Primality's Ultranatural Nature: An Inquiry into Composites

By Arthur V. Shevenyonov

11-12 March 2022

ABSTRACT

The 'gray area,' or indeed overlap, in between primes versus composites pertains to facets spanning beyond their belonging on the natural axis. The former can, for one, best be posited as building blocks with reference to a generalized rho-operation. At this rate, primes build on multiplicity (rho specified anywhere near 0) whereas composites on additivity (rho taken to 1). These result from symmetry (self-duality) versus averaging over the latter's solutions, respectively, while pointing to scenarios such as "either/both" versus "neither/both" (suggesting the number of (#, X) prime materializations/hits for the prime versus [higher] composite subdomains). The "both" overlap (boasting a relatively lower proportion in the prime subdomain) may plausibly amount to the aforementioned inconclusive-necessity area. The prime power (i.e. the number of prime radicals in the putative composite, degenerate/singular not least) of (#, X) tends to that of the input prime/composite in question. Among other regularities, recurring sequences and collated generalizations are observed in a set of follow-on conjectures.

Rehash Tag One: Zeroing in on Meaningful Rigor

It was argued in the previous work (Shevenyonov, 2022) that primality tends to reveal the kind of *self-spawned* or sustained nature that forces it to recur in the **|#±X|** characteristic (the "hash/sharp" indicator pointing to what is known in standard literature as the 'digital root,' or "numerological count" *simpliciter*). If we now zoom in on the more rigorous representation of *X* as the *'floor' function* (as cautioned before, I opt to denote *X=[A/9]* rather than [(A-1)/9] for the 10-basis, if only because in my setup, #9k=**0** per any natural *k*), then it turns out:

$$\forall A \in \boldsymbol{N}\ \exists X \in \boldsymbol{N}\colon\ A \equiv \#A + 9X(A) = \# + 9 * \left[\frac{A}{9}\right] \qquad (1)$$

$$\forall_{\omega}, \omega \gg \frac{1}{2}\colon |\#p_{\omega} \pm X(p_{\omega})| \in \boldsymbol{P}\ \ if\ p_{\omega} \in \boldsymbol{P} \qquad (2)$$

$$|X(p) \pm (p - 9X(p))| \equiv_{\omega} p' \ \ (3)$$

$$p' = 2m \pm p \;\; OR \; p + \begin{cases} \mp 8X = \mp 8 * \left\lfloor \frac{p}{9} \right\rfloor \\ \pm 10X = \pm 10 * \left\lfloor \frac{p}{9} \right\rfloor \end{cases} \equiv p' \quad (4)$$

Evidently, this applies to prior/input primes larger than or equal to 11 (or else the floor makes zero identically, without prime-generation never moving past self-identity). The omega-index points to "[far] more often than not" (most of the time, a material share of instances, spanning a large enough domain), while (2) implying (3)-(4) suggests how the resultant prime generating structure productively informs (and constructively proves/motivates beyond weakly rationalizing) the more 'naïve' ones, notably *p=2m±1* or *p=4m±3* with respect to how *m* is endogenized in terms of the 'floor' metric. Follow-on Conjectures will show how, in a sense, the *omega*-qualifier (domain) is compact in that the resultant, or posterior, primes are largely of the same nature as the prior/input ones (5):

$$p, p' \in \boldsymbol{P}_{\omega} \qquad (5)$$

Rehash Tag Two: Recollections of the Universal Operation

Along the *orduale* lines, one is now poised to deploy the rho-aggregate/residuale (rho-lativity as one ultimate way of making a set or group one, in that a complete and least inconclusive relationship is hence defined therewithin) as a representation of the *generalized operation*, or a universal algebra, as has been attempted before (Shevenyonov, 2019).

$$\forall P_k \in S(n) \in P_{\rho} \; \exists \rho \colon \; n\widehat{P^{\rho}} \equiv \sum_{k=1}^{n} n_k P_k^{\rho} \,, \qquad \sum_{k} n_k \equiv n \quad (6)$$

