

SIMPLE PROOF OF FLT FOR PRIMES $n > 2$ (V. 7)

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ABSTRACT. We parametrize the Fermat equation, $X^n + Y^n = Z^n$, with a form that explicitly shows no integral solution for odd $n > 1$.

1. PROOF OF FLT BY CONTRADICTION FOR ODD $n > 1$

Fermat's last theorem (FLT) says, with integral *triple* $(X, Y, Z) > 0$, integral $n > 0$: Equation $X^n + Y^n = Z^n$ does not hold for $n > 2$.

$$(1) \quad y^n + x^n = ((y^n + x^n)^{\frac{1}{n}})^n. \text{ Notate } (y^n + x^n)^{\frac{1}{n}} \text{ as } z.$$

We consider, for odd $n \geq 1$, that (1), parametrized by x, y , holds for *all* integral (x, y) such that $z \geq y \geq x \geq 0$. We use 0 for completeness.

We define the *general condition* as : If $(A, B, C) > 0$ are real numbers satisfying $A^n + B^n = C^n$, and *any given* two elements of the *associated* (abbreviated as *Asso.*) triple are integers ≥ 0 , then, the 3rd element of this triple, e.g., $(y^n + x^n)^{\frac{1}{n}}$ in (1), is an (*hypothetical*) integer ≥ 0 . The result is the associated (*hypothetical*) integral triple $(x, y, z) \geq 0$.

$$(2) \quad x^n + ((y^n - x^n)^{\frac{1}{n}})^n = y^n. \text{ Notate } (y^n - x^n)^{\frac{1}{n}} \text{ as } w.$$

We consider, for odd $n \geq 1$, that (2), parametrized by x, y , holds for *the same, all integrals* (x, y) we use in (1) such that $y \geq w, x \geq 0$. Per *General condition* : The *Asso.* integral triple is $(x, (y^n - x^n)^{\frac{1}{n}}, y) \geq 0$.

For odd $n \geq 1$, *all* Fermat triples $(X, Y, Z) \geq 0$ in $X^n + Y^n = Z^n$ can be found in the *different* triples that are in (1), and in (2), resp., with (1), (2) each being satisfied by the *same, all* integrals $(x, y) \geq 0$.

The *intersection* of the associated sets of (*hypothetical*) integral triples ≥ 0 in (1),(2) gives the *Asso.* set of (*hypothetical*) integral triples ≥ 0 in (3), below, per simultaneous solution of Eqns. (1), (2).

Below : Eqn. (3) has the same $A^n + B^n = C^n$ form as with (1), (2); in concert, (1), (2) *implies* (3); *it is a true conditional statement* that "If the *general condition* should be met by (1), (2), then, the *general condition* would be met by (3)". Below, for odd $n > 1$: The *general condition* is not met for *any given* *Asso.* triple in (3), so, such condition is not met for the *corresponding*, respective *Asso.* triple in (1), (2).

Factually, Fermat triples exist for $n = 1, 2$. However, (3), below, shows, for integral x , that the Asso. integral triples in (1), (2) are held in common for $n = 1$, but not for $n = 2$, the main reason we consider only odd $n \geq 1$ with (1), (2), making $n \geq 1$ only odd with (3), below.

$$(3) \quad ((y^n - x^n)^{\frac{1}{n}}K)^n + (2^{\frac{1}{n}}xK)^n = ((y^n + x^n)^{\frac{1}{n}}K)^n.$$

For *odd* $n \geq 1$: Equation (3) holds for *all integral* (x, y) ; $y \geq x \geq 0$, such that $K = \frac{1}{2}$ for same parity x, y ; $K = 1$ for opposite parity x, y , with the associated set of triples $((y^n - x^n)^{\frac{1}{n}}K, 2^{\frac{1}{n}}xK, (y^n + x^n)^{\frac{1}{n}}K)$.

As defined, *algebraic identity* (3) is *useful*, so, *is not trivial*, similar to *Euclid's identity*, a well-known parametrization of $x^2 + y^2 = z^2$.

Per the *general condition* : Elements $(y^n - x^n)^{\frac{1}{n}}$ in (1), $(y^n + x^n)^{\frac{1}{n}}$ in (2) each is (hypothetically) integral; hence, in (3), the two parts of the triple in (3), $(y^n - x^n)^{\frac{1}{n}}K, (y^n + x^n)^{\frac{1}{n}}K$, each is also (hypothetically) integral; thus, the 3rd part, $2^{\frac{1}{n}}xK$ in (3), is (hypothetically) integral.

If there exists an integral $(x, y) \geq 0$ satisfying (1), (2) with resp. Asso. sets of (hypothetical) integral $(x, y, (y^n + x^n)^{\frac{1}{n}})$; $(x, (y^n - x^n)^{\frac{1}{n}}, y)$, then, there exists a *same* integral $(x, y) \geq 0$ satisfying (3) with resp., Asso. sets of (hypothetical) integral $((y^n - x^n)^{\frac{1}{n}}K, 2^{\frac{1}{n}}xK, (y^n + x^n)^{\frac{1}{n}}K)$.

Thus, for odd $n \geq 0$, Eqns. (1), (2) *taken together* imply Eqn. (3).

For the subset of odd $n > 1$, and *integral* $y > x > 0$: *Any given* element $2^{\frac{1}{n}}xK$ of the Asso. (hypothetical) integral triple in (3) *is in fact irrational*; hence, in fact, *any given* Asso. (hypothetical) integral triple $((y^n - x^n)^{\frac{1}{n}}K, 2^{\frac{1}{n}}xK, (y^n + x^n)^{\frac{1}{n}}K)$ in (3) is *not positive integral*.

2. RESULTS AND CONCLUSIONS

For odd all $n > 1$ with all integrals $y > x > 0$; $K = 1$ or $1/2$:

1) Equation (1) in concert with (2), having, resp., the Asso. (hypothetical) integrals $(x, y, (x^n + y^n)^{\frac{1}{n}})$; $(x, (y^n - x^n)^{\frac{1}{n}})$, implies (3), with the Asso. (hypothetical) integral $(y^n - x^n)^{\frac{1}{n}}K, 2^{\frac{1}{n}}xK, (y^n + x^n)^{\frac{1}{n}}K$;

2) *Any given* $2^{\frac{1}{n}}xK$ is in fact irrational; so, *any given* Asso. (hypothetical) integral $((y^n - x^n)^{\frac{1}{n}}K, 2^{\frac{1}{n}}xK, (y^n + x^n)^{\frac{1}{n}}K)$ in (3) is *not integral* in fact. Thus, *by contradiction*, the *corresponding*, Asso. (hypothetical) *integral triples* $(x, y, (x^n + y^n)^{\frac{1}{n}})$ in (1) or $(x, (y^n - x^n)^{\frac{1}{n}}, y)$ in (2) is not positive integral in fact. Since the associated (hypothetical) integral (x, y, z) in (1) and (x, w, y) in (2) each includes *all* (integral) Fermat triples, the *corresponding*, associated (hypothetical) integral (x, y, z) in (1) and (x, w, y) in (2) each can not be positive integral *in fact*. So, for primes $n > 2$, Eqn. $X^n + Y^n = Z^n$ has no integral solution. Q.E.D.