**π rational**

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**abstract**

we define \( \pi, e, \text{Li}_s(z) \) rational,

proof the riemann hypothesis,

and create a polynom and formula for \( \pi \)

\[ \pi \text{ rational} \]

Leibniz: \[\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots \]

\[\frac{\pi}{4}\]

Guyer: \[\frac{1}{1} \frac{2}{3} \frac{13}{15} \frac{76}{105} \frac{789}{945} \ldots \]

\[\frac{\pi}{4}\]

“a sum up gives a rational expression”

\[
\frac{1}{1} \quad \frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{7} \quad \frac{1}{9} \\
1 \times 3 \quad - \quad 1 \times 1 \quad = \quad 2 \\
2 \times 5 \quad + \quad 1 \times 3 \quad = \quad 13 \\
13 \times 7 \quad - \quad 1 \times 15 \quad = \quad 76 \\
76 \times 9 \quad + \quad 1 \times 105 \quad = \quad 789 \\
\]

... or step-by-step

**example**

\[ ... + \frac{1}{9} \]

divisor \( \text{lfac} \) \[1 \times 3 \times 5 \times 7 \times 9 = 945\]

dividend \( \text{guyer lfac} \)

\[
945 / 1 = 945 \quad \ast \quad 1 \\
945 / 3 = 315 \quad \ast \quad 1 \\
945 / 5 = 189 \quad \ast \quad 1 \\
945 / 7 = 135 \quad \ast \quad 1 \\
945 / 9 = 105 \quad \ast \quad 1 \\
945 - 315 - 189 - 135 - 105 = 789
\]
\[ e = \sum_{n=0}^{\infty} \frac{1}{n!} = 1 + \frac{1}{1} + \frac{1}{1 \cdot 2} + \frac{1}{1 \cdot 2 \cdot 3} + \cdots \]

\[ e = 2.71828 18284 59045 23536 02874 71352 66249 77572 47093 69995 \ldots \]

Guyer:
\[ \frac{1}{1}, \frac{2}{2}, \frac{5}{2}, \frac{32}{12}, \frac{780}{288}, \frac{93888}{34560}, \ldots \]

"a sum up gives a rational expression"

1 = 1 * 1 + 1 * 1 = 2

+ \frac{1}{1} = 2 * 2 + 1 * 1 = 5

+ \frac{1}{2} = 5 * 6 + 1 * 2 = 32

+ \frac{1}{6} = 32 * 24 + 1 * 12 = 780

+ \frac{1}{24} = 780 * 120 + 1 * 288 = 93888

+ \frac{1}{120} = \frac{93888}{34560} \ldots \]

or step-by-step

\textit{example} \quad \ldots + \frac{1}{120}

divisor \quad \text\textit{!fac} \quad 1 * 2 * 6 * 24 * 120 = 34560

dividend \quad \text\textit{!guyer !fac} \quad 34560 / 1 = 34560 \quad \ast \; 1

34560 / 1 = 34560 \quad \ast \; 1

34560 / 2 = 17280 \quad \ast \; 1

34560 / 6 = 5760 \quad \ast \; 1

34560 / 24 = 1440 \quad \ast \; 1

34560 / 120 = 288 \quad \ast \; 1

34560 + 34560 + 17280 + 5760 + 1440 + 288 = 93888
**Li₅(z) rational**

**example**  \[ Li₅(3) \]

"a sum up gives a rational expression"

\[
3 = 3 \quad 3 \times 2^5 \quad + 3^2 \times 1 = 105 \\
+ 3^3 / 2^5 = 105 / 2^5 \quad 105 \times 3^3 \quad + 3^3 \times 2^5 = 26379 \\
+ 3^3 / 3^5 = 26379 / 6^5 \quad 26379 \times 4^3 \quad + 3^4 \times 6^5 = 27641952 \\
+ 3^4 / 4^5 = 27641952 / 24^5 \quad … \\
+ 3^5 / 5^5 \\
…
\]

or step-by-step

**example**  

\[
\text{divisor} \quad \text{Ifac} \quad 1^5 \times 2^5 \times 3^5 \times 4^5 = 24^5 \\
\text{dividend} \quad \text{guyer Ifac} \quad 7962624 / 1^5 = 7962624 \quad \ast 3 \\
7962624 / 2^5 = 248832 \quad \ast 3^2 \\
7962624 / 3^5 = 32768 \quad \ast 3^3 \\
7962624 / 4^5 = 7776 \quad \ast 3^4 \\
23887872 + 2239488 + 884736 + 629856 = 27641952
\]
... Euler for \( \text{Li}_2(1) = \zeta(2) \)

