A critical analysis of Schwarzschild-like metrics

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Abstract
By neglecting the cosmological constant Λ, Einstein's field equations in absence of matter and other fields read $G_{ik} = 0$, which is not reasonable, since it violates the conservation law of total energy, momentum, and stress, because the gravitational field energy and momentum density cannot be represented by a vanishing Einstein tensor. In order to remedy this shortcoming, we construct a uniform metric, which allows us later to get a more general one, that is asymptotically equal to the Schwarzschild metric. This metric has a plausible energy-momentum density tensor of the gravitational field and correctly describes the effect of light deflection, but the perihelion shift of Mercury is overestimated. Because of the authors' different view in order to overcome the shortcomings, different solutions are obtained.

Keywords
General Relativity, Schwarzschild metric, Podosenov metric, cosmological constant, de Sitter-Schwarzschild metric, homogeneous metric, gravitational field energy and momentum density, light deflection, perihelion shift.

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1. Introduction

Landau and Lifshitz [1] noticed, that there is no gravitational field energy density as a source on the right hand side of Einstein's field equations. In § 95, it is written, that

> Gravitational interaction plays a role only for bodies with a sufficiently large mass (due to the small gravitational constant), ...

and in § 96 regarding the vanishing covariant derivative of the energy-momentum density tensor of matter, $T^k_{i;k} = 0$, that

> In this form, however, this equation does not generally express any conservation law whatever. This is related to the fact that in a gravitational field the four-momentum of the matter alone must not be conserved, but rather the four-momentum of matter plus gravitational field; the latter is not included in the expression for T^k_i.

Indeed, Einstein initially introduced the energy-momentum density tensor of the gravitational field into his field equations [2][1]. However, it turned out, that this quantity, which is calculated from the conservation law of energy and momentum, is not a tensor. In this form, the expression for the gravitational field energy and momentum density was not covariant, which became a serious obstacle for creating the complete field equations. Within only two years, a way out of this dilemma was found, after Einstein simply had removed the gravitational field energy and momentum density from his theory. Then, the field equations became covariant and correctly described the motion of Mercury.

Of course, experiments remain the most important tests for the suitability of Einstein's field equations. But there do not exist enough data of strong gravitational fields. Therefore, additional surveys will expand our understanding of gravity.

A continuous field is uniform in a small neighborhood around any point in space-time. This trivial fact follows from

[1]We can estimate the magnitude of the correction to the Newtonian law of gravity, if we take into account the gravitational mass of the field. This correction is on the same order as the correction of the Newtonian gravitational theory caused by the general theory of relativity [3].

the continuity of a smooth function. Therefore it is natural, that a gravitational field should have a uniform metric in a small neighborhood of points in space-time. However, the approximated metric of the vacuum solution of Einstein's field equations is not homogeneous [4],

$$ds^2 = \left(1+\frac{2\Phi}{c^2}\right)c^2dt^2 - \left(1-\frac{2\Phi}{c^2}\right)d\Omega^2\,, \qquad (1)$$

where Φ is the Newtonian gravitational potential and

$$d\Omega^2 = dx^2+dy^2+dz^2\,.$$

In the beginning of his work on general relativity, Einstein found a relation between time in an accelerated and in an inertial reference frame [5],

$$\delta\tau = \left(1-gx/c^2\right)\delta t\,,$$

where g is the acceleration. However, the exact relation reads

$$d\tau = \exp\left(-\frac{gx}{c^2}\right)dt\,, \qquad (2)$$

which reflects the symmetry of a uniformly accelerated frame of reference [5]:

> From the fact that the choice of the coordinate origin should not affect this relation, we can conclude that it should be replaced by the exact relation.

From the principle of equivalence it follows, that in a uniform gravitational field, Eq. (2) should be satisfied. However, in subsequent works this ratio has not been used.

