in the columns.

\[(6m - 1)(6(5t + 1) - 1) = 5(6m - 1)(6t + 1)\]
If $5(6m - 1)(6t + 1) \leq x$ then

$$t \leq \frac{x - 5(6m - 1)}{30(6m - 1)}$$

but $t \geq 0$ and we want to count the number of columns, so

$$t + 1 \leq \frac{x - 5(6m - 1)}{30(6m - 1)} + 1$$

Because we want integers we define

$$C_{21}(x, m) = \left\lfloor \frac{x - 5(6m - 1)}{30(6m - 1)} + 1 \right\rfloor - \left\lfloor \frac{m - 1}{5} \right\rfloor$$

where the last term eliminates the number of extra columns under the main diagonal. Thus, the total quantity of composite numbers with the form $(6m - 1)(6(5t + 1) - 1)$, lesser or equal to $x$ is

$$\sum_{\substack{m \geq 1 \\ C_{21}(x, m) > 0}} C_{21}(x, m)$$

**C.2)** To find the total quantity of composite numbers with the form $(6(5s + 1) - 1)(6n - 1)$ lesser or equal to $x$ in the rows.

$$(6(5s + 1) - 1)(6n - 1) = 5(6s + 1)(6n - 1)$$

If $5(6s + 1)(6n - 1) \leq x$ then

$$n \leq \frac{x + 5(6s + 1)}{30(6s + 1)}$$

Because we want integers we define
\[ C_{22}(x, s) = \left\lfloor \frac{x + 5(6s + 1)}{30(6s + 1)} \right\rfloor - 5s \]

where the last term eliminates the number of composite numbers before the columns that touches the main diagonal. Thus, the total quantity of composite numbers with the form \((6(5s + 1) - 1)(6n - 1)\), lesser or equal to \(x\) and above the main diagonal is

\[
\sum_{s \geq 0, C_{22}(x, s) > 0} C_{22}(x, s)
\]

C.3) To find the total quantity of composite numbers with the form \((6(5s + 1) - 1)(6(5t + 1) - 1)\) lesser or equal to \(x\) that are intersections between columns and rows.

\[(6(5s + 1) - 1)(6(5t + 1) - 1) = 25(6s + 1)(6t + 1)\]

If \(25(6s + 1)(6t + 1) \leq x\) then

\[ t \leq \frac{x - 25(6s + 1)}{150(6s + 1)} \]

Because we want integers we define

\[ C_{23}(x, s) = \left\lfloor \frac{x - 25(6s + 1)}{150(6s + 1)} \right\rfloor - s + 1 \]

where the last term eliminates the number of composite numbers under the main diagonal. Thus, the total quantity of composite numbers with the form \((6(5s + 1) - 1)(6(5t + 1) - 1)\), lesser or equal to \(x\) and above the main diagonal is

\[
\sum_{s \geq 0, C_{23}(x, s) > 0} C_{23}(x, s)
\]

C.4) To find the total quantity of composite numbers with the form \(5(6n - 1)\) lesser or equal to \(x\) that are numbers in the first row.
If $5(6n - 1) \leq x$ then

$$n \leq \frac{x + 5}{30}$$

Because we want integers we define

$$C_{24}(x) = \left\lfloor \frac{x + 5}{30} \right\rfloor$$

Thus, the total quantity of composite numbers with the form $5(6n - 1)$, lesser or equal to $x$ in the first row is

$$C_{24}(x)$$

**C. Final** The total quantity of composite numbers with the form $(6m - 1)(6n - 1)$, without repetition is

$$C_{(6m - 1)(6n - 1)}(x) = \sum_{\substack{m \geq 1 \\ C_{20}(x, m) > 0}} C_{20}(x, m) - \sum_{\substack{m \geq 1 \\ C_{21}(x, m) > 0}} C_{21}(x, m) - \sum_{\substack{s \geq 0 \\ C_{22}(x, s) > 0}} C_{22}(x, s) + \sum_{\substack{s \geq 0 \\ C_{23}(x, s) > 0}} C_{23}(x, s) + C_{24}(x)$$

**D) Quantity of Composite Numbers in Common with** $(6m - 1)(6n - 1) = (6m + 1)(6n + 1)$

The next theorem shows the structure of the composites with $(6m - 1)(6n - 1) = (6p + 1)(6q + 1)$.

