A note on interpreting special relativity

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March 16th 2022

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Abstract

In this short note I simply try to assess Einstein’s theory of special relativity in my own straightforward way, and decide whether or not it contains inconsistencies, as some authors have claimed. The standard interpretation of SR is consistent in that it deals with the relative motion of two inertial frames of reference usually considered to be moving in a straight line relative to each other. The non-intuitive prediction of clock differences is also logically consistent within the special theory, but in practice, clocks are not properly synchronized, and the theory does not include a general relativity spacetime curvature effect that negates the clock differences due to kinematical relativity alone. I have introduced the modification as a new postulate, but it may just be a consequence of the conservation of energy principle, or the equivalence principle.
1 Introduction

One of the most influential papers written in the history of mathematical physics was Albert Einstein's 1905 paper entitled "Zur Elektrodynamik bewegter Koerper" [1], in which he introduced a revolutionary theory, later to become known as the special theory of relativity (SR). However, over the century and more since its publication, numerous mathematicians and physicists have questioned its validity (see ref. [2] for a summary). For example, a recent paper by Lev Verkhovsky [3] claims there is a contradiction in the theory that invalidates it, while Stephen Crothers also states that SR is logically inconsistent [4], while Laszlo Szabo claims [5] that special relativity theory tells us nothing new about the spatio-temporal features of the physical world, and that the longstanding belief that it does so is the result of a simple but subversive terminological confusion.

In view of this level of scepticism, I decided to examine the situation myself, and try to come to some decision on whether or not SR is consistent. I shall consider essentially three interpretations, and hope to arrive at a logical answer.

Einstein's original paper [1] contains the following transformation of time and space coordinates between two inertial frames of reference, \( O(x, y, z, t) \) and \( O'(x', y', z', t') \), moving at a relative speed \( v \):

\[
x' = \phi(v)\gamma(v) [x - vt]; \quad y' = \phi(v)y; \quad z' = \phi(v)z;
\]

\[
t' = \phi(v)\gamma(v) \left[ t - \frac{vx}{c^2} \right]
\]  \hfill (1)

where \( \gamma = 1/\sqrt{1 - v^2/c^2} \). In his paper, Einstein argued that the function \( \phi(v) \) that appears as a multiplication factor affecting all terms could be set to unity, which meant that it disappeared subsequently from the transformation equations, and has been essentially ignored ever since. This neglect of \( \phi \), however, may have caused some misconceptions over the years, and so I shall review it in this paper, and base some of the analysis on it.
2 Derivation of time dilation equation

One of the main predictions of SR is time dilation, and it is quite simple to show how this comes about by firstly writing an equation for a spherical wavefront propagating in a frame of reference labelled $O$. Imagine that a pulse of light is emitted from the origin of the frame, so that we may write:

$$x^2 + y^2 + z^2 = c^2t^2$$

Using the transforms in Equation 1 to convert to the frame labelled $O'$, and rearranging the terms, we obtain the following:

$$-c^2t'^2 + x'^2 + y'^2 + z'^2 = \phi^2[-c^2t^2 + x^2 + y^2 + z^2] \quad (= 0)$$

where I have included the as-yet unspecified function $\phi(v)$. These equations contain Einstein’s postulate of the invariance of the speed of light, since $c$ is the same in both frames, and we see that the light pulse propagates in both frames as a spherical wavefront, satisfying his relativity principle.

Now writing $x' = y'; z' = z$ for relative motion of the frames in the $x$ or $x'$ direction, we have

$$-c^2t'^2 + x'^2 = \phi^2[-c^2t^2 + x^2]$$

and with $x' = 0$ and $x = vt$, where $v$ is the relative speed of the frames, we have for the time $t'$ in frame $O'$ compared to the time $t$ in frame $O$:

$$\frac{t'}{t} = \phi \sqrt{1 - \frac{v^2}{c^2}} = \frac{\phi}{\gamma}$$

This shall be used as the basis for further analysis in what follows.
3 Discussion

3.1 Interpretation I

So, what does this factor \( \phi(v) \) represent? Imagine you are an observer \( O \) standing on a railway platform, and a train thunders through the station representing frame \( O' \). You also have many compatriots ("book-keepers") positioned at intervals along the very long platform telling you what they observe as the train passes them. It’s always the same information, viz. there is time dilation given by Equation 5, with \( \phi = 1 \), and the proper clock seen in the train \( t' \) is ticking more slowly than the coordinate clocks \( t \) held by observers all along the platform. A train then passes in the opposite direction, and (of course) you measure the same time dilation, irrespective of the direction (because it enters the equation as \( v^2 \)).

