FORMULAE FOR ZETA (odd)

E. ZHOU and M. ZHOU
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ABSTRACT. There are the closed-form representations of the Euler’s Zeta function at odd positive integers in this article.

The work to write down formulae for Euler’s Zeta function at odd integers is still being sorted out and going to take some time, because the workload is huge, so the style of their expressions is first released on this commemorative day.

Euler’s Zeta function is defined as

\[ \zeta(m) = \sum_{n=1}^{\infty} \frac{1}{n^m}, \quad n = 1, 2, 3, \ldots \quad m = 2, 3, \ldots \in \mathbb{N}. \]

For odd positive integers, their closed-form expressions or formulae should be,

\[ \zeta(2k + 1) = \pi^{2k} \ln(2^{r_1}e^{r_2}3^{r_3}\pi^{r_4}), \]

where,

1) \( k = 1, 2, 3, \ldots \in \mathbb{N}; \)
2) \( r_1, r_2, r_3 \) and \( r_4 \) are rational numbers which depend on \( (2k + 1) \) and will be determined.

Although their closed-forms do not behave in the same way as Euler’s zeta functions at even positive integers, from what little information we know, perhaps they are like this. They cannot help being born mysterious.

In addition, similar to the results of \( \zeta(2k + 1) \), the following are formulas of Dirichlet Beta functions for even positive integers,

\[ \beta(2k) = \pi^{2k-1} \ln(2^{r_5}e^{r_6}3^{r_7}\pi^{r_8}). \]

E. ZHOU, UC Berkeley.
M. ZHOU, TJ. pdebox@yahoo.com
说明：前一页为贝塔(双)和泽塔(单)的精确解（形式）结果，它们曾于2020年在某公共信息范围内提及过，因工作量巨大，后续研究一直在间断中进行，想必渐布其所以然之时尚早，……，还是先睹为快舒乐。另外，提供述特亦多项用途为辅。

《卜算子》

與 Beta 雙和 Zeta 單 算卜
2020

從未顯神幽，
還是奇輪偶。
倒零級和似易得，
哪想折磨夠。

門難何全有？
非要文稿厚？
息少條缺律理嬴，
超算須來秀！