**Theorem 4.3.** (Danilo Chávez, January 22, 2022)

Let be $m, n, p, q, r, t, k$ integers. Let be $1 \leq m, 1 \leq n, 1 \leq p, 1 \leq q, 1 \leq r, 1 \leq t, 1 \leq k$.

When

$$(6m - 1)(6n - 1) = (6p + 1)(6q + 1)$$

If

$$p = 5r - 1$$
Proof. Let $b, m, n, p, q, r, t, k$ integers. Let be $1 \leq m, 1 \leq n, 1 \leq p, 1 \leq q, 1 \leq r, 1 \leq t, 1 \leq k$. We take the line $m = 1$ and we have

$$5(6n - 1) = (6p + 1)(6q + 1)$$

or $(6p + 1) = 5t$ or $(6q + 1) = 5t$. By symmetry we can take $(6p + 1) = 5t$ and the same will happen with $(6q + 1) = 5t$ with inverted values.

Let's take $(6p + 1) = 5t$ and we have

$$p = \frac{5t - 1}{6}$$
We are looking for the values where $p$ is integer and we have that $t$ is of the form $6r - 1$ with $r = 1, 2, 3, 4...$

$$p = \frac{5(6r - 1) - 1}{6} = 5r - 1$$

$p$ takes the values $p = 4, 9, 14, 19...$

Now we are looking for the values of $q$ given $p$. We start with the equation

$$5(6n - 1) = (6p + 1)(6q + 1)$$

We can see that

$$q = \frac{5n - p - 1}{6p + 1}$$

As we are looking for the integer values given $p$, we suppose that for $k = 1, 2, 3, 4...$

$$5n - p - 1 = k(6p + 1)$$

giving

$$5n = 6kp + k + p + 1$$

Substituting we have

$$q = \frac{5n - p - 1}{6p + 1} = \frac{6kp + k + p + 1 - p - 1}{6p + 1} = k \left( \frac{6p + 1}{6p + 1} \right) = k$$

that shows that $q$ takes all the values $k = 1, 2, 3, 4...$, for any value of $p$, remember that $p = 5r - 1$.

We conclude that when

$$p = 5r - 1$$

with $1 \leq r$

$$q = 1, 2, 3, 4...$$

or by symmetry if

$$q = 5r - 1$$

then

$$p = 1, 2, 3, 4...$$
Quod erat demonstrandum (Q.E.D.).

To find the total quantity of composite numbers that are in common between the composites with \((6m - 1)(6n - 1) = (6m + 1)(6n + 1)\), we calculate the numbers in the green columns, we add the numbers in the green rows and finally we subtract the numbers between intersections.

**D.1)** To find the total quantity of composite numbers with the form \((6m + 1)(6(5t - 1) + 1)\) lesser or equal to \(x\) in the columns.

\[
(6m + 1)(6(5t - 1) + 1) = 5(6m + 1)(6t - 1)
\]

If \(5(6m + 1)(6t - 1) \leq x\) then

\[
t \leq \frac{x + 5(6m + 1)}{30(6m + 1)}
\]

Because we want integers we define

\[
C_{121}(x, m) = \left\lfloor \frac{x + 5(6m + 1)}{30(6m + 1)} \right\rfloor - \left\lfloor \frac{m}{5} \right\rfloor
\]

where the last term eliminates the number of extra columns under the main diagonal. Thus, the total quantity of composite numbers with the form \((6m + 1)(6(5t - 1) + 1)\), lesser or equal to \(x\) is

\[
\sum_{\substack{m \geq 1 \\\ C_{121}(x, m) > 0}} C_{121}(x, m)
\]

**D.2)** To find the total quantity of composite numbers with the form \((6(5s - 1) + 1)(6n + 1)\) lesser or equal to \(x\) in the rows.

\[
(6(5s - 1) + 1)(6n + 1) = 5(6s - 1)(6n + 1)
\]

