Is the Cosmological Constant really a Universal Constant?

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Abstract

In this article it is demonstrated, that the cosmological constant in Einstein's field equations is not a universal constant but a constant of integration and therefore a constant parameter. The re-declaration of the cosmological constant from a constant parameter to a universal constant in Einstein's general relativity is in fact an unnecessary restriction of the theory. The cosmological constant as a constant parameter opens up the possibility of considering different gravitational systems and thus different metrics of spacetime with different cosmological constants, which is an important step in solving the problems of dark matter, dark energy and of course that of the cosmological constant.

Keywords: General relativity, Cosmological constant, Universal constant, Constant of integration, Constant parameter.

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1. Introduction

In 1917, Einstein made an attempt to describe the universe as a whole [1]. He considered it as homogeneous with a given average density. By considering Newtonian gravity, Einstein replaced the Poisson equation with

$$\Delta\phi - \Lambda\phi = 4\pi G\varrho, \qquad (1)$$

where Λ is a universal constant. In case of a constant mass distribution ϱ_0, the gravitational potential

$$\phi = -\frac{4\pi G}{\Lambda}\varrho_0$$

solves Eq. (1).

Accordingly, Λ is introduced as a universal constant and is designated as the cosmological constant in Einstein's field equations,

$$G_{ik} = \kappa T_{ik} - \Lambda g_{ik}, \qquad (2)$$

where κ is Einstein's gravitational constant. In Eqs. (2), T_{ik} is the energy-momentum density tensor of matter, g_{ik} is the metric tensor, and

$$G_{ik} = R_{ik} - \frac{R}{2}g_{ik}$$

is the Einstein tensor, where R_{ik} is the Ricci tensor and $R = g^{ik}R_{ik}$ is the Ricci scalar.

1.1 Derivation of Einstein's field equations

Einstein's field equations with the cosmological constant (2) can be derived by using the ansatz

$$M_{ik} = \kappa T_{ik} \qquad (3)$$

with the tensor

$$M_{ik} = R_{ik} + c_1 R g_{ik} + c_2 g_{ik}, \qquad (4)$$

which is given in the book of Møller [2]. Einstein's gravitational constant κ must be a universal constant, which assumes the role of a coupling constant, that regulates the strength of the gravitational interaction. It can be determined by reproducing the (modified) Poisson equation in the weak field limit of Newtonian gravity, $\kappa = 8\pi\, G/c^4$.

The tensor M_{ik} depends on the metric tensor g_{ik} and its first and second derivatives only. It fulfills the condition to be linear in the second derivatives of the metric tensor in order to reduce to the (modified) Poisson equation in the weak field limit. Because of Lovelock's theorem [3], Eqs. (3) and (4) show Einstein's field equations in their most possible modified form.

The constants c_1 and c_2 have to be determined in such a way, that

$$M^k_{i;k} = \nabla_k(R^k_i + c_1 R\delta^k_i + c_2\delta^k_i) = 0,$$

because the covariant divergence of the energy-momentum density tensor of matter in Eq. (3) must vanish, $T^k_{i;k} = 0$.

The constant $c_1 = -1/2$ is easily found as it fulfills the Bianchi identity. However, the constant c_2 is declared to be a universal constant: the cosmological constant Λ.

In the following it will be demonstrated, that this declaration of the constant c_2 to be a universal constant Λ in Einstein's general relativity in fact is an unnecessary restriction of the theory, because c_2 and thereby also the cosmological constant Λ turn out to be constant parameters.