„Weil nach Setzen von \( x = 2 \) gilt

\[
1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \text{etc} = \frac{\pi^2}{6},
\]

wobei \( \pi \) die Peripherie des Kreises bezeichnet, dessen Durchmesser 1 ist, wird sein

\[
\frac{4}{3} \cdot \frac{9}{8} \cdot \left(\frac{25}{24} + \frac{49}{48} + \frac{121}{120} + \text{etc}\right) = \frac{\pi^2}{6}.
\]

with wahrheitstabelle for \( \zeta(2) \)

\[
\begin{array}{cccccc}
1 & 5 & 49 & 920 & 21076 & 773136 \\
1 & 4 & 36 & 576 & 14400 & 518400 \\
\end{array}
\]

we see immediately

Euler \( \prod \frac{4}{3} \cdot \frac{9}{8} = \frac{36}{24} \) \( p^2 \) is in direct contradiction to \( \frac{49}{36} \)

and

Euler \( \prod_{1 - \frac{1}{p^s}} \frac{1}{p^s} = \sum \frac{1}{n^s} \) is wrong, we have \( \prod_{1 - \frac{1}{p^s}} \frac{1}{p^s} \neq \sum \frac{1}{n^s} \)

B. Riemann:

Ueber die Anzahl der Primzahlen unter einer gegebenen Grösse

... 

Bei dieser Untersuchung diente mir als Ausgangspunkt die von Euler gemachte Bemerkung, dass das Product

\[
\prod \frac{1}{1 - \frac{1}{p^s}} = \sum \frac{1}{n^s},
\]

wenn für \( p \) alle Primzahlen, für \( n \) alle ganzen Zahlen gesetzt werden. Die Function der complexen Veränderlichen \( s, \ldots \)

is wrong; the Riemann Hypothesis is false.
... if we look closer to Euler's

\[ \frac{p^2}{p^2 - 1} \rightarrow \frac{n^2}{n^2 - 1} = \frac{1^2 \cdot 2^2 \cdot 3^2 \cdot \ldots}{0! \cdot 2! \cdot 3! \cdot \ldots} = 0 \]

if \( n = \frac{1}{1} = \frac{1+1}{1+3} = \frac{1+4}{1+8} = \frac{1+4+9}{1+3+8} = \ldots \)

\[ = \frac{1^2}{1} \cdot \frac{2^2}{3} = \frac{3^2}{6} = \frac{4^2}{10} = \frac{5^2}{15} = \ldots \]

\[ = \frac{n^2}{\text{zeta}(-1) \text{ at } n} \quad [\text{zeta(-1) } = 1 + 2 + 3 + 4 + \ldots] \]

we have conclusion:

Euler\( \prod \) over the primes

\[ \begin{align*}
\frac{4}{3} &\quad \frac{4 + 9}{3 + 8} &\quad \frac{4 + 9 + 25}{3 + 8 + 24} &\quad \ldots \\
Euler\sum &\quad \text{over the primes}
\end{align*} \]

\[ \frac{2^2}{3} \quad \frac{3^2}{6} \quad \frac{11}{11} \quad \ldots \]

; with the o(n)e confusion \( \frac{1^2}{0} \) & \( 1 + 2 \)

in a picture

<table>
<thead>
<tr>
<th>( n^2 )</th>
<th>( 1^2 )</th>
<th>( 2^2 )</th>
<th>( 3^2 )</th>
<th>( 4^2 )</th>
<th>( 5^2 )</th>
<th>( 6^2 )</th>
<th>( 7^2 )</th>
<th>( 8^2 )</th>
<th>( 9^2 )</th>
<th>( 10^2 )</th>
<th>( 11^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>zeta(-1)</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>21</td>
<td>28</td>
<td>36</td>
<td>45</td>
<td>55</td>
<td>66</td>
</tr>
<tr>
<td>Euler( \prod )</td>
<td>( \frac{2^2}{3} )</td>
<td>( \frac{6^2}{24} )</td>
<td>( \frac{30^2}{576} )</td>
<td>( \frac{210^2}{27648} )</td>
<td>( \frac{2310^2}{3317760} )</td>
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<tr>
<td>( \zeta(2) )</td>
<td>( \frac{1}{1^2} )</td>
<td>( \frac{5}{2^2} )</td>
<td>( \frac{49}{6^2} )</td>
<td>( \frac{820}{24^2} )</td>
<td>( \frac{21076}{120^2} )</td>
<td>( \frac{773136}{720^2} )</td>
<td>( \frac{38402064}{5040^2} )</td>
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</tbody>
</table>

example 7 is boring

\[ \begin{align*}
\frac{n^2}{\text{zeta}(-1)} &\quad \frac{7^2}{28} / \frac{4^2 \cdot 6^2}{15 \cdot 35} = Euler\prod \frac{210^2}{27648} \\
\text{or} \quad 28 &\quad 4 + 6 = Euler\sum 18 \\
\end{align*} \]

and

Euler\( \prod \) \( \frac{210^2}{27648} = \zeta(2) \frac{38402064}{5040^2} / 24^2 \)

looks weird
π is something like the “Mülleimer” in Mathematics;

