The expansion of spacetime

A J Owen

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Abstract

In this paper, the physical universe is modelled as an expanding Minkowski space, and this obviates the need for dark energy to be included in the cosmological model. The observed accelerated expansion in the current epoch can be understood purely on the basis of a mass-dominated universe, where deceleration due to gravity is more than compensated for by expansion of the time dimension. In the epoch prior to this, when a linear expansion of the scale factor occurred, the universe was radiation-dominated, and in the very early exponentially expanding universe, cosmic inflation can be attributed to an expanding ensemble of non-interacting particles. This is very different behaviour from that deduced from the currently accepted cosmological model.

1 Introduction

Observations of redshift of light from distant galaxies are interpreted to mean that the universe is expanding. Assuming that the cosmological principle applies, i.e., the universe is homogeneous and isotropic on a large enough scale, the Friedmann-Lemaitre-Robertson-Walker ($FLRW$) metric is conventionally used to describe the expansion of space [1]:

$$d\tilde{s}^2 = -c^2dt'^2 = -c^2dt^2 + a(t)^2ds^2 \qquad [FLRW metric] \qquad (1)$$

where $d\tilde{s}$ is a spacetime increment, c the speed of light, dt' a proper time increment, dt a coordinate time increment, and ds a space increment with a time-dependent scale factor $a(t)$ that describes the expansion. The scale factor is unity in the present era. The time coordinate t is often referred to as cosmic or cosmological time, and the spatial coordinates are comoving coordinates. This metric has been discussed in countless papers and textbooks on cosmology, and is regarded as generic in its description of an expanding universe as well as satisfying Einstein's field equations of general relativity [2].

Using this metric in conjunction with observational evidence of an accelerated expansion, which is thought to have begun about six billion years ago, leads scientists to believe that the universe is composed of almost exactly one-third matter and two-thirds of an unobservable "substance" that is treated as vacuum energy and usually termed dark energy. Although the model is currently adopted by the scientific community, there are several reasons for doubting some of its its validity (see, e.g. [3]). In a review paper by Li *et al*, [4], which contains a comprehensive survey of 652 articles on the subject of dark energy, they conclude their paper by stating that "the problem of understanding the nature of dark energy is as daunting as ever...clearly there is a long way to go for both theorists and experimentalists".

Attempts to detect dark energy have been futile to date and, since there are reasons for believing it is an artefact and will never be detected, the purpose of the present paper is to address this issue and propose a different concept for the way universal expansion occurs that obviates the need to invent dark energy.

2 The modified metric

The first step is to posit that universal expansion is an expansion of spacetime itself. This is based on the insight that space and time are inextricably linked in a four-dimensional framework, as was first realised by Minkowski when he described the Lorentzian symmetry of Einstein's special relativity theory by treating time as a dimension in addition to the three dimensions of space. I shall therefore apply the same scale factor $a(t)$ to a complete spacetime increment, instead of just the spatial part of Equation 1, so that we now have

$$d\tilde{s}^2 = -c^2 dt'^2 = a(t)^2[-c^2 dt^2 + ds^2] \tag{2}$$

Since NASA's *WMAP* observations indicate that the large-scale universe is approximately flat [5], i.e. the spatial increment ds is not curved, the metric will be written using Cartesian coordinates (x, y, z) as

$$d\tilde{s}^2 = a(t)^2[-c^2 dt^2 + dx^2 + dy^2 + dz^2] \tag{3}$$

This is now just a Minkowski space, as encountered in special relativity, that is expanding with time.

We may proceed in the standard way using Lagrangian formalism. The functional is given by

$$L = a(t)^2[-c^2\dot{t}^2 + \dot{x}^2 + \dot{y}^2 + \dot{z}^2] \tag{4}$$

where the *dot* over t, x, y, z refers to differentiation with respect to t' as Lagrangian parameter. One then applies the Euler-Lagrange equation

$$\frac{d}{dt'}\left(\frac{\partial L}{\partial x_a}\right) - \frac{\partial L}{\partial x_a} = 0 \quad [a = t, x, y, z] \tag{5}$$

to obtain the following four geodesic equations:

$$\ddot{t} + \frac{\dot{a}}{a}\dot{t}^2 + \frac{\dot{a}}{ac^2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = 0 \;\; ; \;\; \ddot{x}_i + \frac{2\dot{a}}{a}\dot{t}\,\dot{x}_i = 0 \qquad [x_i = x, y, z] \tag{6}$$