If \(5(6s - 1)(6n + 1) \leq x\) then

\[
n \leq \frac{x - 5(6s - 1)}{30(6s - 1)}
\]
Because we want integers we define

\[ C_{122}(x, s) = \left\lfloor \frac{x - 5(6s - 1)}{30(6s - 1)} \right\rfloor - 5s + 2 \]

where the last term eliminates the number of composite numbers before the columns that touches the main diagonal. Thus, the total quantity of composite numbers with the form \((6(5s - 1) + 1)(6n + 1)\), lesser or equal to \(x\) and above the main diagonal is

\[ \sum_{s \geq 1} C_{122}(x, s) \]

\[ C_{122}(x, s) > 0 \]

**D.3)** To find the total quantity of composite numbers with the form \((6(5s - 1) + 1)(6(5t - 1) + 1)\) lesser or equal to \(x\) that are intersections between columns and rows.

\[(6(5s - 1) + 1)(6(5t - 1) + 1) = 25(6s - 1)(6t - 1)\]

If \(25(6s - 1)(6t - 1) \leq x\) then

\[ t \leq \frac{x + 25(6s - 1)}{150(6s - 1)} \]

Because we want integers we define

\[ C_{123}(x, s) = \left\lfloor \frac{x + 25(6s - 1)}{150(6s - 1)} \right\rfloor - s + 1 \]

where the last term eliminates the number of composite numbers under the main diagonal. Thus, the total quantity of composite numbers with the form \((6(5s - 1) + 1)(6(5t - 1) + 1)\), lesser or equal to \(x\) and above the main diagonal is

\[ \sum_{s \geq 1} C_{123}(x, s) \]

\[ C_{123}(x, s) > 0 \]
D. Final The total quantity of composite numbers with repetition between composite numbers with the form \((6m + 1)(6n + 1)\) and \((6m − 1)(6n − 1)\), is

\[
C_{\text{common}}(x) = \sum_{m \geq 1} C_{121}(x, m) + \sum_{s \geq 1} C_{122}(x, s) - \sum_{s \geq 1} C_{123}(x, s)
\]

E) QUANTITY OF COMPOSITE NUMBERS WITH THE FORM \((6m − 1)(6n + 1) = (6p − 1)(6q + 1)\)

Figure 4: Composite numbers with the form \((6m − 1)(6n + 1) = (6p − 1)(6q + 1)\), the pink numbers are those which are repeated

The next theorem shows the structure of the composites with \((6m − 1)(6n + 1) = (6p − 1)(6q + 1)\).

**Theorem 4.4.** (Danilo Chávez, January 22, 2022)

Let be \(m, n, p, q, r, t, k\) integers. Let be \(1 \leq m, 1 \leq n, 1 \leq p, 1 \leq q, 0 \leq r, 1 \leq t, 1 \leq k\).

When

\[
(6m − 1)(6n + 1) = (6p − 1)(6q + 1)
\]

We have two cases:

**CASE A:**

\[
p = 5r + 1
\]

with \(0 \leq r\), then

22
or if

\[ q = 5r - 1 \]

with \( 1 \leq r \), then

\[ p = 1, 2, 3, 4... \]

CASE B:

If

\[ p = 7r - 1 \]

with \( 1 \leq r \), then

\[ q = 1, 2, 3, 4... \]

or if

\[ q = 7r + 1 \]

with \( 0 \leq r \), then

\[ p = 1, 2, 3, 4... \]

Proof. Let \( b, m, n, p, q, r, t, k \) integers. Let be \( 1 \leq m, 1 \leq n, 1 \leq p, 1 \leq q, 0 \leq r, 1 \leq t, 1 \leq k \).

**CASE A:**
We take the line \( m = 1 \) and we have

\[ 5(6n + 1) = (6p - 1)(6q + 1) \]

or \( (6p - 1) = 5t \) or \( (6q + 1) = 5t \).

