# The Energy-Time Paradox Limits of Special Relativity in Practice

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**ABSTRACT :** When we apply the results of the special theory of relativity specifically to relativistic energy and time dilation together on any physical phenomenon that depends on both, the observer gets contradictory results in the same frame of reference, and thus we have a new paradox in the special theory of relativity revealing to us the limits of special relativity in application, and the reasons for the failure of the combination of Special Relativity with thermodynamics

**Key words**: Paradoxes of modern physics - Inverse Relativity - Relativistic thermodynamics Relativistic Doppler effect - Time dilation - Relativistic Energy - Pair annihilation

## 1 INTRODUCTION

Since the emergence of Special Relativity and it is subject to criticism and testing, it was published in 1905 by the scientist Albert Einstein and is considered one of the basics of modern physics. [1], because the results of the theory are related to many branches of physics, there was a need to combine special relativity with other branches of physics for the relativistic study of these branches, ( i.e. in an inertial reference frame that moves at speeds close to the speed of light ) Such as the Doppler effect of light, Electricity, Quantum mechanics, Thermodynamics [3-2], The combination succeeded with some branches, but others, such as the combination with thermodynamics, produced more than one model for relativistic thermodynamics and the combination with the Doppler effect of light produces for us a model ( Relativistic Doppler effect) that violates the principle of conservation of energy in the redshift, Are there, then, limits to

Special Relativity as a mathematical model in its application to physical phenomena? The answer to the question depends on finding some results of Special Relativity that do not agree together when applied to the same physical phenomena

# 2 METHODS

## 2-1 Electron-positron annihilation phenomenon in reference frames

We assume that we have two reference frames, S and S', from orthogonal cartesian coordinate systems, each reference frame has an observer at the origin O and O', and that the frame S' is moving with a uniform velocity $V_S$ relative to the S frame in the positive direction of the X axis, that is, the reference frames here it is an inertial frames [1], See Figure : 1

$S` \rightarrow x` y` z` t`$

$S \rightarrow x \; y \; z \; t$

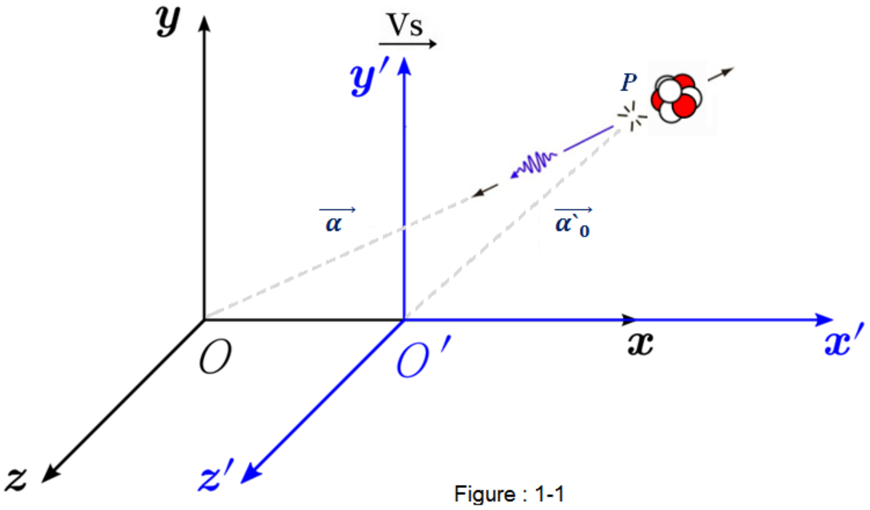


Figure : 1-1

We also assume that while the reference frame S' is moving away from the reference frame S, the phenomenon of annihilation of a pair occurs between an electron and a positron in a rest state [4] in the y'x' plane near the nucleus of a heavy atom at point *P*, where the electron and positron annihilate each other and result in this γ-ray photon according to the following equation

$$e^{-} + e^{+} \rightarrow \gamma \tag{1.1}$$

## 2-2 Relativistic energy and frequency of the photon

The phenomenon of electron-positron annihilation is subject to the laws of conservation of energy and momentum, where the sum of the energy of the photon and the kinetic energy of the nucleus after the phenomenon of annihilation must equal the relativistic total energy of the electron and positron before the phenomenon of annihilation, but the observer O' observes the particles before the phenomenon of annihilation in a rest state relative to the reference frame S', and therefore the energy possessed by the particles here is the energy of the rest mass and the momentum is zero, and here we can also neglect the kinetic energy of the nucleus resulting from the momentum opposite to the momentum of the photon because of the big mass of nucleus

$$\left(\grave{E_0}\right)_{e^-} + \left(\grave{E_0}\right)_{e^+} = \grave{E_\gamma} \qquad (2.1)$$