It is easy to see that, as rho tends to 0, the generalized rho-operation collapses to *multiplication*, such that a *composite* results in the left-hand side (7a). (Note also that **nP** turns out a profitable formulation, in that it likewise connotes the non-prime/composite subdomain of the [odd] natural set.) By contrast, *primality* results as a *self-identity* degenerate prime-root vector as P_k's collapse to P-hat or converge to each other (7b). Interestingly enough, the *n* need not be specified explicitly, as the set can embark on the *entire* prime-radicals axis with the respective n_k powers applying (or amounting to zero identically as if to switch the irrelevant prime roots off.)

$$\hat{P} \xrightarrow[\rho \to 0]{} \Big(\prod_{k=1}^{n} P_k^{n_k}\Big)^{1/n} \in \boldsymbol{nP} \qquad (7a)$$

$$\hat{P} \xrightarrow[\rho \to 1, \;\; P_k \to \hat{P}]{} \sum_{k} \frac{n_k}{n} * \hat{P} \equiv \hat{P} = P \in \boldsymbol{P} \qquad (7b)$$

While the [unrestricted] rho-aggregation (or Lame set/group) could be viewed as an all-comprising operation (indeed generalizing and straddling over multiplication, summation, minimization, finding the norm/distance, etc.), its select corners are of interest, too. As has been suggested, rho=0 results in logs/*multiplicity*, while rho=1 in *additivity*—indeed the core vicinities for composites versus primes, respectively.

What is more, one could think of the composite subdomain as *symmetry* (I have long called it ***cardinalcy*** in contrast to ***ordinalcy/orduality***), or self-duality, whereby (8) holds resulting in 0 and 2 (cf. the “neither” versus “both” scenarios pertaining to the conjectured primality of |*X*±*#*|)—e.g. capturing products and norms. In contrast, primality could be thought of as *not* complying with strong symmetry in general, or as an ‘average case’ in between those corners when it comes to sums building on rho=1 (9). Please note, as per (7c) that there is no way of distinguishing between the [meta] operations of multiplying versus summing between the [dual] operations/actions.

$$\frac{1}{\rho}+\frac{\rho-1}{\rho}\equiv 1, \qquad \rho+\frac{\rho}{\rho-1}=\rho*\frac{\rho}{\rho-1} \qquad (7c)$$

$$\rho=\frac{\rho}{\rho-1}=\begin{cases}0\\2\end{cases} < cardinalcy, composites > \qquad (8)$$

$$\rho\neq\frac{\rho}{\rho-1}\sim\frac{1}{2}*\begin{cases}0\\2\end{cases}\sim 1 \ < orduality, primes > \qquad (9)$$

Conjectures: The “Meseemeth” Swathe

We are now in a position to induce a set of extra or qualifying Conjectures following up on the previous glimpse.

<u>Conjecture 2A</u>. Refer to “either” (*focality*), “neither” (degeneracy, nil-focality), and “both” ([am]bifocality) as scenarios featuring at least one (plus or minus-hit), none, or complete realization of posterior/generated primality as a matter of applying the |*X*±*#*| metric to the prior (ensured primality). It can be argued that “either” dominates for primes whereas “neither” is [relatively] more characteristic of composites. While they allow for the “both” quadrant (which may have a significant overlap with the ‘gray area’ of necessity as opposed to sufficiency), it is far more commonplace for composites (in strong/absolute terms included) and minor (weak or relative) per primes.

<u>Conjecture 2B</u>. The effective *power/rank* of the prime root vector (i.e. the number of primes not switched off in the composite) for the implied X will routinely not exceed that of the input composite.

<u>Conjecture 2C</u>. For input primes (i.e. singular composites), the rank does not exceed one—or the order of comparison may be reversed in case of X proving a multiple of 3^k or 5^k. This partially explains Conjecture 2A or shows its bearing on Conjecture 2B.

<u>Conjecture 2D</u>. The resultant (if any) prime composition of X resembles that of the prior/input (thus again liaising with Conjectures 2B-2C). Somewhat loosely, *#radX(A)=#rad A* overall as a sum and on a piecemeal, per-element basis as one possibility. More loosely, alternate generalizations tend to conform on a piecemeal/weak or complete/strong basis. (The **rad** operator defined as under the ABC conjecture.) This may have some bearing on (or informing for) the above omega-qualifier.

