Why does $0! = 1$?

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Abstract

In this brief note we shall give a natural explanation for why $0! = 1$.

Keywords: Factorial

The definition of factorial is: For any counting number $n$,

$$n! = n(n - 1)(n - 2)(n - 3) \ldots 3 \cdot 2 \cdot 1$$

However, this definition does not show that $0! = 1$. Instead we rely upon the following pattern:

$$3! = \frac{4!}{4} = \frac{24}{4} = 6$$

$$2! = \frac{3!}{3} = \frac{6}{3} = 2$$

$$1! = \frac{2!}{2} = \frac{2}{2} = 1$$

We can continue with this pattern to establish:

$$0! = \frac{1!}{1} = \frac{1}{1} = 1$$

But there is no reason why the pattern must continue to this last case.

Consider, then, by the multiplication property of factorial we have:

$$n! = n(n - 1)(n - 2)!$$

And so we derive the following:

$$\frac{n!}{n - 1} = \frac{n(n - 2)}{n - 1} = \frac{(n - 1) + 1}{n - 1} = (n - 1)(n - 2)!$$

That is,

$$\frac{n!}{n - 1} = (n - 1)! + (n - 2)!, \quad n > 1$$

And now if $n = 2$, we inevitably deduce $1 = 0!$. 

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