DIFFERENTIAL COATING OBJECTIVE

Ervin Goldfain
Welch Allyn Inc.

ABSTRACT

The paper outlines a novel concept on the construction of high resolution objectives. Although similar to binary optics technology, this approach brings additional degrees of freedom to the optical design as far as achieving a finer balance of phase curvature via both refractive properties and relative thickness of the multilayer stack. The differential coating objective can be used in conjunction with diffractive surfaces to optimize the transmission efficiency and control the secondary spectrum correction. The basic equations describing the monochromatic and three-color achromat predesign are presented.

1 INTRODUCTION

The concept discussed below represents an alternative to the so-called hybrid refractive/diffractive optics developed in recent years. Instead of having a discrete phased pattern etched onto one or several surfaces (Kinoform) or creating a volume holographic layer (HOE) on a refractive component, we suggest applying a differential stack of multiple coatings onto a selected number of surfaces derived from the preliminary design.

It is known that optical crystals offer specific benefits over conventional glasses such as unique transmission in both IR and UV regions, a large range of the refractive index, low dispersion, thermal shock resistance, unique relation of refraction and dispersion. Hence by combining optical properties of glass elements with those given by a carefully selected stack of annular crystal layers, new degrees of freedom for performance optimization are introduced.

As in the case of hybrid HOE/glass lenses, annular layers exhibit low scattering features due mainly to the continuous phase shift in the propagating wavefront. Losses of radiant energy occur however at the boundary regions separating adjacent layers (edge diffraction).

Several advantages pertain to this concept such as:

a) lower number of glass elements.
b) reduced refractive curvatures.
c) higher N.A.'s and working distances available.
d) easier apochromatic correction.
e) simplified construction of IR hybrid lenses.

2 MONOCHROMATIC PREDESIGN EQUATIONS

Consider a rotationally symmetric objective consisting of N optical surfaces. The wave aberrations are defined as OPD's of arbitrary skew rays with respect to the principal ray.

A convenient representation of the wave aberrations is the expansion of the wavefront deviation as a polynomial function of the exit pupil variables. The geometry of a ray emerging from a point P of the exit pupil plane is completely specified by the polar coordinates \( r_m, \phi_m \) as well as the unit vector \( \hat{\sigma}_m \): (fig.1)

\[
\hat{\sigma}_m = (L, M, N)_m
\]
In terms of normalized coordinates:

\[ H_m = h_m / h_{\text{max}} \quad \bar{r}_m = r_m / r_{\text{max}} \]  

(2)

where \( h_m \) is the image height \((Q'S)\), \( h_{\text{max}} = (QA')\), \( r_{\text{max}} = CA \) (radius of the pupil). The aberration polynomial can be written as:

\[ W_a (r_m, \phi_m, H_m) = \text{OPD} (r_m, \phi_m, H_m) / (n' \lambda) \]  

(3)

Setting the relative height at a fixed value, \( (3) \) reads:

\[ W_a (r_m, \phi_m) = \sum d C^d_{\text{ef}} r_m \cos^d \phi_m \]  

(4)

where \( C^d_{\text{ef}} \) are the aberrations coefficients listed in \( 4^\text{th} \) and \( 5^\text{th} \). Their magnitude dictate the amount of image degradation in terms of third or higher order aberrations and are assumed to be known as part of system specifications.

If one of surfaces has a certain pattern of annular layers applied on it, the polynomial contains basically three major terms as follows:

1) primary aberrations \((d<3, e+f<4)\) \(\text{OPD}_3\)

2) higher order aberrations \((d<5, e+f>4)\) \(\text{OPD}_h\)

3) phase contribution due to the stack. \(\text{OPD}_{\phi}\)

Since the number and structure of the layers can vary radially (fig. 2), the phase shift introduced becomes a function of aperture. This circumstance implies that the wavefront polynomial can be slightly tuned by adding or substracting an appropriate number of rings from the central area of the surface.