X mathematicians have Y formulas for ≈ Z

Leibniz \[ \frac{1}{1} \frac{1}{3} + \frac{1}{5} \frac{1}{7} + \frac{1}{9} \ldots = \frac{\pi}{4} \]

Euler \[ \frac{1}{1^2} \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} \frac{1}{5^2} + \ldots = \frac{\pi^2}{6} \]

Euler[\prod] \[ \prod \frac{1}{1 - \frac{1}{p^n}} = \frac{\pi^2}{6} \]

Ramanujan \[ \frac{\sqrt{\frac{\pi}{9801}} \sum_{n=0}^{\infty} \frac{(4n)!}{(n)!^4} \frac{26390n}{396^{4n}}}{(2n)!} \approx \frac{1}{\pi} \]

Wallis \[ \frac{2}{1} \frac{2}{3} \frac{4}{5} \frac{4}{5} \frac{6}{7} \frac{6}{7} \frac{8}{9} \ldots = \frac{\pi}{2} \]

...\]

iguier ifac is a formula to generate a rational expression at any point

a approximation

\[ \zeta(2) = \frac{\pi^2}{6} \leftrightarrow \zeta(4) = \frac{\pi^4}{90} \]

is a irrational approach

\[ \pi^2 = \frac{6^2}{1^2} \rightarrow \pi^4 \neq \frac{6^2}{1^2} \]

and irrelevant, but for a solid basis of π we create
Guyer Polynom

example

\[ ! 11 = 39916800 \]

\[ 6 \]

\[ 5 \ast 7 = 35 \quad 6^2 - 1^{[2]} \]

\[ 4 \ast 8 = (\ast)32 = 1120 \quad 6^4 - (5 \ast 6^2) + 2^2 \]

\[ 3 \ast 9 = (\ast)27 = 30240 \quad 6^6 - (14 \ast 6^4) + (49 \ast 6^2) - 6^2 \]

\[ 2 \ast 10 = (\ast)20 = 604800 \quad 6^8 - (30 \ast 6^6) + (273 \ast 6^4) - (820 \ast 6^2) + 24^2 \]

\[ 1 \ast 11 = (\ast)11 = 6652800 \quad 6^{10} - (55 \ast 6^8) + (1023 \ast 6^6) - (7645 \ast 6^4) + (21076 \ast 6^2) - 120^2 \]

\[ \ast 6 \quad = ! 11 \]

we see

\[ \zeta(-2) \quad \& \quad \zeta(2) \]

\[ 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \ldots \]

\[ = 1, 5, 14, 30, 55, \ldots \]

\[ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{6} \]

\[ = 1, 5, 14, 30, 55, \ldots \]

\[ \frac{1}{1^2}, \frac{1}{2^2}, \frac{49}{6^2}, \frac{820}{24^2}, \frac{21076}{120^2}, \ldots = \frac{\pi^2}{6} \]

a Polynom for / with \( \frac{\pi^2}{6} \)

the \(-+n\)

\[ \ldots \]

\[ 0 \ast 12 = 0 \quad 6^{12} - (91 \ast 6^{10}) + (3003 \ast 6^8) - (44473 \ast 6^6) + (296296 \ast 6^4) - (773136 \ast 6^2) + 518400 \]

shortcut \( \pi^2 = G0 \)

and the formula

example \( n 7 \)

\[ 7^{14} - (140 \ast 7^{12}) + (7462 \ast 7^{10}) - (191620 \ast 7^8) + (2475473 \ast 7^6) - (15291640 \ast 7^4) + (38402064 \ast 7^2) - 25401600 \]

\[ = 0 \]

\[ \frac{38402064 + 6}{25401600} = \pi^2 \]
how it looks and how it works:

\[- + 1 \quad h^2 - 1\]
\[- + 2 \quad h^4 - 5h^2 + 4\]
\[- + 3 \quad h^6 - 14h^4 + 49h^2 - 36\]
\[- + 4 \quad h^8 - 30h^6 + 273h^4 - 820h^2 + 576\]
\[- + 5 \quad h^{10} - 55h^8 + 1023h^6 - 7645h^4 + 21076h^2 - 14400\]
\[- + 6 \quad h^{12} - 91h^{10} + 3003h^8 - 44473h^6 + 296296h^4 - 773136h^2 + 518400\]
\[- + 7 \quad \ldots\]

<table>
<thead>
<tr>
<th>example</th>
<th>line calculation</th>
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<tbody>
<tr>
<td>([- + 4]</td>
<td>([- + 5]</td>
</tr>
<tr>
<td>((1*5^2) + 30)</td>
<td>(= 55)</td>
</tr>
<tr>
<td>((30*5^2) + 273)</td>
<td>(= 1023)</td>
</tr>
<tr>
<td>((273*5^2) + 820)</td>
<td>(= 7645)</td>
</tr>
<tr>
<td>((820*5^2) + 576)</td>
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</tr>
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