Introducing the Hubble parameter H, which can be written $H = \dot{a}/a$, where the *dot* over a represents differentiation with respect to t, then we may also write

$$\ddot{t} + H\dot{t}^2 + \frac{H}{c^2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) = 0 \;\; ; \;\; \ddot{x}_i + 2H\,\dot{t}\,\dot{x}_i = 0 \qquad [x_i = x, y, z] \tag{7}$$

The Christoffel connection coefficients Γ^a_{bc} taken from the geodesics are:

$$\Gamma^t_{tt} = H \;; \qquad \Gamma^t_{ii} = H/c^2 \;; \qquad \Gamma^i_{it} = H$$

From these, the Ricci tensor components R_{ab} can be evaluated, where for example

$$R_{tt} = \frac{\partial\Gamma^\gamma_{tt}}{\partial x^\gamma} - \frac{\partial\Gamma^\gamma_{t\gamma}}{\partial t} + \Gamma^\gamma_{tt}\Gamma^\delta_{\gamma\delta} - \Gamma^\gamma_{t\delta}\Gamma^\delta_{t\gamma} \tag{8}$$

Using the additional relationship $\ddot{a}/a = \dot{H} + H^2$, one obtains:

$$R_{tt} = -3\dot{H} \;; \qquad R_{ii} = \frac{(\dot{H} + 2H^2)}{c^2} \;; \;\; R_{it} = 0 \tag{9}$$

and the scalar curvature R, defined as

$$R = \Sigma\frac{R_{aa}}{g_{aa}} \qquad [a = t, x, y, z] \tag{10}$$

where we have here $g_{tt} = -a^2c^2$; $g_{ii} = a^2$; $g_{ti} = 0$). This gives

$$R = \frac{6(\dot{H} + H^2)}{c^2a^2} \tag{11}$$

Next, we obtain the non-zero components of the Einstein tensor G_{ab}, where

$$G_{ab} = R_{ab} - \frac{1}{2}\,R\,g_{ab} \tag{12}$$

This then gives the following 4 diagonal components:

$$G_{tt} = 3H^2 \;\; ; \;\; G_{ii} = -\frac{(2\dot{H} + H^2)}{c^2} \qquad [i = x, y, z] \tag{13}$$

Using the same methodology for the $FLRW$ metric leads to the following non-zero Einstein tensor components, for comparison:

$$G_{tt} = 3H^2 \;\; ; \;\; G_{ii} = -\frac{(2\dot{H} + 3H^2)}{c^2} \qquad [FLRW] \tag{14}$$

3 Interpretation

Einstein's field equations of general relativity are often written in the form

$$G_{ab} + \Lambda g_{ab} = \kappa T_{ab} \tag{15}$$

where Λ is a cosmological constant, $\kappa = 8\pi G/c^4$ is a universal constant and T_{ab} is the stress-energy tensor, which describes the density and flux of energy and momentum in spacetime. In particular the term T_{tt} describes the density of relativistic mass (or energy density divided by c^2), and the terms T_{ii} represent the flux of momentum normal to the i-coordinate direction, which is equivalent to a pressure p.

The cosmological constant Λ needs some explanation. The earliest GR models predicted that the universe would contract due to gravitaional attraction, so Einstein first introduced Λ to counteract this and make the universe static. Subsequently it was found that the universe is expanding, and Λ fell into disrepute for a while. However, when it was realised that the universal expansion is accelerating, Λ was reintroduced, and can be used to describe the universal expansion as caused by some form of vacuum energy. Thus, the idea behind the cosmological constant is that the energy density ϱ and pressure p may be effectively non-zero even for the vacuum of space (where there is no matter).

Writing Equation 15 for zero stress-energy tensor, we thus have

$$G_{ab} + \Lambda g_{ab} = 0 \tag{16}$$

from which we obtain

$$G_{ab} = -\Lambda g_{ab} \tag{17}$$

This means the Einstein tensor for a vacuum (G_{vac}) with a cosmological constant Λ takes the form

$$G_{vac} = \begin{pmatrix} \Lambda c^2 & 0 & 0 & 0 \\ 0 & -\Lambda & 0 & 0 \\ 0 & 0 & -\Lambda & 0 \\ 0 & 0 & 0 & -\Lambda \end{pmatrix}$$

while for a universe with just a matter density ϱ we have:

$$G_{mass} \propto \begin{pmatrix} \varrho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

since $G_{ab} = \kappa T_{ab}$, and the only non-zero component is T_{tt} which is proportional to ϱ.