The next step in the study of a uniform gravitational field was made by Harry Lass [6]. He described the accelerated frame of reference and found its metric,

$$ds^2 = \exp\left(-\frac{2gx}{c^2}\right)\left(c^2dt^2-dx^2\right)-dy^2-dz^2\,.$$

It is true, that this metric satisfies Einstein's field equations in empty space-time, $R_{ik}=0$, but it is not spatially isotropic. This shortcoming has been overcome in Ref. [7],

$$ds^2 = \exp\left(-\frac{2gx}{c^2}\right)\left(c^2dt^2-d\Omega^2\right)\,.$$

Unquestionably, this metric is both, homogeneous and spatially isotropic. Accordingly, metrics of the form

$$ds^2 = \exp\left(\frac{2h}{c^2}\right)\left(c^2dt^2-d\Omega^2\right)\,, \qquad (3)$$

where h is a function, which depends on the coordinates, seem at first sight to be suitable for describing gravitational fields. It is true, that one obtains plausible energy and momentum densities of the gravitational field from metrics of the form shown in Eq. (3), cf. Ref. [8], but in a more precise study, one recognizes, that they are not suitable for describing the gravitational field, because their coordinate velocity is constant, so that consequently the observed light deflection is not obtained within the framework of such metrics.

In the following, we discuss an attempt to modify the Schwarzschild solution within the framework of a uniform metric by using Einstein's field equations.

2. Homogeneous metric

As mentioned above, metrics of the form given by Eq. (3) do not satisfy the principles of the theory of general relativity. Therefore, the metric of the approximated vacuum solution shown in Eq. (1) serves as the starting point for creating a uniform metric by regarding Einstein's exact result, cf. Eq. (2),

$$ds^2 = \exp\left(-\frac{2gx}{c^2}\right)c^2dt^2 - \exp\left(\frac{2gx}{c^2}\right)d\Omega^2\,. \qquad (4)$$

From the geodesic equation, one finds the acceleration,

$$\frac{d^2x}{dt^2} = -c^2\Gamma^1{}_{00} = \frac{c^2}{2}\frac{\partial g_{00}}{\partial x}g^{11} = g\exp\left(-\frac{4gx}{c^2}\right)\,, \qquad (5)$$

and its covariant counterpart with respect to this reference frame,

$$\frac{d^2x_1}{d\tau^2} = -c^2\Gamma^1{}_{00}\frac{g_{11}}{g_{00}} = -g\,,$$

which proofs, that the metric given by Eq. (4) is homogeneous. Moreover, this metric has a positive scalar curvature,

$$R = \frac{2g^2}{c^4}\exp\left(-\frac{2gx}{c^2}\right)\,. \qquad (6)$$

In order to determine the energy-momentum density tensor, we have inserted the metric shown in Eq. (4) into Einstein's field equations, where in absence of matter and other fields, the tensor

$$\widehat{T}_{ik} = \frac{c^4}{8\pi G}G_{ik} = -\frac{g^2}{8\pi G}\,\mathrm{diag}\left(\mathrm{e}^{-\frac{4gx}{c^2}},1,-1,-1\right)$$

solely represents the gravitational field energy and momentum density. Its prefactor on the right hand side resembles the energy density of an electrostatic field and equals the Newtonian gravitational field energy density, cf. Ref. [1].

The dependence of $\widehat{T}_{00}$ on the coordinate arises, because the total energy of bodies in the second approximation of the theory of general relativity depends on the gravitational potential (see Problem 2, § 106 in Ref. [1]). This explains the non-uniform acceleration in a uniform field, cf. Eq. (5). The ambiguity of the asymptotic transition to Eq. (4) does not allow us to state that this homogeneous metric is the final result.

3. Equation of the gravitational field

Since any field should be uniform in the limit of a small neighborhood, consequently also the metric should be the one of a uniform field in this limit. This property is fulfilled by the uniform metric shown in Eq. (4). Accordingly, metrics of the kind

$$ds^2 = \exp\left(\frac{2h}{c^2}\right) c^2 dt^2 - \exp\left(-\frac{2h}{c^2}\right) d\Omega^2 \tag{7}$$

are also uniform in the limit of a small neighborhood around any point in space-time, where h is a continuous function, which depends on the coordinates. It is important to mention, that the form of the metric given by Eq. (7) does not change, if h is a linear transformation. Therefore, h has to be a linear equation. The most suitable candidate for a linear covariant equation is the wave equation[2],

$$\frac{1}{c^2}\frac{\partial^2 h}{\partial t^2} - \Delta h = -4\pi G T\,, \tag{8}$$

where T is the trace of the energy-momentum density tensor of matter. Consequently, a set of continuous real functions h forms an additive group in the sense, that metrics of the kind shown in Eq. (7) can be superposed.