Where

$$\grave{E_0} = \grave{m_0} c^2 \qquad (3.1)$$

Because the rest mass of the electron is equal to the rest mass of the positron, so we deduce from 2.1 and 3.1

$$2\,\grave{E_0} = \grave{E_\gamma} \qquad (4.1)$$

As for the observer O, he observes the particles in a state of motion relative to the reference frame S, and therefore each particle has a relativistic kinetic energy in addition to the energy of the rest mass, We use the same mathematical formulas in equation 4.1 in the reference frame S, According to the principle of special relativity [1] to express the relationship between the particle energy before the annihilation phenomenon and the photon energy after the annihilation phenomenon

$$2\,E_{rel} = E_\gamma \qquad (5.1)$$

From the special theory of relativity, we can get the energy of the electron or the positron before annihilation relative to each frame of reference [1-2], or the relation between the rest mass energy and the relativistic total energy

$$E_{rel} = \grave{E_0}\;\gamma \qquad (6.1)$$

Where γ is the Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{V_S^2}{c^2}}} \tag{7.1}$$

Equation No. 6.1 shows us that the observer O observes the energy of each of the electron or positron increases relative to the reference frame S with increasing the speed of the reference frame $V_S$

Multiply both sides of equation 6.1 by 2

$$2\,E_{rel} \;=\; 2\,E_0^{`}\;\gamma \tag{8.1}$$

Substitution from 5.1 and 6.1 by 8.1

$$E_\gamma = \; E_\gamma^{`}\;\;\gamma \tag{9.1}$$

We get the energy of the photon resulting from the phenomenon of annihilation relative to each reference frame, because special relativity is the theory of transforming a four-dimensional vector from one inertial reference frame to another, so we can represent the energy of the photon relative to each reference frame on the four-dimensional vector, where the photon energy relative to the frame of reference S' is represented on the vector $\overrightarrow{\alpha`_0}$ the zero number symbolizes the occurrence of the annihilation phenomenon in the reference frame S', while the energy of the same photon relative to the frame of reference S is represented on the vector $\overrightarrow{\alpha}$ See Figure:1, Therefore, we can write the previous equation in the following form

$$E_\alpha = \; E_{\alpha_0}^{`}\;\;\gamma \tag{10.1}$$

The energy of the photon according to quantum mechanics [4] is equal to the product of Planck's constant $h$ times the photon frequency $\nu_{\alpha_0}^{`}$

$$E_{\alpha_0}^{`} = h\,\nu_{\alpha_0}^{`} \tag{11.1}$$

We use the same mathematical formula in the frame of reference S according to the principle of special relativity

$$E_\alpha = h\,\nu_\alpha \tag{12.1}$$

Substitution from 11.1 and 12.1 to 10.1

$$h\,\nu_{\alpha} = h\,\grave{\nu}_{\alpha_0}\;\gamma \qquad (13.1)$$

$$\nu_{\alpha} = \grave{\nu}_{\alpha_0}\;\gamma \qquad (14.1)$$

Substitution from 7.1 to 14.1

$$\nu_{\alpha} = \frac{\grave{\nu}_{\alpha_0}}{\sqrt{1-\frac{V_S^2}{c^2}}} \qquad (15.1)$$

Equation 15.1 shows that the frequency $\nu_{\alpha}$ observed by the observer O is greater than the frequency $\grave{\nu}_{\alpha_0}$ observed by the observer O' with increasing the speed of the reference frame $V_S$, due to the increase in the energy or the relativistic mass of each of the electron and positron before the annihilation phenomenon

