A Note on Invo-Regular Rings

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ABSTRACT

In this paper we provide an important and significant observation on a result related to invo-regular rings [1].

Key-words: regular ring, invo-regular ring.

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Introduction

Let $R$ is a unital and associative ring. A ring $R$ is called invo-regular if for each $a \in R$ there exists $b \in Inv(R)$ such that $a = aba$ [1-3]. Here $Inv(R)$ is the set of all involutions. One may note that an element $b$ of $R$ satisfying $b^2 = 1$ is called an involution [1-3].

The proposition 2.5 of [1] states that a ring $R$ is invo-regular iff $R \cong R_1 \times R_2$, here $R_1$ is an invo-regular ring of characteristic two and $R_2$ is an invo-regular ring of characteristic three.

In this note we improve this result by providing a suitable counterexample. In the next section we provide a counterexample for this result.
2. An Important Observation

Let \( R = \begin{pmatrix}
0 & 0 \\
1 & 0 \\
2 & 0 \\
1 & 1 \\
2 & 2 \\
2 & 1 \\
1 & 2 \\
0 & 2 \\
0 & 0 \\
2 & 0 \\
0 & 1 \\
0 & 2
\end{pmatrix} \).

Then \( R \) is a commutative ring of characteristic three under addition and multiplication of matrices modulo three. We note that \( R \) is an invo-regular ring. Now we have the following cases.

**Case I:** \( R \cong R \times \{0\} \). One may note that \( R \) is not a ring of characteristic two.

**Case II:** \( R \cong \{0\} \times R \). It is clear that \( \{0\} \) is not a ring of characteristic two.

**Case III:** \( R \cong R_1 \times R_2 \). Here \( R_1 = Z_3 = R_2 \). We note that the characteristic of \( R_1 = Z_3 \) is not two.

Thus we see that in the above example the characteristic of \( R \) can never be two even though \( R \) is an invo-regular ring. Hence the above example serves as a counterexample for the above result of [1].

**References**


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