Abstract

The quantification of Length and Time in Kepler’s laws implies an angular momentum quantum, identified with the reduced Planck’s constant, showing a mass-symmetry with the Newtonian constant $G$. This leads to the Diophantine Coherence Theorem which generalizes the synthetic resolution of the Hydrogen spectrum by Arthur Haas, three years before Bohr. The Length quantum breaks the Planck wall by a factor $10^{61}$, and the associated Holographic Cosmos is identified as the source of the Background Radiation in the Steady-State Cosmology. An Electricity-Gravitation symmetry, connected with the Combinatorial Hierarchy, defines the steady-state Universe with an invariant Hubble radius 13.812 milliard light-year, corresponding to 70.793 (km/s)/Mpc, a value deposited (1998) in a Closed Draft at the Paris Academy, confirmed by the WMAP value and the recent Carnegie-Chicago Hubble Program, and associated with the Eddington number and the Kotov-Lyuty non-local oscillation. This confirms definitely the Anthropic Principle and the Diophantine Holographic Topological Axis rehabilitating the tachyonic bosonic string theory. This specifies $G$, compatible with the BIPM measurements, but at $6\sigma$ from the official value, defined by merging discordant measurements.

1 The Diophantine Coherence Theorem (DCT)

For connecting different physical measurements, Physics uses multiplication while addition is forbidden. But multiplication is a generalization of addition [26]. This paradox may be suppressed by considering only numerical ratios of the same physical quantity, as in the third Kepler law, introducing Space and Time quanta $L_1$ and $T_1$ [37]. Considered as a Diophantine Equation, which uses only natural numbers $n$, it resolves directly:

$$(T_n/T_1)^2 = (L_n/L_1)^3 \equiv n^6 \Rightarrow T_n = n^3 T_1 ; \quad L_n = n^2 L_1 .$$

(1)

This proceeds from the Holic Principle [32], a Diophantine form of the Holographic Principle, which states that the nature of a physical ratio is related to its exponent identified with its topological dimensions: 3 for Space, 2 for a 2D Time [3], 5 for Mass, and 7 for Field. The Simplest Diophantine Equation
$Y = X^2$ is the basis of the Topological Axis, the skeleton of the cosmic mass spectrum [36].

The $n$-invariant $L_n^2/T_n^2$ is homogeneous to $G m_G$, where $G$ is Newton’s gravitational constant, and $m_G$ is a mass (here the usual central mass is divided by the factor $4\pi^2$). The other Kepler’s law states that the orbital angular momentum per unit mass is an orbital invariant. Since the corresponding term $L_n^2/T_n$ is proportional to $n$, this implies an orbital momentum quantum, identified to the reduced Planck constant, or action quantum $\hbar$, privileged by the the spin concept in particle physics. While the ratio of the kinematic parts of $G$ and $h$ are homogeneous to a speed, these two universal constants presents a symmetry by respect to the mass concept, implying the association of $\hbar$ with a mass $m_h$:

$$L_n^2/T_n^2 = G m_G \quad ; \quad L_n^2/T_n = n \; \hbar/m_h .$$

Any mass pair $(m_G, m_h)$ is associated to a series of Keplerian orbits $(L_n, T_n)$:

$$L_n = \frac{(n \; \hbar)^2}{G m_G m_h^2} ; \quad T_n = \frac{(n \; \hbar)^3}{G^2 m_G^2 m_h^3} .$$

For $n = 1$ and $m_G = m_h = m$, the Special Non-Local Length and Time are:

$$L_{SNL}(m) = \frac{\hbar^2}{G m^3} ; \quad T_{SNL}(m) = \frac{\hbar^3}{G^2 m^5} .$$

Introducing the formal velocity $V_n = L_n/T_n$, this connects the reduced Planck energy $\hbar V_n$ with the gravitational potential energy between the to couple $(m_G, m_h)$ and with the energy $m_h V_n^2$:  

$$V_n = L_n/T_n = G m_G m_h/n \; \hbar \quad \Rightarrow \quad n \; \hbar/T_n = G m_G m_h/L_n = m_h V_n^2 .$$

With the Planck mass $m_P = \sqrt{\hbar c/G}$, where the light speed $c$ is the third universal constant, this reads

$$\frac{n \; \hbar}{T_n} = \frac{G m_G m_h}{L_n} = m_h V_n^2 = m_h \left( \frac{c}{n A} \right)^2 ; \quad A = \frac{m_P}{m_G m_h} .$$

This is called the Diophantine Coherence Theorem (DCT).

### 2 The Atom H and the Holographic Cosmos

Three years before Bohr, Arthur Haas [15] considered the electron orbital period in the Rutherford model, and the corresponding Planck energy $\hbar \nu = n \hbar/T_n = n \; \hbar v_n/L_n$ where $v_n = 2\pi V_n$ is the orbital velocity. The correct Hydrogen spectrum is obtained by equalizing it with the electric potential energy $\hbar c/a L_n$, where $a \approx 137.0359991$ is the electric constant, and the double (virial) kinetic electron energy $m_e v_n^2$ (the useful physical constants are listed in Table 1):

$$n \; \hbar \; \frac{v_n}{L_n} = \frac{\hbar c}{a L_n} = m_e v_n^2 = m_e \left( \frac{c}{n a} \right)^2 .$$

Note that the so-called “properties of vacuum” $\epsilon_0$ and $\mu_0$ are unnecessary:

they are only introduced for historical reasons, leading to the cumbersome, but
official, choice of electrical units, hiding the true "electrical constant" $a$, whose inverse $\alpha$, called "the fine structure constant" is of minor importance. For $n = 1$, this gives the bare Hass-Bohr radius: $r_{HB} = a\lambda_e$, where $\lambda_e \equiv h/(m_e c)$ is the Reduced Electron wavelength (the effective electron mass effect defines the Bohr radius $r_B = r_{HB} \times (1 + 1/p)$). This double equation shows up the same form that the above DCT (6), where additional $2\pi$ factors are integrated in the definitions of $m_G$ and $m_h$. The identification of potential energy terms implies $m_G m_h = m_P^2/a$, thus in this case $A = a$. The simplest choice $m_h = m_e$ implies the following $m_G$, where $m_N = a m_e$ is the Nambu mass, a quasi-quantum in Particle Physics [23]:

$$\begin{align*}
m_h &= m_e \quad ; \quad m_G = \frac{m_P^2}{m_N} \quad ; \quad A = a \ .
\end{align*}$$

(8)

This last mass is $m_G \approx 3.7939 \times 10^{12}$ kg, whose corresponding Special Non Local Length (4) is:

$$d_0 = L_{SNL}(m_P^2/m_N) \approx 3.051 \times 10^{-96} \ \text{meter} .$$

(9)

This is the Cosmic Space Quantum breaking the "Planck Wall" by a factor $10^{61}$, which has been associated to the Cosmos holographic radius $R_{hol}$, defined by the Bekenstein-Hawking Entropy formula [7], where the Planck Length $l_P \equiv (G\hbar/c^3)^{1/2} \equiv L_{SNL}(m_P)$ is a basic holographic length [36]:

$$\pi \left( \frac{R_{hol}}{l_P} \right)^2 = 2\pi \frac{R_{hol}}{d_0} \Rightarrow R_{hol} = 2L_{SNL}(m_N) \approx 18.105 \ \text{Giga light-year (Glyr)},$$