Next, you look along the track at the clock in the receding train. Now you observe that the time dilation has altered, due to the Doppler effect. The light waves are stretched out as the ticking clock recedes, given by the ratio \( c/(c + v) \), where \( v \) is the velocity of the receding train. We then have

\[
\frac{t'}{t} = \left( \frac{c}{c + v} \right) \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{\frac{c - v}{c + v}} \tag{6}
\]

which is the well-known result for the relativistic Doppler effect, in this case a red-shift, and we have an interpretation for the meaning of the function \( \phi \):

\[
\phi = \left( \frac{c}{c + v} \right) \quad \text{[receding]} \tag{7}
\]

It represents the way space has been scaled dynamically due to the relative motion of the frame \( O' \). The equivalent expressions for an approaching frame, where a blue-shift occurs, are obtained by changing
the sign before \( v \), and then we have

\[
\frac{t'}{t} = \left( \frac{c}{c-v} \right) \sqrt{1 - \frac{v^2}{c^2}} = \sqrt{\frac{c+v}{c-v}}; \quad \phi = \left( \frac{c}{c-v} \right) \quad [\text{approaching}]
\]

This is the standard, accepted view, and it can be seen by inserting some convenient values that it leads to a clock difference (see below).

Einstein [1] argued that \( \phi(v) = \phi(-v) \) and \( \phi(v)\phi(-v) = 1 \), which gives \( \phi = 1 \), but it seems he was concerned only with a passing frame, i.e. at right angles to the line of sight, and then it is correct to write \( \phi(v) = 1 \), since at that particular point, \( v = 0 \) in the expression for \( \phi \) (as it suddenly changes from plus to minus, from approaching to receding). The whole essence of his version of SR is that the coordinate frame is equipped with clocks throughout the frame in different spatial locations, but all synchronized with each other, and the motion along the track is then always transverse to the observer.

Adding the Doppler effect into the analysis for assessing the longitudinal or in-line behavior, does not invalidate Einstein’s SR. It seems to remain consistent, and predict a clock difference, as I shall show next.

Take the numerical example of a journey to a distant place 3 light years away at a speed \( \frac{v}{c} = 3/5 \). We have \( \frac{1}{\gamma} = 4/5 \) and \( t = 5 \) years. This gives \( t'/t = 1/2 \), but the observation of this lasts for 8 years, since it takes 5 years on \( O \)'s clock for \( O' \) to arrive at the destination plus 3 years for the light to travel back to the observer, i.e. the \( O' \) clock ticks at half rate for 8 years = 4 years, while the resting observer ages by 5 years. On a return journey, \( O \) observes the \( O' \) clock ticking at double rate for two years (= 5 − 3 years), again 4 years on the \( O' \) clock, instead of 5 years. The age or clock difference occurs on both receding and approaching legs of the journey, so it is theoretically not necessary for the traveller to return to prove he/she has aged differently. Ultimately, the overall ageing effect is given by
the Lorentz factor alone, and the Doppler shift just gives us some ongoing information during the course of the thought experiment.

3.2 Interpretation II

In his paper [3], Verhovsky also considers the factor $\phi$. He claims that SR is contradictory, and that to agree with the invariance of $c$, the length scales in the two frames must be related by the ratio $(c - v)/c$ for a frame approaching the observer (change the sign for receding frames), which implies that $t'/t = 1 - v/c$. Writing this as

$$\frac{t'}{t} = \left(\frac{c - v}{c}\right) = \sqrt{\frac{c - v}{c + v}} \sqrt{1 - \frac{v^2}{c^2}} \quad [\text{approaching}] \quad (9)$$

suggests we have

$$\phi = \sqrt{\frac{c - v}{c + v}} \quad [\text{approaching}] \quad (10)$$

(Note: In Verhovsky’s paper, his function $D$ for Doppler wavelength changes is equivalent to the inverse of my function $\phi$ for frequencies.)