**CASE A1**

Lets take \( (6p - 1) = 5t \) and we have

\[ p = \frac{5t + 1}{6} \]

We are looking for the values where \( p \) is integer and we have that \( t \) is of the form \( 6r + 1 \) with
\( r = 0, 1, 2, 3 \ldots \)

\[
p = \frac{5(6r + 1) + 1}{6} = 5r + 1
\]

\( p \) takes the values \( p = 1, 6, 11, 16 \ldots \)

Now we are looking for the values of \( q \) given \( p \). We start with the equation

\[
5(6n + 1) = (6p - 1)(6q + 1)
\]

We can see that

\[
q = \frac{5n - p + 1}{6p - 1}
\]

As we are looking for the integer values given \( p \), we suppose that for \( k = 1, 2, 3, 4 \ldots \)

\[
5n - p + 1 = k(6p - 1)
\]

giving

\[
5n = 6kp - k + p - 1
\]

Substituting we have

\[
q = \frac{5n - p + 1}{6p - 1} = \frac{6kp - k + p - 1 - p + 1}{6p - 1} = k \left( \frac{6p - 1}{6p - 1} \right) = k
\]

that shows that \( q \) takes all the values \( k = 1, 2, 3, 4 \ldots \), for any value of \( p \), remember that \( p = 5r + 1 \).

We conclude that when

\[
p = 5r + 1
\]

with \( 0 \leq r \)

\[
q = 1, 2, 3, 4 \ldots
\]

CASE A2

Let's take \( (6q + 1) = 5t \) and we have

\[
q = \frac{5t - 1}{6}
\]

We are looking for the values where \( q \) is integer and we have that \( t \) is of the form \( 6r - 1 \) with \( r = 1, 2, 3, 4 \ldots \)

\[
q = \frac{5(6r - 1) - 1}{6} = 5r - 1
\]
$q$ takes the values $q = 4, 9, 14, 19\ldots$

Now we are looking for the values of $p$ given $q$. We start with the equation

$$5(6n + 1) = (6p - 1)(6q + 1)$$

We can see that

$$p = \frac{5n + q + 1}{6q + 1}$$

As we are looking for the integer values given $q$, we suppose that for $k = 1, 2, 3, 4\ldots$

$$5n + q + 1 = k(6q + 1)$$

giving

$$5n = 6kq + k - q - 1$$

Substituting we have

$$p = \frac{5n + q + 1}{6q + 1} = \frac{6kq + k - q - 1 + q + 1}{6q + 1} = k \left( \frac{6q + 1}{6q + 1} \right) = k$$

that shows that $p$ takes all the values $k = 1, 2, 3, 4\ldots$, for any value of $q$, remember that $q = 5r - 1$.

We conclude that when

$$q = 5r - 1$$

with $1 \leq r$

$$p = 1, 2, 3, 4\ldots$$

**CASE B:**

We take the line $n = 1$ and we have

$$7(6m - 1) = (6p - 1)(6q + 1)$$

or $(6p - 1) = 7t$ or $(6q + 1) = 7t$.

CASE B1

Let's take $(6p - 1) = 7t$ and we have

$$p = \frac{7t + 1}{6}$$

We are looking for the values where $p$ is integer and we have that $t$ is of the form $6r - 1$ with
\[ r = 1, 2, 3, 4... \]
\[ p = \frac{7(6r - 1) + 1}{6} = 7r - 1 \]

\( p \) takes the values \( p = 6, 13, 20, 27... \)

Now we are looking for the values of \( q \) given \( p \). We start with the equation
\[ 7(6m - 1) = (6p - 1)(6q + 1) \]

We can see that
\[ q = \frac{7m - p - 1}{6p - 1} \]

As we are looking for the integer values given \( p \), we suppose that for \( k = 1, 2, 3, 4... \)
\[ 7m - p - 1 = k(6p - 1) \]
giving
\[ 7m = 6kp - k + p + 1 \]

Substituting we have
\[ q = \frac{7m - p - 1}{6p - 1} = \frac{6kp - k + p + 1 - p - 1}{6p - 1} = k \left( \frac{6p - 1}{6p - 1} \right) = k \]

that shows that \( q \) takes all the values \( k = 1, 2, 3, 4... \), for any value of \( p \), remember that \( p = 7r - 1 \).