(10)

which is a typical cosmic length. The Cosmos radius $R_C$ has been defined by the natural mono-chromatic holographic extension:

$$\pi \left( \frac{R_{hol}}{l_P} \right)^2 = 2\pi \frac{R_{hol}}{d_0} = 2\pi \frac{R_C}{l_P} \Rightarrow R_C = 2L_{SNL}(m_N^2/m_P) \approx 9.075 \times 10^{86} \ \text{m},$$

(11)

which is related to the above Haas-Bohr radius $r_{HB}$ by the 10 ppm quasi holographic formula:

$$\frac{4\pi^2}{3} \left( \frac{p}{n_e} \right)^2 \frac{R_C}{r_{HB}} \left( \frac{r_{HB}}{l_P} \right)^3 = a^\alpha ,$$

(12)

showing a significative role of the Electric constant $a$, implying it is a calculation basis [39], in concordance with the quantification of Space-Time [13].

Introducing $\lambda_{hol} = \sqrt{l_P L_{SNL}(m_N)} \equiv L_{SNL}(\sqrt{m_N m_P})$, it enters an extension of the Holographic Principle:

$$\left( \frac{\lambda_{hol}}{l_P} \right)^2 = \frac{l_P}{d_0} ,$$

(13)

which is the above basic holic form at the basis of the Topological Axis [34].

This wavelength $\lambda_{hol}$ enters also the following dramatic holographic relations involving the wavelengths of the Electron, the Hydrogen, and the Weak Bosons $W$ (0.3 ppm):

$$4\pi \sqrt{3} \frac{\lambda_{hol} \lambda_W}{\lambda_H} = \frac{4\pi}{3} \left( \frac{\lambda_e}{\lambda_H} \right)^3 \ ,$$

(14)
\[ \pi \approx 3 + (7 + 1/\sqrt{2} \times 137)^{-1}. \] The involved term \( HW \approx Ze^2 (\pi q/\pi)^\sqrt{(p/pw)} \) (30 ppb), implying the corrected \( \pi \) value defined by the adimensional electric charge \( \beta = (4\pi q/a)^{1/2} = W \sin \theta / H^0 \), with \( \cos \theta = W / Z \). This means that the heart of Physics is the following holographic relations:

\[ a + 1 \cdot 4\pi (e^\pi)^2 \approx 4\pi e^\pi \approx 4\pi a^{3/2} L_n (a + 1) \approx 1/2 \sqrt{aa_w / H^0} \approx F / 10a_s \approx 9\mu \sqrt{aa_w} / W \sqrt{137}, \] (15)

where the tau mass appears in the relations \( 9\mu / \tau \approx g_1 / g_2 \equiv \tan \theta \) and \( W^2 \approx 2\pi Z \sin \theta (H^0/a)^{1/2} \).

These formula leads to the following ppb relation, showing a role of the geometrical factor 4\( \pi \):

\[ \left( \frac{P}{a_w} \right)^3 \approx \left( \frac{4\pi}{\sqrt{a}} \right)^3 (pH^2)^5 / 2 \approx \frac{aW}{137Z} (pH)^5 \] (16 ppm), (16)

The Table 2 shows the symmetry between the Nambu mass \( m_N \) and the Planck mass \( m_P \), whose large value is the source of the "Hierarchical Problem" [31].

Now \( \lambda_{hol} = \lambda_{CMB} / 2a_s^2 \), where \( 2a_s^2 \approx a_s \) trying to 0.3 % the strong coupling \( a_s \) and the nominal wavelength \( hc / kT_{CMB} \) of the Cosmic Micro-onde Background (CMB), whose source is lacking in the steady-state cosmology [6], [16]. The simplest hypothesis is that the above Cosmos is this source. Indeed, the Wien CMB wavelength \( \lambda_{CMB} \) enters:

\[ 4\pi \left( \frac{R_{hol}}{\lambda_{CMB}} \right)^2 \approx e^a \] (0.1%). (17)

This perfect holographic formula suggests that the CMB would be coherent, meaning it brings information. This could be the real significan of the CMB Anisotropy Statistics [1].

### 3 The Gravitational Dihydrogen

The Haas method was already applied to the special three-body gravitational dihydrogen [34, p.391):

\[ n \frac{\hbar v_n}{L_n} = \frac{Gm_pm_H}{L_n} = m_e v_n^2, \] (18)

The comparison with the above Haas equation implies the substitution: \( a \rightarrow a_G = m_H / m_p m_H \), corresponding to the following \( m_G \) value:

\[ m_h = m_e; \quad m_G = m_{bc}; \quad A = a_G \] (19)

where \( m_{bc} = m_p m_H / m_e \) is close to the DNA bi-codon mass [34], (DNA = Desoxyribo Nucleic Acid), which shows a central position in the Topological Axis [34], corresponding to the dimension 16. Indeed the corresponding topological term \( e^{16} \) is close to \( pH R_{hol} / RH_2 \), and, more precisely, to \( 2n^4_1 / a^3 \) (0.04 %).

For \( n = 1 \), this Haas-Sanchez radius \( RH_2 \) shows a direct Electricity-Gravitation symmetry, by respect to the Reduced Electron wavelength \( \lambda_e = h / m_e c \), where
\[ m_0 = (m_em_p m_H)^{1/3} \]

\[ r_{HB} = a \lambda_e = \frac{h}{m_e c} \]

\[ R_{H_2} = a_G \lambda_e = \frac{h^2}{G m_e m_p m_H} \equiv L_{SNL}(m_0) \approx 6.906 \text{ Glyr}. \] (20)

Note that \( a \) and \( a_G \) are very close to the last two terms of the Combinatorial Hierarchy \( H_3 \) and \( N_L + 1 \), with \( N_L = 2^{127} - 1 \), the Lucas Number [4]. It was noted that the implied Mersenne numbers 3,7,127 relates the gravitational main large number \( P = m_P/m_e \) with the weak parameter \( a_w = \left( \frac{m_F}{m_e} \right)^2 \) and \( \sqrt{\alpha} \) in the following relation, and the rearranging of the exponents shows the above neutron ratio:

\[ P^{3+7} \approx a_w^7 \sqrt{\alpha}^{7+127} (57 \text{ ppm}) \Rightarrow \frac{P}{a_w} \sqrt{\alpha} \approx \left( \sqrt{\alpha}^{127}/P^3 \right)^{1/7} \approx n_1^4. \] (21)

In the gravitational dihydrogen radius formula \( R_{H_2} \), the speed \( c \) is eliminated: for this reason, a precise approximation was immediately guessed by the \( c \)-free "dimensional analysis", the so-called Three Minutes Formula, from the ternary symmetry Electron-Proton-Neutron (Closed Letter to the Paris Science Academy, March 1998) [33] (Table 2). Indeed, the speed \( c \) is far too small a speed to explain the cosmic coherence manifested by the Foucault pendulum (Mach Principle).