2. Disproof

One begins with Einstein's field equations with the cosmological constant Λ in contravariant representation,

$$G^{\mu\nu} = \kappa T^{\mu\nu} - \Lambda g^{\mu\nu}, \qquad (5)$$

where Λ initially is defined or treated as a universal constant. The trace of Eqs. (5) reads

$$\kappa T + R = 4\Lambda, \qquad (6)$$

which is obtained by multiplying Eqs. (5) with the metric tensor $g_{\mu\nu}$. Eq. (6) can be substituted into Eqs. (5) to replace Λ,

$$G^{\mu\nu} + \frac{R}{4} g^{\mu\nu} = \kappa\left(T^{\mu\nu} - \frac{T}{4} g^{\mu\nu}\right). \quad (7)$$

These are the unimodular field equations, which are also yielded by fixing the determinant of the metric to be a constant in the action principle [4, 5]. However, in order to get the Eqs. (7) in this disproof, this unimodular condition has not been used. Instead, there have only been made algebraic transformations.

By taking the covariant divergence in Eqs. (7) one obtains

$$\nabla_\nu (R g^{\mu\nu}) = -\kappa \nabla_\nu (T g^{\mu\nu}), \quad (8)$$

where one has used $G^{\mu\nu}_{;\nu} = T^{\mu\nu}_{;\nu} = 0$. Eqs. (8) can be simplified by utilizing $g^{\mu\nu}_{;\nu} = 0$,

$$R_{,\nu} g^{\mu\nu} = -\kappa T_{,\nu} g^{\mu\nu},$$

and be multiplied with $g_{\sigma\mu}$, so that [5]

$$R_{,\sigma} = -\kappa T_{,\sigma} \text{ or } \frac{\partial}{\partial x^\sigma}(\kappa T + R) = 0. \quad (9)$$

This differential equation has the solution

$\kappa T + R = 4\Lambda,$

but where Λ now – in contrast to Eq. (6) – is a constant of integration, which can be any constant and is therefore a constant parameter and no universal constant [5].

However, Einstein's field equations with the cosmological constant (5) and the unimodular field equations (7) must be equivalent, because the unimodular field equations (7) are obtained from Einstein's field equations with the cosmological constant (5) by algebraic transformations. In order to achieve this equivalence, the cosmological constant Λ must be a constant parameter and no universal constant.

3. Discussion

Even at a very early stage of the disproof, one can argue that the cosmological constant Λ is not a universal constant, but a constant of integration and therefore a constant parameter. This fact becomes obvious by differentiating Eq. (6), in order to obtain the differential equation (9). Its solution, which is achieved by integration, demonstrates, that it has the same form as Eq. (6), whereby one easily concludes, that the cosmological constant Λ must be a constant of integration, which is a constant parameter and no universal constant.

A coupling constant regulates the strength of a fundamental physical interaction and therefore must be a universal constant and no constant parameter. As has been mentioned earlier, the strength of the interaction in gravity is already regulated by the coupling constant κ. The cosmological constant Λ therefore cannot assume the role of a coupling constant, because it turns out to be a constant parameter. Besides, two or even more coupling constants for a theory of a fundamental physical interaction would indeed make no sense.

The cosmological constant Λ being a constant parameter does of course not contradict to Lovelock's theorem [3].

4. Conclusions and outlook

The cosmological constant Λ in Einstein's field equations is a constant parameter and no universal constant. Even at an early stage of the disproof, it can be argued that the cosmological constant Λ is not a universal constant, but a constant of integration in Einstein's field equations and therefore a constant parameter of a particular problem. The cosmological constant Λ as a constant parameter opens up the possibility of considering different gravitational systems and thus different metrics of spacetime with different cosmological constants, which is an important step in solving the problems of dark matter, dark energy and of course that of the cosmological constant. This also means, that the cosmological constant Λ in general has not necessarily something to do with the universe as a whole.

General relativity is a classical and no quantum theory. Therefore, the cosmological constant Λ has absolutely nothing to do with the energy density of the vacuum, because the latter one can only occur in a quantum theory. Hence, it is no wonder, that there is a huge difference between the theoretical and the observed value of the energy density of the vacuum.

In Ref. [6], one of the authors of this article has found a reasonable physical explanation for the cosmological constant Λ. With this finding, flat rotation curves of spiral galaxies are obtained in Ref. [7].

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