Obviously, in case of large distances from a point-like mass, the uniform metric given by Eq. (7) merges into the metric of the approximated vacuum solution shown in Eq. (1). Thus, the covariance and the correspondence principle are fulfilled.

4. Metric of a point-like mass

In the stationary case, the wave equation (8) becomes the Poisson equation

$$\Delta h = 0\,,$$

which has the spherically symmetric solution

$$h = \frac{C_1}{r} + C_2\,.$$

The boundary conditions of the Cauchy problem can be specified at infinity. They read $C_1 = -GM$ and $C_2 = 0$, so that

$$h = -\frac{GM}{r}\,,$$

which is equivalent to the Newtonian gravitational potential Φ with mass M in the origin. Consequently, the uniform variant of the Schwarzschild metric is given by

$$ds^2 = \exp\left(-\frac{r_g}{r}\right) c^2 dt^2 - \exp\left(\frac{r_g}{r}\right) d\tilde{\Omega}^2\,, \tag{9}$$

where $r_g = 2GM/c^2$ and

$$d\tilde{\Omega}^2 = dr^2 + r^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)\,.$$

[2] Nordström used a similar equation in his first relativistic theory of gravity [8].

The coordinate speed of light, which is given by this metric, is asymptotically equal to the speed of light, that is observed in Shapiro's experiment [9],

$$c\exp\left(-\frac{r_g}{r}\right) \approx c\left(1 - \frac{r_g}{r}\right)\,.$$

The scalar curvature almost coincides with that one of the homogeneous metric in Eq. (6),

$$R = \frac{2g^2}{c^4}\exp\left(-\frac{r_g}{r}\right)\,,$$

where $g = -GM/r^2$. The energy and momentum density of the gravitational field can be obtained by using Einstein's field equations,

$$\widehat{T}_{ik} = -\frac{g^2}{8\pi G}\,\mathrm{diag}\left(\mathrm{e}^{-\frac{2r_g}{r}}, 1, -r^2, -r^2\sin^2\theta\right)\,,$$

where $\widehat{T}_{00}$ at infinity equals the Newtonian gravitational field energy density.

In order to compute the angle of light deflection and the perihelion shift with the metric given by Eq. (9), the relativistic Kepler problem has to be solved by using the geodesic equations with the aid of

$$\left(\frac{ds}{d\lambda}\right)^2 = g_{\mu\nu}\frac{dx^\mu}{d\lambda}\frac{dx^\nu}{d\lambda} = c^2\left(\frac{d\tau}{d\lambda}\right)^2 = \varepsilon\,,$$

where for a massive particle $\lambda = \tau$, while for a massless particle, for which $m = 0$ and $d\tau = 0$, another parameter λ has to be used [10]. The solutions to the geodesic equations read,

$$\begin{aligned} \theta = \frac{\pi}{2}\,, \qquad \ell = r^2\frac{d\phi}{d\lambda}g^{00}\,, \qquad F = \frac{dx^0}{d\lambda}g_{00}\,, \\ \varepsilon = \left[F^2 - \left(\frac{dr}{d\lambda}\right)^2\right]g^{00} - \frac{\ell^2}{r^2}g_{00}\,. \end{aligned} \tag{10}$$

The first result given by Eqs. (10), shows that the orbit is located in the equatorial plane. The second solution represents the constant relativistic angular momentum. The third one is a further orbital constant, and

$$\varepsilon = \begin{cases} c^2 & (m \neq 0) \\ 0 & (m = 0) \end{cases}\,.$$

By using the second and the fourth result from Eqs. (10), the equation of orbital motion is derived,

$$\phi(r) = \int \frac{dr}{r^2}\left[\left(\frac{Fg^{00}}{\ell}\right)^2 - \frac{1}{r^2} - \frac{\varepsilon g^{00}}{\ell^2}\right]^{-\frac{1}{2}}\,. \tag{11}$$

For light, $\varepsilon = 0$, and

$$0 = \left.\frac{dr}{d\phi}\right|_{r=r_0} \tag{12}$$

at the radius of the sun, where $r_0 = R_\odot$. Then, by neglecting terms of second and higher order, we obtain the angle of light deflection

$$\Delta\phi = \frac{4a}{R_\odot} \,,$$

where $a = GM/c^2$. This result conforms to observations.