The Special Non-Local Time \( T_{SNL}(m_0) \) is very close (0.9 %) to the time given by the triplet : \( (\hbar, G_F, \rho_{cr}) \), with the Fermi constant \( G_F \) and the critical steady-state density \( \rho_{cr} = 3c^2/8\pi G R^2 \) with horizon radius \( R = 2R_{H_2} \) : it is \( h^2/G_F^{5/2} \rho_{cr}^{3/2} \), introducing the steady-state Universe of radius \( R = 2R_{H_2} \) discussed now.

4 The Steady-State Universe re-established

A salient feature of the observed Universe is its critical character, relating its horizon radius \( R \) with its mass by \( R = 2GM/c^2 \). However, in the initial "flat universe" model [12], the total mass \( M \) is only matter, while in the present \( \Lambda CDM \) standard model, it is separated between a material part, with relative density \( \Omega_m \), and a so-called "dark energy" part with relative density \( 1 - \Omega_m \) [1].

We have noted that \( \Omega_m \) is compatible with 3/10, which is both the relative density of the classical gravitational energy of a critical homogeneous ball and the relative density of the steady-state non-relativist recession kinetic energy [36].

While the standard cosmology uses an ad-hoc inflation to justify this observed critical condition, we consider rather the Universe as a particle (Topon) in the above Cosmos, with the Topon wavelength \( \lambda_M \equiv h/Mc = 2h G/Rc^2 \equiv 2l_p^2/R \).

Then, the critical condition results from the Bekeinstein-Hawking entropy holographic relation, as above (Eq. 10), where the Topon appears as a Length Quantum, since the wavelength \( \lambda_m \) associated for any particle of mass \( m \) is a whole multiple \( n_m \) of the Topon, in conformity with the Field Quantum Theory.

The geometrical interpretation is clear : it is a sphere area described by a whole
number of sweeping circles, illustrating the fact that multiplication is a series of additions, an approximation supporting the vastness of the world [36]:

$$4\pi \left( \frac{R_{HB}}{lp} \right)^2 = \pi \left( \frac{R}{lp} \right)^2 = 2\pi \frac{R}{\lambda_M} \equiv 2\pi \frac{\lambda_m}{\lambda_m} \Rightarrow M = \frac{Rc^2}{2G} \equiv \frac{R_{HB}c^2}{G},$$

(22)

identifying twice the above Haas-Sanchez’s gravitational radius $R_{HB}$ with $R$, the steady-state Universe horizon radius, which is also the limit of a theoretical star radius when its number of atoms shrinks to one [10], a central length in astrophysics which induces a coefficient-free Universe Mass Relation:

$$R = 2 \frac{\hbar^2}{Gm_e m_p m_H} \approx 13.812 \text{ Glyr} \Rightarrow M = \frac{m_p^4}{m_e m_p m_H}.$$  (23)

This is called the Machian Formula. Recall that the standard General Relativity in unable to explain the Mach Principle. With the effective electron mass $m'_e = m_e m_p/(m_p + m_e) \equiv M/n_e$, this introduces $n_e$, the Universe Electron Quantum Number, canonical in Quantum Field Theory. The Eddington’s Electron-Proton symmetry shows up in the following resolution of the so-called Large Number Problem, where $\lambda_{pH}$ is the geometrical mean of the reduced wavelengths of the proton and Hydrogen:

$$\frac{m_p^2}{m_p m_H} \equiv \sqrt{n_e} \equiv \frac{R}{2\lambda_{pH}},$$

(24)

which is extended by very precise dramatic expressions involving the symmetry between the weak bosons of masses $m_W = W m_e$ and $m_Z = Z m_e$, specifying the known relation $a_G \approx W^8$ [9]:

$$\sqrt{n_e} \approx (WZ)^4 \frac{aZ}{W} \approx (\frac{m_p^2}{m_p m_H})^7 \left( \frac{aZ}{W} \right)^3,$$

(25)

where appears as well a Planck-Fermi symmetry, enlighting the “Hierarchical problem” [31].

In the Topological Axis, the above Topon corresponds to the orbital number $k = 7$, while the gauge bosons corresponds to $k = 3$ (weak bosons W,Z) and $k = 5$ (strong GUT boson X), letting a single place $k = 1$ for a non-standard massive Gluon [36].

The particular values of the topological function $f(k) = e^{2k+1/2}$ for $k = 7$ and 6 show up in (0.06%):

$$\sqrt{n_e}/153 \approx \sqrt{f(7)} \equiv f(6) \approx 6R/\lambda_e.$$

(26)

implying that $m_p/m_e \approx 1836 \equiv 6 \times 2 \times 153$, the Diophantine approximation of the Wyler formula $p_W = 6\pi^5$ [43]. This is a dramatic confirmation of the Topological Axis pertinence. The spectroscopic number associated to $k$ is $2(2k+1)$, where 2 is the spin degeneracy and $2k + 1$ the number of magnetic states [37]. For $k = 6$, this is 26, the canonical dimension in the bosonic string theory [31].

This invariable Universe radius $R \approx 13.812$ Giga light-year (Glyr) of Eq. (23) is close to $c$ times the variable standard Universe age. So the standard theoretical approach is correct, but not its Big Bang interpretation : a confusion
is made between Time and Length, which readily occurs by putting \( c = 1 \).

Moreover, the corresponding Hubble constant \( c/R \) is 70.793 (km/s)/Mpc, which is compatible with both the WMAP and the Carnegie-Chicago Hubble Program recent direct measurements (Table 3).

The above Universe gravitational potential energy (3/10)\( M_c^2 \) shows a Neutron Quantum Number (the number of neutron masses) very close (0.05 \%) to the large Eddington Number [36]. So it has nearly anticipated the correct Hubble Constant value (Table 3).

The theoretical prediction [9] that \( a \) is the order of \( \ln a \) was specified in the Single Electron Cosmical Radius \( R_1 \), [38], leading to the 0.4 ppm connection: \( R_1 \approx (RR_{hol}(3))^{1/2}p_c/p_W \), with \( p_W = 6\pi^5 \) and \( p_G = m_p/\sqrt{N_L}m_e \approx 1831.530547 \), which shows the following ppb relation, a symmetric extension of \( a^2 \approx 137^2 + \pi^2 \):

\[
p^2_G \approx pH - 137^2 - \pi^2 - e^2. \tag{27}
\]

The cosmic radius \( R_c \) and the holographic mass \( M_{hol} = R_{hol}c^2/2G \) connect with the ratio \( R/\lambda_c \equiv T/t_c \) through the Cosmos-Universe couple MLT Formula (1\%) [37]:

\[
\left[ \ln \left( \frac{R_c}{\lambda_c} \right) \right]^2 \approx \left[ \ln \left( \frac{M_{hol}}{m_e} \right) \right]^2 + \left[ \ln \left( \frac{R}{\lambda_c} \right) \right]^2 + \left[ \ln \left( \frac{T}{t_c} \right) \right]^2 . \tag{28}
\]