We conclude that when
\[ p = 7r - 1 \]
with \( 1 \leq r \)
\[ q = 1, 2, 3, 4... \]

CASE B2

Let's take \( (6q + 1) = 7t \) and we have
\[ q = \frac{7t - 1}{6} \]

We are looking for the values where \( q \) is integer and we have that \( t \) is of the form \( 6r + 1 \) with \( r = 0, 1, 2, 3... \)
\[ q = \frac{7(6r + 1) - 1}{6} = 7r + 1 \]
$q$ takes the values $q = 1, 8, 15, 22...$

Now we are looking for the values of $p$ given $q$. We start with the equation

$$7(6m - 1) = (6p - 1)(6q + 1)$$

We can see that

$$p = \frac{7m + q - 1}{6q + 1}$$

As we are looking for the integer values given $q$, we suppose that for $k = 1, 2, 3, 4...$

$$7m + q - 1 = k(6q + 1)$$

giving

$$7m = 6kq + k - q + 1$$

Substituting we have

$$p = \frac{7m + q - 1}{6q + 1} = \frac{6kq + k - q + 1 + q - 1}{6q + 1} = k \left( \frac{6q + 1}{6q + 1} \right) = k$$

that shows that $p$ takes all the values $k = 1, 2, 3, 4...$, for any value of $q$, remember that $q = 7r + 1$. We conclude that when

$$q = 7r + 1$$

with $0 \leq r$

$$p = 1, 2, 3, 4...$$

Quod erat demonstrandum (Q.E.D).

To find the total quantity of composite numbers with the form $(6m - 1)(6n + 1)$, we calculate the total composite numbers with that form lesser or equal to $x$. We will calculate ROW BY ROW, beginning from $m = 2$. We subtract the numbers with the form $(6m - 1)(6(5t - 1) + 1)$. We subtract the numbers with the form $(6m - 1)(6(7t + 1) + 1)$. We add the intersections between the numbers with the forms $(6m - 1)(6(5t - 1) + 1)$ AND $(6m - 1)(6(7t + 1) + 1)$, we always avoid the rows with $m \equiv 1 \text{mod}(5)$ and $m \equiv -1 \text{mod}(7)$. We subtract every composite numbers lesser or equal to $x$ with $m \equiv 1 \text{mod}(5)$ OR $m \equiv -1 \text{mod}(7)$. We add every composite numbers lesser or equal to $x$ that are in the intersection where $m \equiv 1 \text{mod}(5)$ AND $m \equiv -1 \text{mod}(7)$. Finally we subtract the composite numbers in the column $n = 1$ that are in the row $m = 1$ (it has repetition).
E.0) To find the total quantity of composite numbers with the form \((6m - 1)(6n + 1)\) lesser or equal to \(x\).

If \((6m - 1)(6n + 1) \leq x\) then

\[
n \leq \frac{x - (6m - 1)}{6(6m - 1)}
\]

Because we want integers we define

\[
C_{30}(x, m) = \left\lfloor \frac{x - (6m - 1)}{6(6m - 1)} \right\rfloor
\]

Thus, the total quantity of composite numbers with the form \((6m - 1)(6n + 1)\), lesser or equal to \(x\) is

\[
\sum_{m \geq 1} \sum_{C_{30}(x, m) > 0} C_{30}(x, m)
\]

E.1) To find the total quantity of composite numbers with the form \((6m - 1)(6(5t - 1) + 1)\) lesser or equal to \(x\) in the columns. Adding from \(m = 2\) and avoiding \(m \equiv 1 \mod(5)\) AND \(m \equiv -1 \mod(7)\).

\[
(6m - 1)(6(5t - 1) + 1) = 5(6m - 1)(6t - 1)
\]

If \(5(6m - 1)(6t - 1) \leq x\) then

\[
t \leq \frac{x + 5(6m - 1)}{30(6m - 1)}
\]

Because we want integers we define

\[
C_{31}(x, m) = \left\lfloor \frac{x + 5(6m - 1)}{30(6m - 1)} \right\rfloor
\]

Thus, the total quantity of composite numbers with the form \((6m - 1)(6(5t - 1) + 1)\), lesser or equal to \(x\), adding from \(m = 2\) and avoiding \(m \equiv 1 \mod(5)\) AND \(m \equiv -1 \mod(7)\) is
\[
\sum_{m \geq 2 \atop m \not\equiv 1 \mod(5) \atop m \not\equiv -1 \mod(7) \atop C_{31}(x,m) > 0} C_{31}(x,m)
\]

