Cracking the Enigma of the Sagnac Effect

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21 May 2022

Abstract

One of the daunting problems in searching for a correct model of the speed of light is the contradiction between the Michelson-Morley experiment and the Sagnac effect. I have been working on a new theory called Apparent Source Theory (AST), which is based on three assumptions: 1. The effect of absolute motion of an inertial observer is to create an apparent change in the time of light emission. 2. The center of the light wave fronts moves with the same velocity as the absolute velocity of the inertial observer and the velocity of light depends on the mirror velocity relative to the observer. 3. Two observers/detectors that happen to be at the same point in space at the same time instant and moving with equal velocities will observe identical physical phenomena (for example, fringe position). The third postulate is used to analyze light speed problems involving accelerating observers/detectors. AST has been successful in providing consistent explanations for many light speed experiments. However, the precise application of AST to the Sagnac effect has been a challenge for AST. In this paper, a new analysis of Sagnac effect based on AST is presented. One of the unexpected findings is that the light beam propagating in the same direction as the observer will take less time to reach the observer than the light beam propagating in the opposite direction. Unconventionally, the fringe shift in the Sagnac effect is not due to a difference in path lengths of the counter-propagating light beams, but due to difference in their velocities according to the ballistic hypothesis. Experimental testing of this claim is proposed.

Introduction

The problem of absolute motion and the speed of light has confounded physicists for centuries and decades. Many experiments have been performed to reveal the fundamental nature of light and the correct model underlying the speed of light, but these have only added more confusions than clarity. All classical and modern theories, including ether theory, emission theory, special relativity, and their variations, have failed to provide a consistent explanation and resolve the contradictions.

The special relativity theory is based on the assumptions of non-existence or non-detectability of absolute motion and constancy of the speed of light. While there are experiments that appear to support these, these assumptions are increasingly being challenged by other experimental evidences. Absolute motion effect has been detected in the Silvertooth, the Marinov and the Roland De Witte experiments. Apparent non-constancy of the speed of light has also been observed in the Venus planet radar ranging experiment and the Lunar Laser Ranging experiment, hinting that mirror velocity adds to the velocity of light.

One of the daunting problems in searching for a correct model of the speed of light is the contradiction between the Michelson-Morley experiment and the Sagnac effect. I have been working on a new theory called Apparent Source Theory (AST) [1][2][3][4][5][6][7], which has been successful in providing consistent explanations for many light speed experiments.
However, the precise application of AST to the Sagnac effect has been a challenge for AST. In this paper, a new analysis of Sagnac effect based on AST is proposed.

We will first introduce Apparent Source Theory and then apply it to analyze the Sagnac effect.

Apparent Source Theory (AST)

We know that the Silvertooth, the Marinov and the Roland De Witte experiments have shown the existence of absolute motion. The question is: why did the Michelson-Morley experiment give a null result?

Consider the Michelson-Morley experiment (Fig.1).

Let us first see an intuitive explanation of why the Michelson-Morley experiments fail to detect absolute motion. According to AST, the effect of absolute motion of the Michelson-Morley experiment is to cause an apparent change in the time of emission of light [7]. The question follows: will change in time of emission of light cause a fringe shift? The answer is obviously: No. This is because both the longitudinal and transverse light beams will be delayed (or advanced) equally, hence no fringe shift occurs. So, relative to the detector of the Michelson-Morley experiment, the effect of absolute motion is only to create an apparent change in the time of light emission. Relative to the co-moving detector, neither the path length nor the speed of light is affected by absolute motion.
Consider a light source and an observer (Fig. 2). Assume that the light emits a short light pulse at time \( t = 0 \). The observer is passing through point \( O \) at \( t = 0 \), moving with absolute velocity \( V_{\text{abs}} \) to the right.

Conventionally, the observer will meet the light pulse at point \( O' \). Distances \( D' \) and \( \Delta \) are determined by the fact that the time it takes the observer to move from \( O \) to \( O' \) is equal to the time it takes light to move from \( S \) to \( O' \)

\[
\frac{D'}{c} = \frac{\Delta}{V_{\text{abs}}} \quad \ldots \quad (1)
\]

But,

\[
\Delta = D \cos \theta - \sqrt{(D')^2 - (D \sin \theta)^2} \quad \ldots \quad (2)
\]

\( D' \) and \( \Delta \) can be determined from the last two equations.

From the conventional/classical analysis above, we will only adopt the values of \( \Delta \) and \( D' \) in Apparent Source Theory.

We introduce the assumptions of Apparent Source Theory as follows.

1. The effect of absolute motion of an inertial observer is to create an apparent change in the time of light emission.
2. The center of the light wave fronts moves with the same velocity as the absolute velocity of the inertial observer and the speed of light depends on mirror velocity relative to the observer.
3. Two observers/detectors that happen to be at the same point in space at the same time instant and moving with equal velocities will observe identical physical phenomena (for example, fringe position).
The third postulate is used to analyze light speed problems involving accelerating observers/detectors.

Assume that the source emits a light pulse at \( t = 0 \), relative to observer’s at absolute rest. Conventionally, the time of light emission is the same for all observers, regardless of their velocities. According to AST, the time instant of light emission depends on the absolute velocity of the observer. The instant of light emission for a moving observer differs from that of an observer at rest. The same event (emission of light) apparently occurs at different time instants depending on the absolute velocity of the observer.

