SIMPLE PROOF OF FLT FOR PRIMES $n > 2$ (V. .DESC)

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ABSTRACT. We parametrize the Fermat equation, $x^n + y^n = z^n$, with a form that explicitly shows no integral solution for odd $n > 1$.

1. Proof of FLT By Contradiction for Odd $n > 1$

Fermat’s last theorem (FLT) says, with integral triple $(x, y, z) > 0$, integral $n > 0$: Equation $x^n + y^n = z^n$ does not hold for $n > 2$.

(1) $y^n + x^n = ((y^n + x^n)^{1/n})^n$. Notate $(y^n + x^n)^{1/n}$ as $z$.

We consider, for odd $n \geq 1$, that (1), parametrized by $x, y$, holds for all integral $(x, y)$ such that $z \geq y \geq x \geq 0$. We use 0 for completeness.

We define the general condition as: If $(A, B, C) > 0$ are real numbers satisfying $A^n + B^n = C^n$, and any given two elements of the associated (abbreviated as Asso.) triple are integers $\geq 0$, then, the 3rd element of this triple, e.g., $(y^n + x^n)^{1/n}$ in (1), is an (hypothetical) integer $\geq 0$. The result is the Asso. (hypothetical) integral triple $(x, y, z) \geq 0$.

(2) $x^n + ((y^n - x^n)^{1/n})^n = y^n$. Notate $(y^n - x^n)^{1/n}$ as $w$.

We consider, for odd $n \geq 1$, that (2), parametrized by $x, y$, holds for the same, all integrals $(x, y)$ we use in (1) such that $y \geq w, x \geq 0$. General condition: The Asso. integral triple is $(x, (y^n - x^n)^{1/n}, y) \geq 0$.

For odd $n \geq 1$, with all integral $x, y \geq 0$, all (hypothetical) Fermat triples $\geq 0$ are in (1), and in (2): For different $(x, y)$, every $(x, y, z)$ can be found in $\{x, w, y\}$, and every $(x, w, y)$ can be found in $\{x, y, z\}$.

The intersection of the associated sets of (hypothetical) integral triples $\geq 0$ in (1),(2) gives the Asso. set of (hypothetical) integral triples $\geq 0$ in (3), below, per simultaneous solution of Eqns. (1), (2).

Below : Eqn. (3) has the same $A^n + B^n = C^n$ form as with (1), (2); in concert, (1), (2) implies (3); it is a true conditional statement that "If the general condition should be met by (1), (2), then, the general condition would be met by (3)". Below, for odd $n > 1$: The general condition is not met for any given Asso. triple in (3), so, such condition is not met for the corresponding, respective Asso. triple in (1), (2).
Factually, Fermat triples exist for \( n = 1, 2 \). However, (3), below, shows, for integral \( x \), that the Asso. integral triples in (1), (2) are held in common for \( n = 1 \), but not for \( n = 2 \), the main reason we consider only odd \( n \geq 1 \) with (1), (2), making \( n \geq 1 \) only odd with (3), below.

\[
(3) \quad ((y^n - x^n) + \frac{2}{\sqrt{3}}xK)^n + (\frac{2}{\sqrt{3}}xK)^n = ((y^n + x^n) + \frac{2}{\sqrt{3}}K)^n.
\]

For odd \( n \geq 1 \): Equation (3) holds for all integral \((x, y)\); \( y \geq x \geq 0 \), such that \( K = \frac{1}{2} \) for same parity \( x, y \); \( K = 1 \) for opposite parity \( x, y \), with the associated set of triples \(((y^n - x^n) + \frac{2}{\sqrt{3}}K, 2\frac{2}{\sqrt{3}}K, (y^n + x^n) + \frac{2}{\sqrt{3}}K)\).

As defined, algebraic identity (3) is useful, so, is not trivial, similar to Euclid’s identity, a well-known parametrization of \( x^2 + y^2 = z^2 \).

Per the general condition: Elements \((y^n - x^n) + \frac{2}{\sqrt{3}}K\) in (1), \((y^n + x^n) + \frac{2}{\sqrt{3}}K\) in (2) each is (hypothetically) integral; hence, in (3), the two parts of the triple in (3), \((y^n - x^n) + \frac{2}{\sqrt{3}}K, (y^n + x^n) + \frac{2}{\sqrt{3}}K\), each is also (hypothetically) integral; thus, the 3rd part, \(2\frac{2}{\sqrt{3}}K\) in (3), is (hypothetically) integral.

If there exists an integral \((x, y) \geq 0\) satisfying (1), (2) with resp. Asso. sets of (hypothetical) integral \((x, y, (y^n + x^n) + \frac{2}{\sqrt{3}}K)\); \((x, (y^n - x^n) + \frac{2}{\sqrt{3}}K, y)\), then, there exists an integral \((x, y) \geq 0\) satisfying (3) with the resp., Asso. sets of (hypothetical) integral \(((y^n - x^n) + \frac{2}{\sqrt{3}}K, 2\frac{2}{\sqrt{3}}K, (y^n + x^n) + \frac{2}{\sqrt{3}}K)\).

Thus, for odd \( n \geq 0 \), Eqns. (1), (2) taken together imply Eqn. (3).

For the subset of odd \( n > 1 \), and integral \( y > x > 0 \): Any given element \(2\frac{2}{\sqrt{3}}K\) of the Asso. (hypothetical) integral triple in (3) is in fact irrational; hence, in fact, any given Asso. (hypothetical) integral triple \(((y^n - x^n) + \frac{2}{\sqrt{3}}K, 2\frac{2}{\sqrt{3}}K, (y^n + x^n) + \frac{2}{\sqrt{3}}K)\) in (3) is not positive integral.

2. Results and Conclusions

For odd all \( n > 1 \) with all integrals \( y \geq x \geq 0 \); \( K = 1 \) or \( 1/2 \):

1) Equation (1) in concert with (2), having, resp., the Asso. (hypothetical) integrals \((x, y, (x^n + y^n) + \frac{2}{\sqrt{3}}K)\); \((x, (y^n - x^n) + \frac{2}{\sqrt{3}}K)\), implies (3), with the Asso. (hypothetical) integral \(((y^n - x^n) + \frac{2}{\sqrt{3}}K, 2\frac{2}{\sqrt{3}}K, (y^n + x^n) + \frac{2}{\sqrt{3}}K)\);

2) Any given \(2\frac{2}{\sqrt{3}}K\) is in fact irrational; so, any given Asso. (hypothetical) integral \(((y^n - x^n) + \frac{2}{\sqrt{3}}K, 2\frac{2}{\sqrt{3}}K, (y^n + x^n) + \frac{2}{\sqrt{3}}K)\) in (3) is not integral in fact. Thus, by contradiction, the corresponding, Asso. (hypothetical) integral triples \((x, y, (x^n + y^n) + \frac{2}{\sqrt{3}}K)\) in (1) or \((x, (y^n - x^n) + \frac{2}{\sqrt{3}}K, y)\) in (2) is not positive integral in fact. Since the associated (hypothetical) integrals \(\{x, w, y\}\) in (1), (2), resp., have every value in common, the corresponding, associated (hypothetical) integrals \(\{x, y, z\}\) in (1) and \(\{x, w, y\}\) in (2) each can not be positive integral in fact.

So, for primes \( n > 2 \): \(x^n + y^n = z^n\) has no integral solution. Q.E.D.