The New Notation for Hyperoperation of a Sequence

Kyumin Nam

February 12, 2022

Abstract

For a sequence \( a_1, a_2, \ldots, a_n \), we define the exponent, tetration and pentation of a sequence \( a_n \) as

\[
\begin{align*}
\varepsilon_k (a_k) &= a_1[3]a_2[3] \cdots [3]a_n, \\
\end{align*}
\]

Also, we define the \( i \)-th hyperoperation of a sequence \( a_n \) as

\[
H_i (a_k) = a_1[i]a_2[i] \cdots [i]a_n.
\]

Contents

1. Introduction .......................................................... 2
2. Definition .......................................................... 2
3. Examples ......................................................... 2
4. Conclusion .......................................................... 2
5. Reference .......................................................... 3
Introduction

In this paper, we provide the notation for hyperoperations of a sequence. This notation will make it easier to write hyperoperations of sequence that are very long, and difficult to write.

Definition

The notation for summation and product of sequence $a_n$ is already defined as

$$\sum_{k=1}^{n} (a_k) = a_1[1]a_2[1] \cdots [1]a_n = a_1 + a_2 + \cdots + a_n \ [1]$$

$$\prod_{k=1}^{n} (a_k) = a_1[2]a_2[2] \cdots [2]a_n = a_1a_2 \cdots a_n \ [2]$$

using the uppercase of $\sigma$, and $\pi$ because the first letter of ‘summation’, and ‘product’ corresponds to $\sigma$, and $\pi$.

Now, we will define the new notations for exponent, tetration, pentation, and hyperoperation of sequence $a_n$ by the same way as a Capital-sigma notation and Capital-pi notation.

For a sequence $a_1, a_2, \ldots, a_n$, we define the notation for exponent of a sequence $a_n$ as

$$\varepsilon_{k=1}^{n} (a_k) = a_1[3]a_2[3] \cdots [3]a_n = a_1 \uparrow a_2 \uparrow \cdots \uparrow a_n$$

using the uppercase of $\varepsilon$ because the first letter of ‘exponent’ corresponds to $\varepsilon$.

Also, we define the notation for tetration and pentation of a sequence $a_n$ as

$$\tau_{k=1}^{n} (a_k) = a_1[4]a_2[4] \cdots [4]a_n = a_1 \uparrow\uparrow a_2 \uparrow\uparrow \cdots \uparrow\uparrow a_n$$

$$\phi_{k=1}^{n} (a_k) = a_1[5]a_2[5] \cdots [5]a_n = a_1 \uparrow\uparrow\uparrow a_2 \uparrow\uparrow\uparrow \cdots \uparrow\uparrow\uparrow a_n$$

using the uppercase of $\tau$, and $\phi$ because the first letter of ‘tetration’, and ‘pentation’ corresponds to $\tau$, and $\phi$.

Moreover, we define the notation for $i$-th hyperoperation of a sequence $a_n$ as

$$H_i_{k=1}^{n} (a_k) = a_1[i]a_2[i] \cdots [i]a_n$$

using the uppercase of $\eta$ because the first letter of ‘hyperoperation’ corresponds to uppercase of $\eta$. 
Examples

(1) $\prod_{k=1}^{4} (2k) = 2 \uparrow 4 \uparrow 6 \uparrow 8 = 2^{4^{6^{8}}}$

(2) $\prod_{k=1}^{3} (k - 4)^2 = 9 \uparrow \uparrow 4 \uparrow \uparrow 1 = 9^{9^{9^9}}$

(3) $\prod_{k=1}^{n} k = 1 + 2 + ... + n = \frac{1}{2}n(n + 1)$

(4) $\prod_{k=1}^{n} k = 1 \times 2 \times \cdots \times n = n!$

Conclusion

In this paper, we provided the new notation for hyperoperation of a sequence. We defined the exponent, tetration, and pentation of a sequence $a_1, a_2, \ldots, a_n$ as $\prod_{k=1}^{n} a_k = a_1[3]a_2[3] \cdots [3]a_n$, $\prod_{k=1}^{n} (a_k) = a_1[4]a_2[4] \cdots [4]a_n$, $\prod_{k=1}^{n} (a_k) = a_1[5]a_2[5] \cdots [5]a_n$, for a sequence $a_1, a_2, \ldots, a_n$. Also, we defined the $i$-th hyperoperation of a sequence $a_1, a_2, \ldots, a_n$ as $\prod_{k=1}^{n} (a_k) = a_1[i]a_2[i] \cdots [i]a_n$ for a sequence $a_1, a_2, \ldots, a_n$.

Reference