The perihelion shift is calculated in the same manner by using $\varepsilon = c^2$ in Eqs. (11) and (12) respectively, where in this case $r_0 = q$ in the perihelion and $r_0 = Q$ in the aphelion of the orbit. The constant quantities in the equation of orbital motion (11) read

$$\chi \equiv \frac{F^2}{\ell^2} = \frac{g_{00}(q)/q^2 - g_{00}(Q)/Q^2}{g^{00}(q) - g^{00}(Q)} \,,$$
$$\zeta \equiv \frac{c^2}{\ell^2} = \chi g^{00}(q) - \frac{g_{00}(q)}{q^2} = \chi g^{00}(Q) - \frac{g_{00}(Q)}{Q^2} \,.$$

The metric coefficient g^{00} as well as its square, which appear in Eq. (11), can be expanded until second order with high enough accuracy. Then, with the aid of the constant auxiliary quantities

$$\alpha = a^2(8\chi - 2\zeta) - 1 \,, \quad \beta = 2a(2\chi - \zeta) \,, \quad \gamma = \chi - \zeta \,,$$

the equation of orbital motion (11) can be written in the form

$$\phi(Q) - \phi(q) = \int_q^Q \frac{dr}{r} \left(\alpha + \beta r + \gamma r^2\right)^{-\frac{1}{2}}$$

and be solved analytically, see e.g. formula 2.266 in Ref. [11]. The perihelion shift is then given by

$$\Delta\phi = 2\left[\phi(Q) - \phi(q)\right] - 2\pi \,.$$

In contrast to Ref. [12], it turns out, that the computed perihelion shift of Mercury exceeds by far the observed one. This result is not surprising by realizing the differences in the accelerations, cf. Fig. 1.

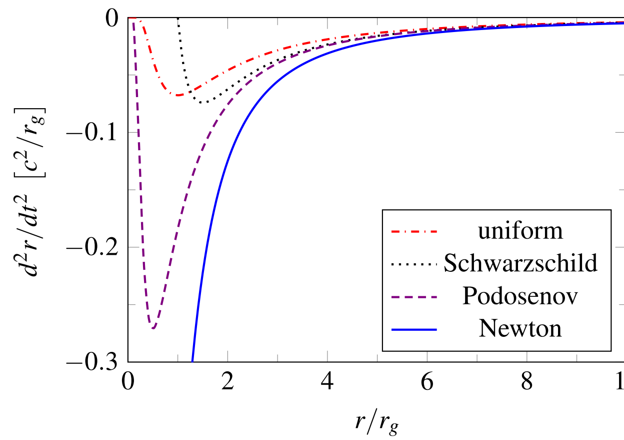

Figure 1. Comparison of the accelerations in different metrics, $d^2r/dt^2 = -c^2\Gamma^1{}_{00}$.

5. Discussion

The metrics of uniformly accelerated systems, which have been strictly obtained in Refs. [6, 7] are not applicable to the theory of gravity. In fact, the metric, which is shown in Eq. (3), describes space-time with a constant speed of light. This shortcoming has been overcome with the metric given by Eq. (7), which is homogeneous with respect to the covariant acceleration.

There exists another metric, which is homogeneous with respect to the contravariant acceleration, the Podosenov metric [13], that reads in spherical coordinates

$$ds^2 = \exp\left(-\frac{r_g}{r}\right) c^2 dt^2 - d\tilde{\Omega}^2 \,.$$

The spherical Podosenov metric shows gravitational accelerations, which are asymptotically equal to the Schwarzschild metric, cf. Fig. 1. However, the speed of light, which depends on the gravitational potential, is obtained incorrectly. In addition, this metric has a zero value of G_{00}.