Moreover, the Cosmos radius connects with the above radius \( R_{hol} \) and \( R \) by (-0.7 ppm and 0.6 ppm), with the deviant forms (Archimède) \( \pi_{Arc} = 22/7 \) and Ptolémée, where 17 is replaced by \( \pi/\mu := \pi_\tau/\mu = 3 + (7 + \mu/\tau)^{-1} \):

\[
\beta R_c \left( \frac{m_e}{m_p} \right)^2 \approx R_{hol} \left( \frac{W}{H} \right)^2 \approx R(2F Z^2/3)(4\pi_{Arc}/3) \left( \frac{4\pi_{Arc}/3}{\pi_\tau/\mu} \right) . \tag{29}
\]

implying the following confirmation of the Holographic Principle, stronger than the analytic \( \pi \), where \( 4\pi_{Arc}/3 \approx \sqrt{n_\pi}/4\pi p_W (\sin \theta)^2 \) (0.15 ppm):

\[
u = \frac{R_{hol}}{R} = \frac{pK}{a^3} \approx \frac{(4\pi_{Arc}/3)\lambda^3_e/\lambda_F \lambda^2_Z}{2\pi_{\tau/\mu}(\lambda_{hol}/\lambda_e)^2} . \tag{30}
\]

With Eq (13), the elimination of \( Z/W \) implies a new 0.3 ppm formula for \( R \) (Table 3). Taking account of the above relation \( WH \approx Ze^{\sigma} \), this leads to the 0.6 ppm relation:

\[
\frac{6F}{(e^2)^2} \approx \frac{(H + 1)^2}{a^3} . \tag{31}
\]

The ratio \( u = R_{hol}/R \equiv pH/a^3 \approx 1.310 841 007 \) shows high correlation with Particle Physics:

\[
u \approx \frac{\pi Z}{eW} \approx \left( \frac{F \sin \theta}{W} \right)^{1/2} \approx \frac{a_Z Z}{2F} \approx \frac{qd_2 \sqrt{a}}{e} \approx \frac{3 \times 137 \times H^{(0)}}{\pi a Z} \approx \frac{4\pi g_0}{\sqrt{a}} . \tag{32}
\]

where \( g_3 = g_1 g_2/g_0 \), with \( g_0 = \frac{ppG}{2a^3} \), confirming the holistic character of Quantum Cosmology.

7
5 The Cosmic Microwave Background (CMB)

This Universe radius $R = 2R_H$ enters a 1D-2D holographic relation: $2\pi R/\lambda_e = 4\pi \lambda_p \lambda_H/\ell_p^2$. The extension to the 3D holographic relation using $\lambda_{H_e}$, the reduced wavelength of the dihydrogen molecule $H_2$, involves the reduced wavelength of the Cosmic Microwave Background (CMB) $\lambda_{CMB} = h/c/kT_{CMB}$:

$$\frac{2\pi R}{\lambda_e} = 4\pi \frac{\lambda_p \lambda_H}{\ell_p^2} \approx \frac{4\pi}{3} \left(\frac{\lambda_{CMB}}{\lambda_{H_e}}\right)^3,$$  \hspace{1cm} (33)

leading to $T_{CMB} \approx (8G \ h^4/3\lambda_p^6)^{1/3}/k \approx 2.729$ Kelvin, which is once more, apart the holographic factor $8/3$, a $c$-free three-fold (Mass, Length, Time) dimensional analysis, giving the energy $kT_{CMB}$ from the constants $G$, $h$, $\lambda_p$. Moreover, by substituting $a_G = R/2\lambda_e$ with the above Lucas Number $N_L$, this leads to a new holographic expression (analog to the area of a 4D sphere), which gives $T_{CMB}$, compatible with the measured value $2.7255(6)$ Kelvin, which defines a mammal temperature $T_{mm} = jT_{CMB} \approx 310.50 \ K = 37.350° \ C$, with $j = 8\pi^2/\ln 2$ [34], with dramatic connections involving $T_{H,O}$, $\lambda_W$ and $\lambda_Z$:

$$N_L \approx 2\pi^2 \lambda_{CMB}^3/\lambda_e \lambda_H^2 \Rightarrow T_{CMB} = hc/k\lambda_{CMB} \approx 2.7258205 \ Kelvin
$$

$$\sqrt{R_{bol}p} \approx \lambda_{H,O} \quad ; \quad \sqrt{R_{pl}} \approx (p/H)\lambda_{CMB}/(j + 1) \quad (20 \ ppm)
$$

$$R \approx (32\beta^2/\pi^3)\lambda_{CMB}^3/\lambda_Z^2 \quad (1.5 \ ppm)
$$

$$\lambda_{CMB}^3/\lambda_W \lambda_Z \approx (R_{bol}/2\beta^2)^3/N_L \lambda_e \quad (0.6 \ ppm)$$  \hspace{1cm} (34)

The standard Cosmology predicts a Neutrino background with temperature $T_{CMB} = T_{CMB}/(4/11)^{1/3} \approx 1.946 \ Kelvin$. The total CMB photon number is $n_{ph} = ((3/\pi)(R/\lambda_{CMB}))^3$, exceeding the total Hydrogen number $n_H = M/m_H = R_{H}/2\ell_p^2$. But, in term of energy, the matter dominates. So one must consider also the ratio between the critical energy density $u_{cr} = 3c^4/8\pi GR^2$ and the total background energy density $u_{ cmb +cnb} = y \ u_{ cmb}$, with $y = 1 + (21/8)(4/11)^{3/2} \approx 1.681322$ [44] and $u_{ cmb} = (\pi^2/15)hc/\lambda_{CMB}^3$. We observed that these ratios are tied by an Eddington’s type relation [38]:

$$(2n_{ph}/n_{H})^{1/2} \approx u_{cr}/u_{ cmb +cnb} \Rightarrow T_{CMB} \approx 2.724 \ Kelvin .$$  \hspace{1cm} (35)

This confirms the existence of the Neutrino background. Now assuming that the total background Photon + Neutrino is the result of an on-going Hydrogen-Helium transformation, producing $e_{He} = 6.40 \times 10^{14}$ Joule by kilogram of Helium, i.e. an efficiency $\epsilon_{He} = e_{He}/c^2 \approx 1/140$. The Helium mass density is $Y \rho_{ bar}$; with the standard evaluation of baryonic density $\epsilon_{bar} = \rho_{bar}/\rho_{cr} \approx 0.045$ and $Y \approx 0.25$ [1], this leads to:

$$(\lambda_{CMB}^3/\ell_p R)^2 \approx 8\pi^3 y/45Y\epsilon_{bar}e_{He} \approx 1.15 \times 10^5 \Rightarrow T_{CMB} \approx 2.70 \ Kelvin .$$  \hspace{1cm} (36)

In the standard model, the Universe age in far too small to explain a large Helium large density resulting from stellar activities [8]. Thus, it is not a real problem in the steady-state model.
6 The Electron and the Kotov Non-Local Period