Now write the terms of both tensors in the same units $\Lambda c^2 = \varrho_\Lambda$ and $\varrho = \varrho_m$, and assume the universe contains some combination of mass energy ϱ_m and vacuum energy ϱ_Λ (as a cosmological constant) (i.e. neglecting any radiation energy density), we see that the tt-component of the combined Einstein tensor is proportional to the sum of these densities for mass-energy (ϱ_M) and the cosmological constant (ϱ_Λ), whereas the diagonal components G_{ii} are proportional to just $-1/c^2$ times ϱ_Λ:

$$G_{tt} \;\propto\; (\varrho_M + \varrho_\Lambda) \;\; ; \;\; G_{ii} \;\propto\; -\varrho_\Lambda/c^2 \tag{18}$$

For the $FLRW$ model (Equation 14) this means we have

$$\varrho_m + \varrho_\Lambda \;\propto\; 3H^2 \;\; ; \;\; \varrho_\Lambda \;\propto\; 2\dot{H} + 3H^2 \tag{19}$$

and if we take the $WMAP$ result for the composition of the universe (one third matter/two-thirds vacuum energy) as $\varrho_m : \varrho_\Lambda = 1 : 2$, we then have by rearranging these expressions:

$$\dot{H} = -\frac{1}{2} H^2 \tag{20}$$

Integrating this equation gives

$$H = \frac{2}{t} \tag{21}$$

and with $H = \dot{a}/a$ we obtain the following relationship between scale factor and time:

$$a \sim t^2 \tag{22}$$

This reverse calculated expression now reproduces the accelerated expansion that has been inferred from $WMAP$ data combined with the $FLRW$ metric.

In summary, since an accelerated expansion ($a \sim t^n$ with $n > 1$) cannot be explained by gravitational attraction between masses using the $FLRW$ metric (where $a \sim t^{2/3}$), a vacuum energy term was introduced as a cosmological constant (where $a \sim e^t$) to produce a negative pressure that provides the expansion, and the term dark energy was invented to give it a name.

Next, substituting this observed time-dependence of H, viz. $H = 2/t; \dot{H} = -4/t^2$ (Equation 21) into my expression for the Einstein tensor (Equation 13) gives $G_{ii} = 0$, i.e. the cosmological constant term ϱ_Λ becomes zero, implying that there is no vacuum energy and only gravitational mass needs to be considered for describing the universal expansion occurring as $a \sim t^2$. Thus, stretching the time dimension by the same scale factor a produces the accelerated expansion, outweighing the deceleration caused by the attractive force of gravity between masses.

4 The equivalent of the Friedmann equations

Using the $FLRW$ metric and the resulting expressions for G_{ab}, the following two equations, called the Friedmann equations [6], are conventionally obtained:

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\varrho}{3} + \frac{\Lambda c^2}{3} \tag{23}$$

from the G_{tt} component and

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}\left(\varrho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3} \tag{24}$$

from the G_{ii} components. It then follows that

$$\dot{H} = -4\pi G\left(\varrho + \frac{p}{c^2}\right) \tag{25}$$

Differentiating Friedmann's first equation and substituting the above equation then leads to

$$\dot{\varrho} = -3\frac{\dot{a}}{a}\left(\varrho + \frac{p}{c^2}\right) \tag{26}$$

A cosmological equation of state is then introduced in the form

$$p = w\varrho c^2 \tag{27}$$

where w is a dimensionless parameter to obtain the following relationships:

$$\varrho \sim a^{-3(1+w)} \;\; ; \;\; a \sim t^{\frac{2}{3(1+w)}} \quad [FLRW/Friedmann] \tag{28}$$

A mass-dominated universe, for instance, has a pressure that is considered negligible compared to the mass-energy density, so the parameter

$w = 0$, and $\varrho \sim a^{-3}; a \sim t^{2/3}$. This $t^{2/3}$-behaviour expresses the deceleration in expansion you would expect solely on the basis of gravitational attraction. A cosmological constant has a value $w = -1$ in the equation of state, and this produces an exponential expansion of a with time. To obtain $a \sim t^2$ the parameter $w = -2/3$, which is another way of saying that the universe consists of 1/3 matter ($w = 0$) and 2/3 vacuum energy ($w = -1$) on the basis of the $FLRW$/Friedmann model.