Another disagreement with observations is, that the uniform variant of the Schwarzschild metric (9) as well as the Podosenov metric do not show black hole solutions, see Fig. 1.

It is irrefutable, that the covariance of the gravitational field equation and the correspondence to Newton's theory are not enough to consistently describe the gravitational field. The reason for this is, that in empty space-time there is no gravitational energy and momentum density in Einstein's field equations without the cosmological constant, i.e. $G_{ik} = 0$. Therefore, an additional condition or principle is required, which is able to repair this shortcoming.

In this article, we have considered metrics, which are consistent with Einstein's exact relation shown in Eq. (2). Unfortunately, all of them turned out to have significant flaws. Perhaps there exists another metric, which gives results, that are in agreement with observations. Future studies will show this.

This is the status of research of the first version of this work from December 31, 2019, which has also been translated into Russian language on January 11, 2020. To that time, the authors could not obtain satisfying results, which are able to overcome the above demonstrated shortcomings. However, meanwhile enormous progress has been made and further articles have been published, whereby these problems can be solved.

6. Conclusions

Much time is passed since the first version of this article has been written. Naturally, meanwhile the authors have obtained different solutions to the above shown problems because of their different view. Therefore, the authors consider it appropriate to outline their different ideas and results.

6.1 Rüster's solution

In contrast to the whole article, throughout this subsection the metric signature $(-,+,+,+)$ is used.

Einstein's field equations with the cosmological constant in their mixed-tensor representation form the necessarily existing conservation law of *total* energy, momentum, and stress in the theory of general relativity [14],

$$\kappa^{-1}\Lambda\,\delta^i_{\ k} = T^i_{\ k} - \kappa^{-1}G^i_{\ k}\,. \tag{13}$$

Hence, the respective energy and momentum densities can be assigned to the respective tensors in Eqs. (13):

- $T^i_{\ k}$ is the energy-momentum density tensor of matter,
- $-\kappa^{-1}G^i_{\ k}$ is the energy-momentum density tensor of the gravitational field,
- $\kappa^{-1}\Lambda\,\delta^i_{\ k}$ is the total energy-momentum density tensor,

which means, that the total energy-momentum density tensor equals the energy-momentum density tensor of matter plus the energy-momentum density tensor of the gravitational field, see Refs. [14, 15].

Up to now, the cosmological constant erroneously is related to the energy density of the vacuum. Thereby, the cosmological constant problem arises, see e.g. Ref. [16]. There appears a huge mismatch between the theoretical and the observed value of the vacuum energy density, which cannot be overcome by keeping this hypothesis. Moreover, the vacuum energy is defined as the difference of the non-vanishing ground-state energy of a quantum-mechanical particle system and the minimum of the energy of this system, if it would be described classically [17]. Since the theory of general relativity is a classical gravitational and no quantum theory, it is questionable why the cosmological constant Λ should be related to the energy density of the vacuum [18]. The findings in Ref. [14] give a logical and reasonable explanation for the cosmological constant being a constant parameter, which is proportional to the total energy density with respect to the metric under consideration, which in consequence also means, that there are different cosmological constants with respect to different metrics.

By considering the empty space-time around a star for example, the total energy density equals the energy density of the gravitational field,

$$\kappa^{-1}\Lambda = -\kappa^{-1}G^0_{\ 0}\,. \tag{14}$$

The Newtonian value of the energy density of the gravitational field around a star is always negative. Consequently, also the energy density of the gravitational field is negative around a star in the theory of general relativity, $-\kappa^{-1}G^0_{\ 0} < 0$. Hence, the total energy density and thereby the cosmological constant with respect to the metric of a star or of any other celestial object is negative, $\Lambda < 0$, cf. Eq. (14). This fact explains the dark matter phenomenon [14]. With this finding, flat rotation curves of spiral galaxies are obtained in Ref. [19].

By making use of the Friedmann-Lemaître-Robertson-Walker (FLRW) metric, its positive value of the cosmological constant explains the accelerated expansion of our universe and also the dark energy phenomenon being caused by its total energy density, which is represented by its cosmological constant [14].