This study confirms the central role of $\lambda_e$, the unit length in the Topological Axis and in the Single Electron Universe [34]. So we look for a Diophantine series giving it for $n = 1$. This means:

$$\lambda_e \equiv \hbar/m_e c = h^2/Gm_G m_h^2 \Rightarrow A \equiv m_p^2/m_G m_h = m_h/m_e$$  \hspace{1cm} (37)

so that the fundamental ($n = 1$) energy is: $E \equiv m_h c^2/A = m_e c^2/A$. There is an elimination of $c$ by considering the term $A^2$ as the product of the above gravitational constant $a_G = \hbar c/Gm_G m_h$ and the electro-weak one $a_w = \hbar^3/e G_F m_e^2$ [9], where $G_F$ is the Fermi constant:

$$A^2 = a_G a_w \Rightarrow E = m_e c^2/\sqrt{a_G a_w}$$  \hspace{1cm} (38)

with $t_e \equiv h/m_e c^2$ the electron period, this corresponds to the time:

$$t_e \sqrt{a_G a_w} \approx 9600.60 \text{ s}.$$  \hspace{1cm} (39)

The identification with the Kotov $P_0$ period $t_K \approx 9600.606(12) \text{ s}$ [19], [20] corresponds to $G \approx 6.6754527 \text{ SI}$, specified to $10^{-8}$ by the above Single-Electron Radius $R_1 \approx (4\pi p/p)^2a_w c t_K$ [36] and consistent with the BIPM measurements [29], but at $6 \sigma$ from the official value, an unusual mean between discordant measurements. With the Fermi mass $m_F = m_e \sqrt{a_w}$, close to the mean nucleotide mass [34], the Lepton Mu mass $m_\mu = R_{hol}/R$, the critical density $\rho_{cr} = 3c^2/8\pi G R^2$, and $m_{GF} = (m_F m_F)^{1/2}$, this defines our optimal strong coupling $a_s$, in the natural process of optimal correlations [39], where $\pi_\tau/\mu = 3 + (7 + \mu/\tau)^{-1}$, $1/90 = 1 + g_1^2 + g_2^2 \approx pp_G/2a^3$, with $g_1 = Z \sin\theta/H^{(0)}, g_2 = W/H^{(0)}$ [36]:

$$m_G = m_e m_p m_H/m_e^2 ; \sqrt{G_G} \equiv (h/m_G)^2 = (h/\sqrt{m_p m_H})(\lambda_e^2/t_K)$$
$$m_h/m_p = m_F/\sqrt{m_p m_H} \equiv m_p^2/m_e m_N \equiv 2\pi a_s m_p m_H/m_e m_F$$
$$G_F/Gm_p^2 c_p \approx a m_p m_p / m_e^2 \hspace{0.5cm} (0.2\%); \sqrt{pW/m_\tau \lambda_e^2 c_p^2 / (2\pi)^2 R c t_K} = 0.8 \text{ ppm}$$
$$h/(G_F \rho_{cr})^{1/2} \approx \lambda_e^2/u^{1/16} l_P \hspace{0.5cm} (0.01\%); (4\pi q/3)(aa_w^3) \approx 4\pi_\tau/\mu (r_e/l_P)^2$$
$$\frac{(Z/H^{(0)})^2}{1/g_0 - 1} \approx \frac{a/137\sqrt{\beta}}{\pi q/\pi} \approx 0.4 \text{ ppm} \hspace{0.5cm} (3\text{ppb})$$

exhibiting a symmetry between canonical area speeds. Note that $2c t_K \approx L_SNL(m_{bc}),$ confirming once more the bi-codon mass, which enters also a relation involving the Cosmos, the Photon and Graviton masses [36] (Table 3).

Moreover, with the precise variant (0.14 ppm) of the Golden Number: $\Phi_0 = P/(a_w H)^3 \approx ((4\pi/3)(H/p)^2)^{1/3}$, one observes (15 and 74 ppm):

$$\frac{\text{LSNL}(m_{GF})}{\tau_{HB}} \equiv \frac{P}{F^{5/2}} \frac{1}{a} \approx \Phi_0^2$$
$$c T_{SNL}(m_{GF}) \equiv l_P \frac{P}{F} \approx \left(\frac{R_{hol} \lambda_e}{2}ight)^{1/2} \frac{1}{d_e^2} \hspace{1cm} (41)$$

9
This specifies the holographic relations $a^2 \approx (4\pi/3)p^{3/2}$ and $F^5/P a^3 \approx \eta$, with
\[ \eta = 1 + 2/(3 \times 139) \text{ (ppb precision)} \] [37], where 139 is the complete Atiyah form [2], adding the dimensions of the four algebra (octonion, quaternion, complex, real): $139 = 137 + 2 = 2^7 + 2^4 + 2^3 + 2^0 \approx i - i\pi$, and $3 \times 139 + 2 = 419$, the positive crystallographic number [41] in the superstring dimensions 10D and 11D [31], see Table 7 in [37]. Moreover, $T_{SNL}(m_{GF}) \approx 19.14$ ms, typical of the Human nervous system, and the third octave down the flat La tone (Lab) for $La(A_4) = 442.9$ Hz, an anthropic argument far more pertinent and precise than the rough standard ones, principally based on a cosmic Big Bang scenario [9].

7 Discussion

The Pythagoras Principle stating that all is ruled by whole numbers has been forgotten during centuries. This resulted in the failure of Poincaré to resolve the apparently most difficult problem of modern physics, the apparition of quanta [27]. He ought to have remember that the more difficult is a problem, the more basic feature must be revisited, in particular the Kepler laws, leading to an elementary Diophantine equation, of trivial resolution, which implies directly an angular momentum quantum, identifiable with the reduced Planck’s constant $\hbar$. Interestingly enough, in the same London conference (p. 102-103), Poincaré explained that cosmology cannot be entirely founded on differential equations. Since the main scientific criteria is the repeatability of experiments, this implies the Perfect Cosmological Principle founding the steady-state model [6], and Poincaré could have concluded that cosmology, hence the whole physics, must be tied to the Number Theory [37].

This approach leads to the Diophantine Coherence Theorem (DCT) which has the same structure than the Hass formulation for the Hydrogen atom spectrum problem. This shows that the real invariant quantity is the Frequency, so that the Energy conservation would mean a Frequency Accordance, or “Coherence Principle”, mandatory in Practical Holography, and conform with the Harmony Principle of Pythagoras, the father of Natural Philosophy, the very root of Science. This confirms the pertinence of the Quantum Field Theory, where any Particle Field is defined by a whole number, entering the Holographic principle in the revisited critical steady-state Universe. In particular, both the Electron Quantum Number $n_e$ and the Neutron Quantum Number $n_t$ play a central role. The Universe Length Quantum (Topon) is associated to a Universe Time quantum (“Chronon” $t_M = \lambda_M/c$), which may be looked as the period of the Permanent Bang matter-antimatter oscillation [35].