## 2-3 Relative time and frequency of the photon

According to the Special Theory of Relativity, time is also relative, where each reference frame has its own time, and therefore the periodic time of the photon will differ from one reference frame to another, and we can get the periodic time of the gamma photon relative to each reference frame, through the time dilation equation [1-5]

$$\Delta t_{\alpha} = \Delta \grave{t}_{\alpha_0}\,\gamma \qquad (16.1)$$

Where the observer O observes the period $\Delta t_{\alpha}$ of the photon wave relative to the reference frame S is longer than the periodic time $\Delta \grave{t}_{\alpha_0}$ of the wave of the same photon observed by the observer O' with relative to the reference frame S', from classical mechanics we can obtain the relation between the frequency of a wave photon periodic time

$$\grave{\nu}_{\alpha_0} = \frac{1}{\Delta \grave{t}_{\alpha_0}} \qquad (17.1)$$

We use the same mathematical formula to obtain the relation between frequency and periodic time in the frame of reference S according to the principle of special relativity

$$\nu_{\alpha} = \frac{1}{\Delta t_{\alpha}} \qquad (18.1)$$

Substitution from 17.1 to 18.1

$$\nu_{\alpha} = \frac{1}{\Delta t\grave{}_{\alpha_0}\gamma} \quad (19.1)$$

Substitution from 17.1 to 19.1

$$\nu_{\alpha} = \grave{\nu}_0 \ \gamma^{-1} \quad (20.1)$$

Substitution from 7.1 to 20.1

$$\nu_{\alpha} = \grave{\nu}_{\alpha_0} \sqrt{1 - \frac{V_S^2}{c^2}} \quad (21.1)$$

Equation 1.21 shows that the frequency $\nu_{\alpha}$ observed by the observer O decreases from the frequency $\grave{\nu}_{\alpha_0}$ observed by the observer O' with increasing the speed of the reference frame Vs, due to the time dilation

The relativistic Doppler effect is ignored here in order to explain the effect of time dilation only on the photon frequency, but when the Doppler effect is taken into account, The value of the decrease in photon frequency $\nu_{\alpha}$ is greater than the value calculated in equation 21.1, because the reference frame S' moves away from S, or equal to the same value when the light source (Point *P*) is in the closest visual approach to the observer O [5]

$$\nu_{\alpha} \leq \grave{\nu}_{\alpha_0} \sqrt{1 - \frac{V_S^2}{c^2}} \quad (22.1)$$

## 3 RESULTS

We conclude, then, from equations 15.1 and 21.1, that an increase in the relativistic energy or mass leads to an increase in the photon's frequency relative to the frame of reference S on the vector $\overrightarrow{\alpha}$ , while the time dilation leads to a decrease in the frequency and energy of the photon relative to the reference frame S on the vector $\overrightarrow{\alpha}$ , and therefore the increase of relativistic energy with the dilation of time represents a paradox in the special theory of relativity because we get contradictory results relative to the same reference frame, but rather on the same four-dimensional vector in the reference frame S, and that is from the purely theoretical side, but from the experimental side, observer O will definitely get only one result, but we cannot determine here which of the results of equations 15.1 and 21.1 are to be achieved by experiment, and we also cannot justify the occurrence of this result and the non-occurrence of the other result, because both results depend on the same laws and concepts of special relativity

## 4 DISUSSIONS

In spite of that, we cannot say that this paradox represents a mistake in the special theory of relativity, because both the relativistic mass or relativistic total energy increase and the dilation of time have been tested a number of times, but both were tested separately from the other, and the paradox arises when the physical phenomenon depends on energy Or relativistic mass and time dilation together, as in the phenomenon of electron-positron annihilation, and therefore, this paradox reveals to us only the limits of the special theory of relativity as a mathematical model in application and practice to physical phenomena.

The paradox of energy and time also provides an explanation for the failure of physicists over the course of a hundred years to combine special relativity with thermodynamics or consensus on one model of relativistic thermodynamics, because the phenomena of thermodynamics depend on both relativistic energy and time dilation together, which made physicists obtain Mathematical models in which the temperature decreases and others in which the temperature increases in the same frame of reference, as falling into the same paradox [2-6]. If the mathematical models consider the particle's velocity to be the only thermal variable, the results depend on the time dilation, but if the mathematical models consider the particle's mass to be the only thermal variable, the results here depend on the relativistic energy [7-8-9-10-11]

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