The above could be induced from inspecting a sample of composites.

143=11*13=*(#,X)=(*2^3, 3*5)=(8, 15): 15+8=23, 15-8=7 <“both”=max rank below 2 at min>

221=13*17=(5, 2^3*3)=(5, 24): 24+5=29, 24-5=19 <“both”=max rank below 2 at min>

247=13*19=(2^2, 3^3)=(4, 27): 27+4=31, 27-4=23 <“both”=max rank below 2 at min>

323=17*19=(2^3, 5*7)=(8, 35): 35+8=43, 35-8=27=3^3 <“either”=rank below 2 due to singularity: rad 3^3=3<3^3>

817=19*43=(7, 2*3^2*5)=(7, 90): 90+7=97, 90-7=83 <“both”= max rank below 2 at min >

96773=29*47*71=(5, 2^8*3*7)=(5, 10752): 10757=31*347, 10747=11*977 <“neither,” max rank below 3>

Not only are the total #-scores equal identically in the priors versus their #’s, or left-characteristics (that being tautological), so too are they with respect to X, or the right-characteristic in the latter case: 5=#5=#10752. Moreover, this one is of special interest (as is 221) in just how the generalized/lax representations of the conventional values suggest a one-to-one correspondence between A and X(A) per element or even sequence-wise!

I will denote ***{B}=A***, meaning that “A is a specification of B” without going into much detail. Consider {21}=221=13*17={3}{7} (i.e. 221, 13, 17 are particular specifications of 21, 3, and 7 respectively). Alternative generalizations may remind the rationale of Roman numerals: e.g. 47={7-4}={3}, *Y9={Y}*, even though *aB={B-a}* may be at odds with *1B={B}* as a special case. As per 96773=29*47*71={2}{3}{7} cf. rad(2^8*3*7)=2*3*7. Likewise, 19*43=817 suggests {1}{4+3}={7}, with (81)7 acting as if analogous to (9)7={7}, #9=#81=0.

Now consider still-higher power cases of 4 and 5.

3147941=17*23*83*97=(2, 349771)=(2, 19*41*449): minus-outcome 7*29*1723, plus-outcome 3*31*3761 <“neither,” max rank under 4>

173518049=23*29*37*79*89=(2, 19279783)=(2, 463*41641): minus 43*448367, plus 3*5*7^2*17*1543 <“neither,” max rank equaling 5 at max>

After all, *rad* is but one way of generalizing values alongside # and *{}* being alternate ways. It may come as much of a surprise they measure up as it should not seem counterintuitive!

Whilst at it, somewhat informally, **Conjecture 2E** could be proposed surmising that [higher] composites [of smaller primes <53] boost the frequency of # making 2^k, if only because the odds of stumbling into *#- even* matches increase with the rank given that the weight of even-# primes-*values* is disproportionately larger amid the even/odd product-*match* proportion being symmetric from within the basis’ structure alone (cf. the Vedic square). In such instances, X encompasses *odd-only* prime roots, whilst *#-odd* necessarily and sufficiently factors in 2^k (strong-evens) on the X make-up.

Lest AAA (arbitrarily-abductive-adhockery) might seem outmatching, consider an informal alteration of formal estimates as in **Conjecture 2F**: The frequency/size of similar digital sequencing-match between prior/input composites versus posterior X-metrics or |X±#| outcomes tends to grow for higher composites. To illustrate (beyond the rad and {} and # matches): ..79.. as of 173518049, ..17*23.. versus ..1723.. as per 3147941, ..7*7.. in 96773, etc.

References

Shevenyonov, Arthur V. (2022). Primality’s Unwinding Nature: Drawing upon the Identity Tweak. *viXra: 2203.0054*

Shevenyonov, Arthur V. (2019). Orduality versus *F*lausible Falsifiability & Inference Criteria Explicated: None (All) is a Proof? *viXra: 1906.0111*