Using the same numerical example as previously, we have $t'/t = 8/5$ for receding frames and $t'/t = 2/5$ for approaching frames (compared to $t'/t = 2$ and $1/2$ previously), which Verkhovsky interpreted to mean that clock differences disappear altogether. (Presumably, adding the two ratios for receding and approaching, gives 10 years time lapse on both clocks.) This interpretation also changes the usual formula for relativistic mass from $m = \gamma m_0 = m_0/\sqrt{1 - v^2/c^2}$ to the following: $m = m_0(1 - v/c)$, for an approaching mass, and where the sign before $v$ is changed for a receding mass.

I have to admit that this doesn’t seem right to me, since we expect $m = \gamma m_0$ to hold for a mass, say, in a particle accelerator or electron microscope, whereas this interpretation gives $m = m_0$ as it passes the observer. Secondly, it seems that the length contraction given in
Equation 9 is incorrect, since it doesn’t include Lorentz contraction of distances in the \( O' \) frame.

### 3.3 Interpretation III

In this section I shall propose a different postulate that also negates a clock difference.

It is always assumed in such thought experiments that clocks are identical and have been synchronized with each other to tick at the same rate when compared side-by-side. However, this condition has not been specifically met here with regard to the clock in the frame \( O' \) compared to clocks in the \( O \) frame. To be precise we must firstly synchronize clocks \( O \) and \( O' \) in the same spatial location on the imagined station platform - and then consider what happens.

Subsequently, twin \( O' \) boards the initially stationary train, which then accelerates up to the cruising speed \( v \). The train engines convert chemical energy into mechanical work on everything in the train (all the masses) to produce an acceleration, and this increases the kinetic energy of the train and its contents, including the clock, relative to \( O \).

It is well-known from Einstein’s other relativity theory, general relativity (*GR*), that an additional time dilation effect occurs in a curved spacetime, such as a gravitational field. For sufficiently weak gravitational potential changes \( U(\tilde{r}) \), one has the relation

\[
t' = t \sqrt{1 - \frac{v^2}{c^2} + \frac{2U}{m_0 c^2}}
\]

where \( m_0 \) is the rest mass, and \( \tilde{r} \) is a radial coordinate (see, for example, ref. [6]).

There is of course no gravitational field *per se* in the present thought experiment we are discussing, but it is suggestive to think that there
is an equivalence between an increase in potential energy and an increase in kinetic energy. Thus, I shall make this additional postulate: an increase in kinetic energy results in an increase in clock rate identical to that caused by an equivalent increase in potential energy.

We may then write \( U = \frac{1}{2} m_0 v^2 \) in Equation 11 and the total time dilation becomes zero. In this case, then, an increase due to increased energy exactly offsets the decrease in clock rate that occurs due the Lorentzian symmetry of spacetime. The result is that no change in clock rate occurs, on the proviso that all clocks have been initially synchronized at rest beforehand.

Kinematical time dilation is, of course, an experimentally measured fact. The reason it is observed in the decay of elementary particles, such as cosmic muons passing through the Earth’s atmosphere, is that their subatomic clocks were never synchronized with cosmic muons in the laboratory - they were effectively created at high speed - and then the relativistic kinematical time dilation effect of \( \text{SR} \) becomes specifically apparent. Many scientists believe that experiments with clocks in aeroplanes, such as the Hafele-Keating experiment, provide unequivocal proof of the age differences predicted by \( \text{SR} \) in the so-called twin paradox. However, I do not think they are a good test, for various reasons: (a) the circular motion around the Earth is not the standard linear configuration of \( \text{SR} \), (b) the Sagnac effect plays a vital rôle, i.e., the Earth itself is rotating, and (c) gravitational time dilation itself is the dominant effect.

In conclusion, I have introduced a theoretical modification as a new postulate, because the kinematical approach to time dilation in \( \text{SR} \) does not include all the ingredients necessary for understanding the described thought experiment. The new postulate is possibly just a consequence of the conservation of energy principle, or the equivalence principle.
4 Acknowledgments

I thank Lev Verkhovsky for private correspondence that provided the stimulus for writing this short paper.

References