**E.2)** To find the total quantity of composite numbers with the form \((6m - 1)(6(7t + 1) + 1)\) lesser or equal to \(x\) in the columns. Adding from \(m = 2\) and avoiding \(m \equiv 1 \mod(5)\) AND \(m \equiv -1 \mod(7)\).

\[(6m - 1)(6(7t + 1) + 1) = 7(6m - 1)(6t + 1)\]

If \(7(6m - 1)(6t + 1) \leq x\) then

\[t \leq \frac{x - 7(6m - 1)}{42(6m - 1)}\]

Because we want integers we define

\[C_{32}(x,m) = \left\lfloor \frac{x - 7(6m - 1)}{42(6m - 1)} \right\rfloor\]

Thus, the total quantity of composite numbers with the form \((6m - 1)(6(7t + 1) + 1)\), lesser or equal to \(x\), adding from \(m = 2\) and avoiding \(m \equiv 1 \mod(5)\) AND \(m \equiv -1 \mod(7)\) is

\[
\sum_{m \geq 2 \atop m \not\equiv 1 \mod(5) \atop m \not\equiv -1 \mod(7) \atop C_{32}(x,m) > 0} C_{32}(x,m)
\]

**E.3)** To find the total quantity of composite numbers with the form \((6m - 1)(6(35t - 6) + 1)\), that are the intersections between the numbers with form \((6m - 1)(6(5t - 1) + 1)\) and the form \((6m - 1)(6(7t + 1) + 1)\), lesser or equal to \(x\) in the columns. Adding from \(m = 2\) and avoiding \(m \equiv 1 \mod(5)\) AND \(m \equiv -1 \mod(7)\).

\[(6m - 1)(6(35t - 6) + 1) = 35(6m - 1)(6t - 1)\]

If \(35(6m - 1)(6t - 1) \leq x\) then
Because we want integers we define

\[ C_{33}(x, m) = \left\lfloor \frac{x + 35(6m - 1)}{210(6m - 1)} \right\rfloor \]

Thus, the total quantity of composite numbers with the form \((6m - 1)(6(35t - 6) + 1)\), lesser or equal to \(x\), adding from \(m = 2\) and avoiding \(m \equiv 1 \mod(5)\) AND \(m \equiv -1 \mod(7)\) is

\[ \sum_{m \geq 2 \atop m \not\equiv 1 \mod(5) \atop m \not\equiv -1 \mod(7) \atop C_{33}(x, m) > 0} C_{33}(x, m) \]

**E.4)** To find the total quantity of composite numbers with the form \((6m - 1)(6n + 1)\) lesser or equal to \(x\), in the rows where \(m \equiv 1 \mod(5)\) and not in the column \(n = 1\), we have

If \((6m - 1)(6n + 1) \leq x\) then

\[ n \leq \frac{x - (6m - 1)}{6(6m - 1)} \]

Because we want integers we define

\[ C_{34}(x, m) = \left\lfloor \frac{x - (6m - 1)}{6(6m - 1)} \right\rfloor - 1 \]

Thus, the total quantity of composite numbers with the form \((6m - 1)(6n + 1)\), lesser or equal to \(x\), in the rows where \(m \equiv 1 \mod(5)\) and not in the column \(n = 1\), is

\[ \sum_{m \geq 2 \atop m \equiv 1 \mod(5) \atop C_{34}(x, m) > 0} C_{34}(x, m) \]
E.5) To find the total quantity of composite numbers with the form \((6m - 1)(6n + 1)\) lesser or equal to \(x\), in the rows where \(m \equiv -1 \mod(7)\) and not in the column \(n = 1\), we have

If \((6m - 1)(6n + 1) \leq x\) then

\[
 n \leq \frac{x - (6m - 1)}{6(6m - 1)}
\]

Because we want integers we define

\[
 C_{35}(x, m) = \left\lfloor \frac{x - (6m - 1)}{6(6m - 1)} - 1 \right\rfloor
\]