Among the two main cosmological models, the steady-state one is by far the most easily refutable, so the most scientific, in the Popper sense [28]. It is why it was thought as being refuted by hasty observations, however these so-called refutations were much debated [21]. This article firmly re-establishes the steady-state cosmology. It is now mandatory to measure more precisely the Universe Temperature at any distance, in order to compare it with the microwave background one. The galaxy recession means not at all an Universe expansion: it has been shown that the repulsive force explains at last the acceleration of the galaxy recession, the so-called "dark energy" being a trivial problem, resolved by simply considering the no-relativistic potential and kinetic galaxy...
population energy. This repulsive force exceeds the Newtonian attraction for larger distances than a million years, typical of a galaxy group, which shows no internal recession [36], and the renewal of matter inside such a group could be attributed to the giant black holes.

The Cosmic Length Quantum breaks the "Planck wall" by the factor $10^{-61}$. The DCT shows that the Haas-Bohr radius is a secondary length quantum, while the Universe itself appears as a ternary length quantum in the Cosmos, defined by the Holographic Principle where the Planck length is an intermediate holographic length, instead of the standard Length quantum. The unifying length quantum is the reduced Electron Wavelength which shows up in the DCT, the Single Electron Model and the Cosmos-Universe MLT Formula.

The Kotov non-local period induces a symmetry between gravitation and electroweak interaction. The Kotov-Lyuty Non-Doppler oscillation plays a central role, but was overlooked: it is however a clear sign of the non-local character of Quantum Cosmology which is patent in the Foucault pendulum. It is mandatory to check the Lyuty Non-Doppler Quasar measurements [19]. The standard speed limit $c$ excludes any explanation of the wave packet reduction phenomena, which requires a non-local or tachyonic Physics. So, it is logical that the bosonic string theory, which introduces tachyon, is confirmed by the Diophantine Topological Axis. Indeed, the central bosonic dimension $d = 26$ corresponds unambiguously to the non local universe whose radius is given by the Machian Formula (23).

The Planck mass enters naturally in the DCT, while incompatible with the standard in Particle Physics. However, the standard spin formulation rejoins our conclusion that the reduced Planck constant $\hbar$ plays a more fundamental role than $\hbar$.

8 Conclusions

On the basis of the invariance of physical laws, the Mach Principle cannot enter an evolutionary Universe, so only the steady-state model is really compatible with scientific cosmology [6]. The invariant Universe radius is tied to 33 formula in the Table 3, whose 22 are in the $10^{-4}$ precision defining $R \approx 13.812$ Glyr, in the same way that Jean Perrin [24] collected 14 formula to demonstrate definitely the atom existence. This is an historical parallel between the quantification of matter and the quantification of space-time, a complete rehabilitation of the historical numerical empirical method, which has been greatly overlooked by an excess of formalism.

The International System must come back the three basic units (instead of seven): Mass, Length, Time. Also, it must define electronic units by using only the electrical constant $a$. In particular, Particle Physics must suppress the use of eV unit, and Astrophysics suppress the Parsec unit. Also the two intercorrelated measures, the non-local Kotov-Lyuty period and $G$, whose standard value is now erroneous by 6 $\sigma$ must be revisited.

The Quantum Physics is not separated from Cosmology, which gets definitely the status of a real science. The object "Universe" is well defined: inside an external Cosmos, it is both a mono-atomic star, a quasi-homogeneous black hole, a gravitational molecule, a nuclear fusion reactor, a thermal machine and a particle, the ultimate gauge boson, as shown by the Topological Axis. The latter
rehabilitates the bosonic string theory, but the connection with the Periodic Table must be explained [37].

This is a decisive confirmation of the Holographic Principle, including canonical deviations from the mathematical $\pi$, including the one associated to the electrical charge. Also the DNA bi-codon mass is central, so the DNA could be an helix-hologram, opening the way towards bio-computing [25].

The $c$-free Elementary Non-Local Three Minutes Formula giving the Universe half-radius is now fully established: this means a tight harmony between the Universe and Human Consciousness, a special and decisive manifestation of the real Permanent Anthropic Principle.

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tance and radial velocity among extra-galactic nebulae. A. J., 61:97–162,
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sant rendant compte de la vitesse radiale des nebuleuses extra-galactiques.
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Found, for the Det. of the Hubble Cst and Stronger Evidence for Physics