In the model presented in this paper, however, the metric in Equation 2 leads to different Einstein tensor components from the currently accepted ones and, therefore, different Friedmann-type equations. What we obtain now is

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\varrho}{3} \tag{29}$$

from the G_{tt} component and

$$\frac{\ddot{a}}{a} = \dot{H} + H^2 = -\frac{4\pi G}{3}\left(\varrho + \frac{3p}{c^2}\right) \tag{30}$$

from the G_{ii} components. A cosmological constant is now of no consequence, and can be dismissed as unreal or having no physical meaning. The two new Friedmann-like equations then lead to the following expression

$$\dot{\varrho} = -3\frac{\dot{a}}{a}\left(\frac{\varrho}{3} + \frac{p}{c^2}\right) \tag{31}$$

and with the equation of state this gives

$$\dot{\varrho} = -\frac{\dot{a}}{a}\left(1 + 3w\right)\varrho \tag{32}$$

From this we then obtain

$$\varrho \sim a^{-(1+3w)} \;\; ; \;\; a \sim t^{\frac{2}{(1+3w)}} \tag{33}$$

The following table shows some of the relevant exponents for the way the scale factor a increases with time, for comparison:

w	a (*this model*)	a ($FLRW$)	*type of universe*
-1	–	e^t	cosmological constant
$-\frac{1}{3}$	e^t	t	non-interacting particles
0	t^2	$t^{2/3}$	mass-dominated
$+\frac{1}{3}$	t	$t^{1/2}$	radiation-dominated

5 Cosmological time dilation

Time is a relative quantity, and in Einstein's theory of special relativity, for example, if a time interval dt' is measured between two events in an inertial frame of reference travelling at a speed v relative to another inertial frame where the time interval between the same events is recorded as dt, one may write the relationship between the time intervals as

$$dt' = \left(\sqrt{1 - v^2/c^2}\right) dt \tag{34}$$

Writing the coordinate time interval dt as $dt = dt'_{v=0}$, we then have

$$dt'_v = \left(\sqrt{1 - v^2/c^2}\right) dt'_0 \ < \ dt'_0 \tag{35}$$

Since clocks indicate proper time, this is interpreted to mean that a clock in a moving frame appears to tick more slowly (less time passes) than a clock in the observer's frame. This effect is called kinematic time dilation.

As another example, an increase in clock rate occurs (more time passes) when a clock is raised in a gravitational field. For a point mass as the source of gravity, a time increment may be written using general relativity as

$$dt' = \sqrt{1 - \frac{2GM}{c^2 r}}\, dt$$

or we could write

$$dt'_r = \sqrt{1 - \frac{2GM}{c^2 r}}\, dt'_\infty \tag{36}$$

where r is the distance from the mass M causing gravity, and a clock at a distance r shows a proper time increment dt'_r. Since the function under the square root is an increasing function of r, this is inferred to mean that a clock near a mass runs at a slower rate than a clock further away. The interpretation is correct, and is the basis of gravitational time dilation, which is observed in practice, e.g., in the Global Positioning System.

Now consider what can be called cosmological time dilation. For the metric of Equation 2, for $ds = 0$ we have

$$c^2 dt'^2 = a^2 c^2 dt^2 \tag{37}$$

or

$$dt' = a\, dt \tag{38}$$

Now imagine you could observe a clock in the past and compare it with an identical clock in the present. The previous equation may be rewritten as

$$dt'_{(a<1)} = a\, dt'_{(a=1)} \tag{39}$$

A scale factor $a = 1$ represents the present era, so in the past when $a < 1$ a time interval indicated on a clock in the past is less than in the present. By analogy with the previous two examples, this must mean that a clock in the past will tick more slowly than a present-day clock, and relative to a present-day clock it will speed up into the future.

One can now treat the situation as if there were two time scales. Call the time in the Friedmann metric t_F and the time in my metric t_M. They are related by $dt_F = a\, dt_M$. The question now is, which time scale relates to the experimental observation that $a \sim t^2$? In terms of t_M, writing $a \sim t_M^2$, we would have $t_M \sim t_F^{1/3}$ and $a \sim t_F^{2/3}$. This describes a mass-dominated universe, and I consider this to be correct, with no need to introduce vacuum energy. Checking by relating the experimental observations to t_F gives a nonsensical result. Thus, t_M must be the time scale that relates actual observations to theory, while t_F leads to false conclusions.