Thus, the total quantity of composite numbers with the form \((6m - 1)(6n + 1)\), lesser or equal to \(x\), in the rows where \(m \equiv -1 \mod(7)\) and not in the column \(n = 1\), is

\[
 \sum_{\begin{subarray}{c} m \geq 2 \\ m \equiv -1 \mod(7) \\ C_{35}(x, m) > 0 \end{subarray}} C_{35}(x, m)
\]

E.6) To find the total quantity of composite numbers with the form \((6m - 1)(6n + 1)\) lesser or equal to \(x\), in the rows where \(m \equiv 1 \mod(5)\) AND \(m \equiv -1 \mod(7)\) and not in the column \(n = 1\), we have

If \((6m - 1)(6n + 1) \leq x\) then

\[
 n \leq \frac{x - (6m - 1)}{6(6m - 1)}
\]

Because we want integers we define

\[
 C_{36}(x, m) = \left\lfloor \frac{x - (6m - 1)}{6(6m - 1)} - 1 \right\rfloor
\]

Thus, the total quantity of composite numbers with the form \((6m - 1)(6n + 1)\), lesser or equal to \(x\), in the rows where \(m \equiv 1 \mod(5)\) AND \(m \equiv -1 \mod(7)\) and not in the column \(n = 1\), is
\[
\sum_{\substack{m \geq 2 \\
m \equiv 1 \mod(5) \\
m \equiv -1 \mod(7) \\
C_{36}(x, m) > 0}} C_{36}(x, m)
\]

\textbf{E.6) To find the total quantity of composite numbers which lies in the first row } m = 1 \text{ and in the columns } n = 7s + 1, \text{ which has repetition in the rows with } m = 5t + 1 \text{ and in the columns } n = 1, \text{ we have}

\[
7(6(5t + 1) - 1) = 35(6t + 1)
\]

Because we want integers we define

\[
C_{37}(s) = 35(6s + 1)
\]

Thus, the total quantity of composite numbers which lies in the row } m = 1, \text{ which has repetition in the column } n = 1, \text{ lesser or equal to } x \text{ is}

\[
\sum_{s \geq 1 \atop C_{37}(s) < x} 1
\]

\textbf{E. Final) The total quantity of composite numbers with the form } (6m - 1)(6n + 1), \text{ without repetition is}

\[
C_{(6m-1)(6n+1)}(x) = \sum_{m \geq 1 \atop C_{30}(x, m) > 0} C_{30}(x, m) - \sum_{m \geq 2 \atop m \not\equiv 1 \mod(5)} C_{31}(x, m) - \sum_{m \geq 2 \atop m \not\equiv -1 \mod(7)} C_{32}(x, m)
\]

\[
+ \sum_{m \geq 2 \atop m \not\equiv 1 \mod(5), m \not\equiv -1 \mod(7)} C_{33}(x, m) - \sum_{m \geq 2 \atop m \equiv 1 \mod(5)} C_{34}(x, m) - \sum_{m \geq 2 \atop m \equiv -1 \mod(7)} C_{35}(x, m)
\]

\[
+ \sum_{m \geq 2 \atop m \equiv 1 \mod(5), m \equiv -1 \mod(7)} C_{36}(x, m) - \sum_{s \geq 1 \atop C_{37}(s) < x} 1
\]
F) QUANTITY OF PRIME NUMBERS WITHOUT THE FORM $6k + 1$ or $6k - 1$

The prime numbers without the form $6k + 1$ or $6k - 1$ under $x$ are the numbers 2 and 3, two of them, so we need to add a constant at the end to complete the task.

Now we define

$$C_2 = 2$$

G) THE FUNCTION $\pi(x)$

Finally we have all the elements to formulate the function $\pi(x)$ if $25 \leq x \leq 846$.

$$\pi(x) = C_{6k+1}(x) + C_{6k-1}(x) - C_{(6m+1)(6n+1)}(x) - C_{(6m-1)(6n-1)}(x) + C_{common}(x) - C_{(6m-1)(6n+1)}(x) + C_2$$

5  Python Code to Test the Formula

With the next python code you can test the formula of $\pi(x)$. You need to download python from their website https://www.python.org/ and then install it. Download the pi_N_846.py file in this address

https://www.mediafire.com/file/m7qaxi5vge3dc8s/pi_N_846.py/file

After installing the software you can run the file, just double clicking on it.

References