<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Symbol and Value</th>
<th>Unit</th>
<th>ppb (10^{-6})</th>
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</thead>
<tbody>
<tr>
<td>Optimal Gravitation Constant [36]</td>
<td>$G \approx 6.67434272 \times 10^{-11}$</td>
<td>kg$^{-1}$m$^3$s$^{-2}$</td>
<td>2.7 × 10³</td>
</tr>
<tr>
<td>BIPM Gravitation Constant [29]</td>
<td>$6.674548(18) \times 10^{-11}$</td>
<td>kg$^{-1}$m$^3$s$^{-2}$</td>
<td>1.7 × 10⁶</td>
</tr>
<tr>
<td>Official Gravitation Constant</td>
<td>$6.67430(15) \times 10^{-11}$</td>
<td>kg$^{-1}$m$^3$s$^{-2}$</td>
<td>exact</td>
</tr>
<tr>
<td>Planck constant $h = 2\pi \hbar$</td>
<td>$h \approx 6.62607015 \times 10^{-44}$</td>
<td>J s</td>
<td>exact</td>
</tr>
<tr>
<td>Optimal Fermi Constant</td>
<td>$G_F \approx 1.43580499 \times 10^{-52}$</td>
<td>J m$^3$</td>
<td>exact</td>
</tr>
<tr>
<td>Speed of light in vacuum</td>
<td>$c = 299792458$</td>
<td>m s$^{-1}$</td>
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<tr>
<td>Electrical Constant</td>
<td>$a_c \approx 137.035999084(21)$</td>
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<td>0.15</td>
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<tr>
<td>Single Electron Universe radius</td>
<td>$R_e \approx 1.492365473 \times 10^{-8}$</td>
<td>m</td>
<td>-</td>
</tr>
<tr>
<td>Electron Excess Magnetic moment</td>
<td>$d_e \approx 1.00115965218096$</td>
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<tr>
<td>Electron mass</td>
<td>$m_e \approx 9.10983877015 \times 10^{-31}$</td>
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<tr>
<td>Electron Classical Radius $r_e = \lambda_e / a$</td>
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<td>0.3</td>
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<tr>
<td>Planck/Electron mass ratio $P = m_P / m_e$</td>
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<td>-</td>
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<tr>
<td>Reduced Electron Wavelength $\lambda_e = h / m_e c$</td>
<td>$\lambda_e \approx 3.8615922675 \times 10^{-14}$</td>
<td>m</td>
<td>0.3</td>
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<tr>
<td>Proton/Electron mass ratio $p = m_p / m_e$</td>
<td>$p \approx 1836.15267343$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Wyler Proton/Electron mass ratio $p_W = 6\pi^2$ [43]</td>
<td>$p_W \approx 1836.110819$</td>
<td>-</td>
<td>exact</td>
</tr>
<tr>
<td>Neutron/Electron mass ratio $n_e = m_n / m_e$</td>
<td>$n_e \approx 1838.6836617$</td>
<td>-</td>
<td>0.5</td>
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<tr>
<td>Hydrogen/Electron mass ratio $H = m_H / m_e$</td>
<td>$H \approx 1837.15266014$</td>
<td>-</td>
<td>0.06</td>
</tr>
<tr>
<td>Hydrogen correction factor $\beta = 1/(1 - p)$</td>
<td>$\beta \approx 1.0000006$</td>
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<td>-</td>
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<tr>
<td>Opt. Weak Coupl. Ct. $a_w = F^2 = h^2 / 8\pi G m_e^2$ [37]</td>
<td>$a_w \approx 3.28335406 \times 10^{-11}$</td>
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<tr>
<td>Official Strong Coupling constant</td>
<td>$a_s \approx 8.48(7)$ (8)</td>
<td>-</td>
<td>7.6 × 10⁹</td>
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<td>Optimal Strong Coupling Constant [37]</td>
<td>$a_s \approx 8.434502914$</td>
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<td>-</td>
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<tr>
<td>Optimal Muon/Electron mass ratio $\mu = m_\mu / m_e$ [36]</td>
<td>$\mu \approx 206.768260$</td>
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<td>-</td>
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<tr>
<td>Optimal Tau/Electron mass ratio $\tau = m_\tau / m_e$ [36]</td>
<td>$\tau \approx 3477.441701$</td>
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<td>-</td>
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<tr>
<td>Opt. Higgs Boson mass ratio $H^{(0)} = m_{H^{(0)}} / m_e$ [37]</td>
<td>$H^{(0)} \approx 492$</td>
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<td>-</td>
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<tr>
<td>W-boson mass ratio $W = m_W / m_e$</td>
<td>$W \approx 157340.1093$</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Z-boson mass ratio $Z = m_Z / m_e$</td>
<td>$Z \approx 17851.7520$</td>
<td>-</td>
<td>-</td>
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<tr>
<td>Adimens. El. Charge $q = W \sin \theta / H^{(0)} = \sqrt{4\pi a_e / a}$</td>
<td>$q \approx 0.3029732863$</td>
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<td>-</td>
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<tr>
<td>Boltzmann Constant (conversion factor)</td>
<td>$k = 1.380649010^{-23}$</td>
<td>J K$^{-1}$</td>
<td>exact</td>
</tr>
<tr>
<td>Measured CMB temperature</td>
<td>$T_{\text{CMB}} \approx 2.7255(6)$</td>
<td>Kelvin</td>
<td>-</td>
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<tr>
<td>Optimal CMB Temperature</td>
<td>$T_{\text{CMB}} \approx 2.725820138$</td>
<td>Kelvin</td>
<td>-</td>
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<tr>
<td>Optimal CMB Wien wavelength</td>
<td>$\lambda_{\text{W}} \approx 1.063082472 \times 10^{-4}$</td>
<td>m</td>
<td>-</td>
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<tr>
<td>Optimal CMB reduced wavelength $\lambda_{\text{CMB}} = h c / k T_{\text{CMB}}$</td>
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<td>m</td>
<td>-</td>
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<tr>
<td>Optimal CMB Temp. $T_{\text{CMB}} \equiv T_{\text{CMB}}(11/4)^{-1/4}$</td>
<td>$T_{\text{CMB}} \approx 1.945597$</td>
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<tr>
<td>Water Triple Point Temperature</td>
<td>$T_{\text{H}_2O} \approx 273.16$</td>
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<td>Optimal critical density $\rho_c = 3e^2 / 8\pi G a_e^2$</td>
<td>$\rho_c \approx 9.41179317 \times 10^{-47}$</td>
<td>kg m$^{-3}$</td>
<td>-</td>
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<td>Kotov $I_K$ period $I_K$</td>
<td>$I_K \approx 9600.600(12)$ [20]</td>
<td>s</td>
<td>1200</td>
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Table 2: Values of the DCT Fundamental (n = 1) Radius $h^2/Gm_Gm_h^2$ for specific values of $m_G$ and $m_h$. Holographic ratio $u = R_{hol}/R$ approx 14.310 841 007.

Nambu mass: $m_N = am_e$. Bicodon mass $m_{bc} = m_p m_H/m_e$. Photon mass $m_{ph} = h/c^2 t_K \approx 1.2222 \times 10^{-55}$ kg. Graviton mass: $m_{gr} = m_{ph}/a_w \approx 3.7223 \times 10^{-67}$ kg [36].

<table>
<thead>
<tr>
<th>$m_G$</th>
<th>$m_h$</th>
<th>Length</th>
<th>Symbol</th>
<th>Precision/offset</th>
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<td>$m_P/m_N$</td>
<td>$m_P/m_N$</td>
<td>Cosmic Space Quantum</td>
<td>$d_0$</td>
<td>exact</td>
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<td>$m_P/m_0$</td>
<td>$m_P/m_0$</td>
<td>Universe Space Quantum (Topon)</td>
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<td>exact</td>
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<td>$m_{bc}/a_w$</td>
<td>$m_e\sqrt{a_w a_G}$</td>
<td>Reduced Electron Wavelength</td>
<td>$\lambda_e$</td>
<td>exact</td>
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<td>$m_P/m_N$</td>
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<td>Hass-Bohr radius $r_{HB} = a\lambda_e = r_B/(1 + 1/p)$</td>
<td>$r_{HB}$</td>
<td>exact</td>
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<tr>
<td>$a^2 m_P$</td>
<td>$\sqrt{m_p m_{N}}$</td>
<td>Background Wien Wavelength</td>
<td>$\lambda_W$</td>
<td>$3.2 \times 10^{-4}$</td>
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<tr>
<td>$m_{bc}$</td>
<td>$m_{bc}$</td>
<td>Twice Kotov Length</td>
<td>$2l_K$</td>
<td>$6.3 \times 10^{-3}$</td>
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<tr>
<td>$m_{H_g}$</td>
<td>$m_{H_g}$</td>
<td>$R\lambda_e/\lambda_{CMB}$</td>
<td>$R_{H_g} \equiv R/2$</td>
<td>exact</td>
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<tr>
<td>$m_{bc}$</td>
<td>$m_e$</td>
<td>Half Universe Radius</td>
<td>$R_{H_u}$</td>
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<td>$m_N$</td>
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<td>Half Holographic Cosmos radius</td>
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<td>$m_{N}/m_P$</td>
<td>$m_{N}/m_P$</td>
<td>Half Cosmos Radius</td>
<td>$R_{C}/2$</td>
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<tr>
<td>$u \times m_{bc}$</td>
<td>$\sqrt{m_{ph} m_{gr}}$</td>
<td>Cosmos radius</td>
<td>$R_{C}$</td>
<td>$1.7 \times 10^{-3}$</td>
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Table 3: Implication of Eddington Number ($N_E = 136 \times 2^{256}$) and Holophysics formula for the invariant Hubble radius $R \approx 13.812$ Giga light-year (Gly = 1 billion light-year, Human year, 365.25 days) and the corresponding Hubble constant $H_0 = c/R$, which uses the length unit Megaparsec, compared to the main measurements. Lucas Number $N_L = 2^{127} - 1$. Topological Function $f(k) \equiv 2^{k+1/2}$. Holographic ratio $\alpha = R_{\text{hol}}/R$. Mammal temperature $T_m \equiv hc/k\lambda_m = \beta T_{\text{CMB}}$, with $\beta = 8\pi^2/112 [34]$. The optimal WZ value (Table 1) is defined from the identification to the Central Formula $R = 2 \times$ gravitational $H_2$ radius, which is also $2 \times$ the "Three Minutes Formula" (closed draft 1998) where the neutron mass is replaced by the hydrogen mass. The last Euler idenone number is $s_{65} = 1848$. For comparison, the so-called standard "Universe Age" is also presented, with unit in the $c$ ratio (Gyr = $1 \times 10^9$ year).