To evaluate the OPD associated with the multilayer stack, an arbitrary skew ray may be traced through the surface (fig. 3). Assuming a number of "k" rings at the intercept \( A_0 \):

\[ n_0 \sin \theta_0 = n_1 \sin \theta_1 = ... = \sin \theta_k \]  

(5)

\[ \text{OPD}_s = \sum_i (A_i A_{i+1}) n_{i+1} - \sum_i (A'_i A'_{i+1}) \quad (i = 0, k-1 \quad A'_0 = A_0) \]  

(6)

Processing \( (6) \) one obtains:

\[ \text{OPD}_s = \sum_i t_{i+1} [n_{i+1} - \cos^{-1}(\theta'_i - \theta_{i+1})] / \cos \theta_{i+1} \]  

(7)

The Seidel monochromatic aberrations can be easily derived as functions of system's specifications taking into account that the rings are optically equivalent to thin meniscus lenses:

\[ (O_i + O^*_j) = S + S^* \quad \text{(spherical)} \]

\[ (O_iU_j + O^*_U^*) = C + C^* \quad \text{(coma)} \]

\[ (O_iU_i^2 + O^*_U^2) = A + A^* \quad \text{(astigmatism)} \]

\[ (P_i + P^*_j) = P + P^* \quad \text{(Petzval)} \]  

(8)
\[(Q_{11}U_{1j}^3 + P_{1j}U_{1j}) + (Q_{j}^*U_{1j}^3 + P_{j}^*U_{1j}) = D + D^*\] (distortion) (9)

where \(j=1,2,3...p\) and the asterisk refer to the layers located on the optical axis and:

\[O_i = Q_{si}^2 h_i^4 (n_{i-1} s_i)^{-1} + (n_i s_i')^{-1}\]
\[P_i = R_i^{-1} (n_{i-1} - n_i)\]
\[Q_{si} = n_{i-1} (R_i + s_i^{-1}) = n_i (R_i - s_i')^{-1}\]

In which \(s_i, s_i'\) are the paraxial conjugates for an object having the paraxial height \(h_i\) and \(R_i\) stand for curvature radii. In a similar way:

\[U_i = (Q_{si}^2 h_i^2)^{-1} [1 + Q_{si}^2 h_i^2 \sum t_i (n_{i1} h_{i1} h_{i1+1})^{-1}] \quad (l = 1,2,\ldots,i-1)\] (11)
\[U_j = (Q_{sj}^2 h_j^2)^{-1} (1 + Q_{sj}^2 \sum t_j n_j) \quad (j = 1,2,\ldots,p)\] (12)

Where \(t_i\) designates layers thicknesses.

Note that in equations (8) to (11) the index \(i\) specifies the sequence of all optical surfaces \((i = 1,2,\ldots,N)\).

Consolidating the above, one can rewrite (3) as:

\[W_a(r_m, \phi_m) = (\text{OPD}_3 + \text{OPD}_h + \text{OPD}_s)m/\lambda\] (13)
\[\text{OPD}_3 = (S + C + A + P + D) + (S^* + C^* + A^* + P^* + D^*)\] (14)

It is apparent that the number and structure of the layers located on axis can be used as degrees of freedom to control \(\text{OPD}_h\) by imposing the overall \(W_a\). For a diffraction limited objective \(W_a = 1/4\) (Rayleigh).

An alternate way to compute \(\text{OPD}_s\) is to operate directly with an arbitrary ray transverse aberrations in the image plane, i.e. (fig.1)):

\[\text{OPD}_s = -\sin u' \int_0^r (\Delta x' \sin \phi + \Delta y' \cos \phi) \, dr\] (14)

in which \(u'\) represents the slope of marginal ray in the image space.