6 Discussion

The main evidence for an expanding universe comes from observations of the redshift of light from distant galaxies. In the framework of the *FLRW* metric the redshift (or increase in wavelength of light) is attributed to the expansion of space while the light is in transit. However, if the same argument is applied to a model where time expands commensurately with space (Equation 2), then the redshift seems to be annulled - which is contrary to observational findings. Thus, if the model proposed in this paper is correct, the redshift must have a different explanation from the conventional one. Several other mechanisms for redshift have been proposed in the past, the most well-known being Zwicky's idea of "tired light", in which light loses energy (its frequency decreases) through scattering processes as it is transmitted through space [7].

The model presented here does indeed suggest a different rationale for understanding the observed redshift, one that originates from the cosmological time dilation explained in the previous section. A clock in the past ticks more slowly compared with an identical clock in the present, and we can infer that any periodic phenomenon will behave in the same way, including atomic electron transition frequencies, such as those in a cesium atomic clock or in a hydrogen atom. The observed redshift of light is then a manifestation of the reduced atomic frequencies when they emitted their light in the past, relative to the present day. Due to their great distance, we are only receiving this information now. The same scale factor-redshift relation appears in my model, only the interpretation differs.

Clifton *et al* [2] previously calculated the solutions to the Friedmann equations by transforming the *FLRW* time to a different time quantity called the conformal time τ, defined in terms of the cosmic time t as

$$\tau = \int_0^t \frac{dt}{a(t)} \tag{40}$$

A distance $c\tau$ represents the comoving distance travelled by a photon in the time t. Rewriting this as $a\,d\tau = dt$ then allows the *FLRW* metric to be reformulated as

$$d\tilde{s}^2 = a(\tau)^2[-c^2 d\tau^2 + ds^2] \tag{41}$$

which appears mathematically to be the same as my metric in Equation 2 with my time quantity equal to the conformal time. Their expressions in terms of the conformal time are physically equivalent to the *FLRW* model, because they were obtained via a coordinate transformation of the *FLRW* metric. My expressions, on the other hand, are not physically equivalent to that model, because they involve a different spacetime, which introduces new physics through an expansion of the time coordinate.

The accelerated expansion discussed above is a phenomenon that appeared relatively late on in the development of the universe (in the past 6 billion years out of a total age of aproximately 14 billion years). In the very early stages of the universe, cosmic inflation occurred, in which the universe is thought to have increased rapidly in size by several orders of magnitude. It is usually stated that this inflation was driven by a negative pressure or vacuum energy density, and modelled using the *FLRW* metric as a de Sitter space that acts like a cosmological constant and causes an exponential growth of the scale factor. However, the model of an expanding spacetime proposed here offers a different explanation. In the Table it can be seen that an exponential growth is characteristic of the spreading out of an ensemble of non-interacting particles. For a constant time scale, non-interacting particles would spread out linearly with time ($a \sim t$), but including expansion of the time coordinate turns this into an observed exponential or inflationary expansion, which is not part of the currently adopted cosmological model. Prior to the accelerated expansion, the scale factor did increase approximately linearly with time, which in the present model is attributable to a radiation-dominated universe.

About the author

The author is formerly a lecturer in the Department of Physics, University of Regensburg, Germany, and the University of Wales, Newport, as well as having been a research scientist in the Physics Department at the University of Bristol and the University of Cambridge, Cavendish Laboratory.

Email: ajowen@physics.org

References

[1] http://en.wikipedia.org/wiki/*FLRWmetric*

[2] T.Clifton, P.G.Ferreira, A.Padilla, C.Skordis, *Modified Gravity and Cosmology*, Physics Reports, 2012

[3] http://en.wikipedia.org/wiki/*Lambda-CDM model*

[4] M.Li, X-D.Li, S.Wang, Y.Wang, *Dark Energy*, arXiv:1103.5870 (March 2011)

[5] http://en.wikipedia.org/wiki/*WMAP*

[6] http://en.wikipedia.org/wiki/*Friedmann equations*

[7] F.Zwicky, *On the red-shift of spectral lines through interstellar space*, PNAS 15:773 (1929)