<table>
<thead>
<tr>
<th>Date</th>
<th>Source</th>
<th>$R = 2GM/c^2$</th>
<th>Hubble radius</th>
<th>Hubble Cst. km s$^{-1}$/Mpc</th>
<th>Univ. &quot;Age&quot; Gyr</th>
</tr>
</thead>
<tbody>
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<td>1945</td>
<td>Eddington Nb. $N_E \equiv 11$; $N_E \approx (3/10)M_{mp}/m_{H}m_n$</td>
<td>13.812</td>
<td>70.793</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1927</td>
<td>Lemaître [22]</td>
<td>1.6</td>
<td>620</td>
<td></td>
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<tr>
<td>1929</td>
<td>Hubble [17]</td>
<td>1.8</td>
<td>540</td>
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<td></td>
</tr>
<tr>
<td>1956</td>
<td>Humason, Maydal and Sandage [18]</td>
<td>5.4</td>
<td>180</td>
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<td></td>
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<tr>
<td>1958</td>
<td>Sandage [40]</td>
<td>13</td>
<td>75</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1998</td>
<td>$2R^2/Gm_p m_n m_n$ TWICE &quot;3 MN FORMULA&quot;</td>
<td>13.800</td>
<td>70.852</td>
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<tr>
<td>2000</td>
<td>$2R^2/Gm_p m_p m_n$ [33]</td>
<td>13.800</td>
<td>70.852</td>
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<td></td>
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<tr>
<td>2000</td>
<td>$2N_L \lambda_s$ [33]</td>
<td>13.899</td>
<td>70.400</td>
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<td></td>
</tr>
<tr>
<td>2017</td>
<td>$(WZ)^4(\lambda_s \lambda_H)^{1/2}$ [9] [34]</td>
<td>13.812</td>
<td>70.793</td>
<td></td>
<td></td>
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<tr>
<td>2017</td>
<td>$\lambda_s f(0)/6$ [34]</td>
<td>13.821</td>
<td>70.747</td>
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<tr>
<td>2017</td>
<td>$(2 \lambda_s/3) (\lambda_{CMB}/\lambda_{H})^{1.5}$ [34] Holography Eq. (30)</td>
<td>13.857</td>
<td>70.360</td>
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<td>2017</td>
<td>$\lambda_s (3/2)^{1/2}/u$ [34] From $R_{\text{hol}}/\lambda_s \approx (3/2)^{1/2}$</td>
<td>13.812</td>
<td>70.796</td>
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<tr>
<td>2017</td>
<td>$2R^2/Gm_p m_p m_H$ [34] MACHIAN FORMULA</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$(32\beta^2/p^4) \lambda_{CMB}^5/\lambda_2^2$ From Eq. (31)</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$(2\beta^2)/u(N_{L, \lambda_s \lambda_{CMB}^2}/\lambda_2)^{1/2}$ From Eq. (31)</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$2(\epsilon t^2)/a_{w, \lambda_s}$ [34] Non-Local Oscillation</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$\lambda_s(H/pw)(2\pi^2 a)^{3/2}$ [34] Holic Principle</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$(he/kT_{H,O})^2/ul_p$ [34] From $\sqrt{R_{\text{hol}}/\lambda_s} \approx W_{\text{H,O}}$</td>
<td>13.840</td>
<td>70.650</td>
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<td>2017</td>
<td>$(R_{\text{p}}/(1 + 1/2)\lambda_{mm}^2)/l_p$ From Eq. (31)</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$(\sqrt{n_{w, pw}^2/\lambda_{w, t}^2}/l_p)$ From Eq. (37)</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$\lambda_s(2/\pi)\lambda^{2/\pi} \times 3\times 5\times 7$ [36] Complete Holic Principle</td>
<td>13.776</td>
<td>70.978</td>
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<tr>
<td>2017</td>
<td>$(6/\pi)\epsilon u/\lambda_s$ [37]</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$\lambda_s(n_{1/2})\pi/1.5 \times 3/2$ [37]</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$\lambda_p (d_2/2\pi h)^{3/4}$ [37]</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$2A_{L}(1837 + s_{65})/2 + 1)^{\sqrt{\pi}}$ [37] $s_{65} = 1848$</td>
<td>13.812</td>
<td>70.793</td>
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<td>2017</td>
<td>Cosmos-Universe Couple MLT Formula (28) [37]</td>
<td>13.726</td>
<td>71.276</td>
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<tr>
<td>2017</td>
<td>$(3R_{\text{hol}} \lambda_{L, \lambda_s^{1/2}})^{1/2}$</td>
<td>13.852</td>
<td>70.772</td>
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<td>2017</td>
<td>$\lambda_s(a - 136)^{1/2}(e^{\pi e})\sqrt{\pi/2}$</td>
<td>13.814</td>
<td>70.783</td>
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<tr>
<td>2017</td>
<td>$(2\pi/3)(H/p)\lambda_s a_2^{1/4}$</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$\lambda_s/(1 + 1/a)^{3/2} \times 18a_w N_{L, p}^2$</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$R_{L}^2 \lambda_s^2 l_{p_{1/2}}/R_{\text{hol}} \lambda_2^2$ From $R_1 \approx (R_{\text{hol}})^{1/2}$</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$(2\epsilon t^4)/(4\pi^2 p w)^2$ From $R_1 \approx c t K_{w, aw}(4\pi^2 p w)^4$</td>
<td>13.812</td>
<td>70.793</td>
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</tr>
<tr>
<td>2017</td>
<td>$N_1(4\pi)^4 a_2(1837/\alpha)^2/R_{\text{hol}}$</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$(N_{L, u})(2^2/3)a_{w}/(a + 1)$</td>
<td>13.812</td>
<td>70.793</td>
<td></td>
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</tr>
<tr>
<td>2017</td>
<td>$(N_{L, w})(2^2/3)a_{w}/(a + 1)$</td>
<td>13.812</td>
<td>70.793</td>
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</tr>
<tr>
<td>2017</td>
<td>$(2\epsilon/\epsilon)(\lambda)^{3/2}$</td>
<td>13.812</td>
<td>70.793</td>
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<tr>
<td>2017</td>
<td>$2N_{L, \lambda_s}(1 - (137^2 + \pi^2 + e^2)/p H)$ From ppb Eq. (27)</td>
<td>13.812</td>
<td>70.793</td>
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