The Statistical Origin of Einstein’s Famous Formula

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In this paper, it will be demonstrated that the formula \( e=mc^2 \) can be derived very simply using a model of a particle undergoing an inhomogeneous random walk at the speed of light. It was shown in 1995 by G.N. Ord that the Schrödinger equation including its potential term can be derived from the model of an inhomogeneous random walk, without resorting to analytical continuation (1). The equation derived was as follows:

\[
i \frac{\partial}{\partial t} \Psi = -D \frac{\partial^2}{\partial x^2} \Psi + \nu(x) \Psi
\]

Note that the Schrödinger equation can be expressed as

\[
i \frac{\partial}{\partial t} \Psi = -\hbar/(2m) \frac{\partial^2}{\partial x^2} \Psi + \frac{1}{\hbar} \nu(x) \Psi
\]

Therefore the two equations are the same as long as one allows for certain re-scalings by \( \hbar \).

Comparing the coefficients on the first term, one finds that

\[
D = \hbar/(2m)
\]

\[
2mD = \hbar
\]

\[
m = \hbar/(2D)
\]

Ord’s definition of \( D \) is \( D = \delta^2/(2\epsilon) \), where \( \delta \) is the jump size and \( \epsilon \) is the time between jumps. Therefore, we can write

\[
m = \hbar/(2 \delta^2/(2\epsilon))
\]

\[
m = \hbar/(\delta^2/\epsilon)
\]

\[
m = \hbar \epsilon/\delta^2
\]

We can also write this as

\[
m = \hbar \epsilon/\delta
\]

If we further make the assumption that the speed of the process is “\( c \)”, allowing \( \delta/\epsilon = c \), then we have

\[
m = \hbar \epsilon/c \delta
\]

Multiplying both sides by a factor of \( c \) we obtain

\[
m c^2 = \hbar \epsilon/(\delta/c)
\]

\[
m c^2 = \hbar \epsilon/(\delta/\epsilon)
\]

\[
m c^2 = \hbar \epsilon/\delta
\]
If we then assume the De Broglie frequency is related as

\[ f = \frac{1}{(2\pi\epsilon)} \]

So that

\[ 2\pi\epsilon f = 1 \]

\[ \epsilon = \frac{1}{(2\pi f)} \]

\[ \frac{1}{\epsilon} = 2\pi f \]

Then we have

\[ mc^2 = \hbar(2\pi)f \]

\[ mc^2 = \hbar f \]

\[ e = mc^2 \]

In this way, we have provided a simple proof of energy-mass equivalence without resorting to actually using the special theory of relativity. Note that the assumption that the particle’s speed is equal to c was discussed in a separate paper by G. N. Ord but as a replacement of the equation for diffusivity instead of being used in addition to it (2). Note that in demonstrating the statistical nature of \( e=mc^2 \), it is possible to envision how the formula might “break down” under certain circumstances, for example if there are a very small number of steps. This will be the subject of future discussions.

**References:**


2) The Schrodinger and Dirac Free Particle Equations Without Quantum Mechanics, G. N. Ord.