Hence, the cosmological constant in general has absolutely nothing to do with cosmology except in case the whole universe is considered by using the FLRW metric.

Since the existing condition to Einstein's field equations in empty space-time, $G_{ik} = 0$, would not fulfill the conservation law given by Eqs. (13), it has to be novated, because in empty space-time the total energy-momentum density tensor equals the energy-momentum density tensor of the gravitational field,

$$\kappa^{-1}\Lambda\,\delta^i_{\ k} = -\kappa^{-1}G^i_{\ k}\,,$$

which can simply be transformed into

$$G_{ik} = -\Lambda g_{ik}\,, \qquad R_{ik} = \Lambda g_{ik}\,.$$

The metric of a point-like mass is then no longer given by the Schwarzschild metric, but by the de Sitter-Schwarzschild metric, see Ref. [19],

$$ds^2 = -\left(1+\frac{2\tilde{\Phi}}{c^2}\right)c^2dt^2 + \frac{dr^2}{1+\frac{2\tilde{\Phi}}{c^2}} + r^2\left(d\theta^2 + \sin^2\theta\; d\phi^2\right)\,, \tag{15}$$

where

$$\tilde{\Phi} = -\frac{GM}{r} - \frac{\Lambda c^2 r^2}{6}$$

is the modified Newtonian gravitational potential of a point-like mass [18].

Eq. (15) is a metric, which is not homogeneous with respect to the covariant or contravariant accelerations, but it is homogeneous because of its *constant scalar curvature*, $R = 4\Lambda$, see Sec. 5.2 in Ref. [20]. This latter criterion for the homogeneity of a metric is decisive, since constant accelerations in the neighborhood of a point in space-time of a metric can only be achieved in the flat space-time of the Minkowski metric. The accelerations would of course vanish in this case. This is the reason why it is in fact senseless to search for metrics with such a property. The homogeneity of a metric must only be related to its scalar curvature.

Because of the conservation law (13), the contribution of the cosmological constant cannot be neglected in principle, otherwise one would violate it, so that the Schwarzschild metric is only a very good approximation, which is valid only on "short" distances, and the de Sitter-Schwarzschild metric (15) is the exact solution.

A tiny value of the cosmological constant would barely alter the perihelion shift of a planet in a solar system in comparison when the cosmological constant is neglected [21]. When light deflection is considered, one recognizes, that it has absolutely no effect on the deflection angle at all [21].

Considerations about the cosmological constant in the context of compounded celestial objects and gravitational waves are made in Ref. [18]. The cosmological constant can be neglected by regarding gravitational waves, because it is no source of them. It only contributes as a potential to the solution of the wave in the linearized Einstein field equations, which is of less importance. Gravitational waves curve the background, see §35.13. in Ref. [22]. The energy and momentum density carried by gravitational waves is up to a constant given by the Einstein tensor of the background. Since the Einstein tensor up to a constant represents the energy and momentum density of the gravitational field, this demonstrates that gravitational waves are oscillations of the energy and momentum density of the gravitational field, which are caused by oscillating mass densities.

6.2 Morozov's solution

First of all, the boundary conditions [23] are specified, instead of the Schwarzschild boundary conditions

$$g_{00} \approx 1 + \frac{2\varphi}{c^2}\,.$$

As the limiting value at infinity of the solution of the lumped-mass problem, only the flat metric is proposed, that is consistent with the Schwarzschild condition,

$$ds^2 = \left(1 \pm \frac{2\varphi}{c^2}\right) c^2 dt^2 - \left(1 \pm \frac{2\varphi}{c^2}\right)^{-1} dx^2 - dy^2 - dz^2\,.$$

This metric satisfies the equivalence principle for the Schwarzschild solution, since the Schwarzschild metric transforms into this metric in an infinitely small volume.

Moreover, according to Morozov, the gravitational field equation should have the form [24],

$$\frac{\partial \Gamma^{\alpha}{}_{\mu\nu}}{\partial x_\alpha} - \Gamma^{\alpha}{}_{\mu\beta}\Gamma^{\beta}{}_{\nu\alpha} = \kappa\left(T_{\mu\nu} - \frac{T}{2} g_{\mu\nu}\right)\,.$$

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