3 APOCHROMAT PREDESIGN EQUATIONS

It is quite obvious that a differentially coated singlet cannot substitute an achromatic doublet because there is not enough refractive power available in the layer stack. Considering the power and lateral chromatic constraints for a thin achromatic doublet:

\[1/f' = 1/f_{\text{ref}} + 1/f_s\]
\[1/f_{\text{ref}} = 1/f' [V_{\text{ref}}/(V_{\text{ref}} - V_s)]\]
\[1/f_s = -1/f_{\text{ref}} (V_s/V_{\text{ref}}) = \sum n_i (n_i - 1) / n_i (t_i / R_i^2) + 0\]

\[\rightarrow V_s \rightarrow 0 \quad \text{(infinite dispersion)}\]

Chromatic correction becomes however possible in a three-color achromat where small amounts of unbalanced chromatic power can be conveniently cancelled using the contribution of additional layer(s).
In the above formulas "ref" stands for refractive and "s" for the layer(s) stack. The three equations defining a thin lens apochromat read:

\[
\frac{1}{f'} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_s} \quad \text{(power)}
\]

\[
(\frac{1}{f_1}V_1) + (\frac{1}{f_2}V_2) + (\frac{1}{f_s}V_s) = 0 \quad \text{(chromatism)}
\]

\[
P_1(\frac{1}{f_1}V_1) + P_2(\frac{1}{f_2}V_2) + P_s(\frac{1}{f_s}V_s) = 0 \quad \text{(secondary spectrum)}
\]

in which \(P_1, P_2, P_s\) represent relative partial dispersions. Two case can be derived from here namely:

a) differentially coated achromatic doublet

\[
f_s = f'E(V_1-V_s) / [(P_1-P_2)V_s]
\]

\[
f_1 = f'E(V_1-V_s) / [(P_2-P_s)V_1]
\]

\[
f_2 = f'E(V_1-V_s) / [(P_s-P_1)V_2]
\]

\[
\frac{1}{(f_s V_s)} = \sum_i [(n_i-1)^2 / n_i(t_i/R^2)V_i]
\]

\[
P_s(\frac{1}{f_s}V_s) = \sum_i [(n_i-1)^2 / n_i(t_i/R^2)V_i]P_i
\]

\[
E = [V_1(P_2-P_s)+V_2(P_s-P_1)+V_s(P_1-P_2)]/(V_s-V_2)
\]

where \(i=1,2,3,...,M\) specifies the layers index. A number of iterative steps are required to maintain appropriate balance between geometric, two-color and three-color aberrations.

b) hybrid HOE/differentially coated singlet

It is known that the large separation between holographic partial dispersion and standard glasses partial dispersions lead to apochromatic HOE/glass designs with reduced refractive curvatures. This configuration potentially eliminates the need for low dispersion glasses like fluorite which involves major difficulties in polishing, cementing, and introduces sensitivity to shock and thermal stress. It has been shown that hybrid HOE/glass achromatic doublets yield good correction for both visible and IR imaging applications.

A natural extension of the above would be a hybrid HOE/differentially coated singlet. In this approach a holographic coating is created on one surface of the crown element while the second surface is differentially coated. By varying the power distribution among HOE, crown and the layers stack for a given overall focal length, a three-color design can be achieved via tuning in additional layers as required.

Following equations describe the predesign:

\[
f_{\text{HOE}} = \frac{1}{r_x^{\text{eff}}} - 1 = \frac{E(V_{\text{HOE}} H_{\text{HOE}})}{f'(P_{\text{glass}} - P_s)V_d}
\]

\[
f_{\text{glass}} = \frac{E(V_{\text{HOE}} H_{\text{HOE}})}{f'(P_{\text{glass}} - P_s)V_d}
\]

\[
f_s = \frac{E(V_{\text{HOE}} H_{\text{HOE}})}{f'(P_{\text{HOE}} - P_{\text{glass}})V_d}
\]

where \(n_x^{\text{eff}}\) stands for the HOE effective refractive index at a given focal length \(f_x\):
\[ n_x^{\text{eff}} = 1 + 1/C_0 f_x \]  

(19)

\( C_0 \) is a constant related to the wavefront curvature and wavelength used in the hologram recording process, \( V_d(\text{HOE}) \) is the effective HOE V-number:

\[ V_d^{\text{HOE}} = \frac{\lambda_d}{(\lambda_F - \lambda_C)} = -3.452 \]  

(20)

and \( P_{\text{HOE}} \) represents the HOE relative partial dispersion, generically given by:

\[ P_{\text{HOE}}^\lambda = \frac{(\lambda_1 - \lambda_2)}{(\lambda_3 - \lambda_4)} \]  

(21)

4 REFERENCES