Suppose that the observer is at point O (Fig.3), moving to the right with absolute velocity \( V_{abs} \), at \( t = 0 \). According to AST, the light for the moving observer is not emitted at \( t = 0 \), but at an earlier time \( t = -t_1 \). Therefore, by the time light is emitted (\( t = 0 \)) for all observers at rest, the light emitted for the moving observer will have already travelled distance SP in the moving observer’s reference frame.

However, as in classical/conventional thinking, we assume that the moving observer and the observer at rest at point O’ will detect the light simultaneously.

It follows that, if the light for the moving observer is emitted at an earlier time \( t = -t_1 \), then the moving observer is at point O” when light is emitted for the moving observer. Thus, for the moving observer, light is emitted at time \( t = -t_1 \) and from point S’, with the center of the light wave fronts moving with the same velocity as the absolute velocity of the observer (\( V_{abs} \) to the right). Since the center of the light wave fronts moves with the same velocity as the velocity of the observer, it follows that the speed of light is equal to \( c \) relative to the moving observer, independent of the absolute velocity of the observer.

It follows that, if the moving observer and the stationary observer at point O’ are to detect the light simultaneously, then the distance PO should be equal to the distance SO’.
The time of light emission \((-t_1)\) is determined as follows.

\[
    t_1 = \frac{\text{distance } SO}{c} - \frac{\text{distance } SO'}{c} = \frac{D}{c} - \frac{D'}{c} = \frac{\text{distance } SP}{c} \quad \ldots \quad (3)
\]

where \(D'\) is determined from equations (1) and (2).

At the instant of light emission for the moving observer \((t = -t_1)\), the (moving) observer is at point \(O''\) where:

\[
    \text{distance } OO'' = V_{abs} t_1 \quad \ldots \quad (4)
\]

Now consider the specific case where the observer is moving directly towards or away from the light source (Fig. 4). Consider an observer moving towards a light source with absolute velocity \(V_{abs}\) Suppose that the source emits a short light pulse at \(t = 0\), while the moving observer is just passing through point \(O\).

Classically, we know that the observer meets the light pulse at point \(O'\). To determine \(\Delta\), as before, we note that the time taken by the light pulse to move from \(S\) to \(O'\) equals the time taken by the observer to move from \(O\) to \(O'\), i.e.

\[
    \frac{D - \Delta}{c} = \frac{\Delta}{V_{abs}} \implies \Delta = D \frac{V_{abs}}{c + V_{abs}} \quad \ldots \quad (5)
\]

We will use this classically obtained value for \(\Delta\) in the following formulation of AST.

According to AST, as stated above, the time of emission of light \((t = 0)\) applies only for observers at absolute rest. For the moving observer, light is emitted at an \textit{earlier} time \((t = -t_1)\), just as the observer is passing through point \(O''\) (Fig. 5), where:
distance $O''O = V_{abs} t_1$ \ldots (6)

And

$$t_1 = \frac{\text{distance } SO}{c} - \frac{\text{distance } SO'}{c} = \frac{D}{c} - \frac{D'}{c} = \frac{x}{V_{abs}} \ldots (7)$$

$$\Rightarrow t_1 = \frac{D}{c} - \left( \frac{D - \Delta}{c} \right) = \frac{\Delta}{c} = \frac{1}{c} D \frac{V_{abs}}{c + V_{abs}} \ldots (8)$$

From which,

$$x = V_{abs} t_1 = V_{abs} \frac{1}{c} D \frac{V_{abs}}{c + V_{abs}} = \frac{V_{abs}}{c} \Delta \ldots (9)$$

Now consider the case of an observer moving directly away from a light source with absolute velocity $V_{abs}$ (Fig. 6). Suppose that the source emits a short light pulse at $t = 0$, while the observer is just passing through point O. Classically, we know that the observer meets the light pulse at point O’.

To determine $\Delta$, we note that the time interval taken by the light pulse to move from S to O’ equals the time taken by the observer to move from O to O’, i.e.

$$\frac{D + \Delta}{c} = \frac{\Delta}{V_{abs}} \Rightarrow \Delta = D \frac{V_{abs}}{c - V_{abs}} \ldots (10)$$

As before, we will use this value for $\Delta$ in the formulation of AST below.
According to AST, as stated above, the time of emission of light ($t = 0$) applies only for observers at absolute rest. For the moving observer, light is emitted at a later time ($t = t_1$), just as the observer is passing through point $O''$ (Fig. 7), where:

$$distance \, OO'' = V_{abs} \, t_1 \quad \ldots \quad (11)$$

And

$$t_1 = \frac{distance \, SO'}{c} - \frac{distance \, SO}{c} = \frac{D'}{c} - \frac{D}{c} = \frac{x}{V_{abs}} \quad \ldots \quad (12)$$

$$\Rightarrow t_1 = \frac{D + \Delta}{c} - \frac{D}{c} = \frac{\Delta}{c} = \frac{1}{c} \, \frac{D}{c - V_{abs}} \quad \ldots \quad (13)$$

From which,

$$x = V_{abs} \, t_1 = V_{abs} \, \left( \frac{1}{c} \, D \frac{V_{abs}}{c - V_{abs}} \right) = \frac{V_{abs}}{c} \, \Delta \quad \ldots \quad (14)$$
General formulation of modified Apparent Source Theory

Now we will present the quantitative analysis for the case of an inertial observer moving in an arbitrary direction relative to the light source. We have already seen the qualitative analysis (Fig. 3).

Consider a light source and an observer moving with absolute velocity $V_{abs}$ to the right (Fig. 8). Let us start with the conventional view again. At the instant of light emission ($t = 0$), the distance between the source and the observer is $D$. However, this statement is based on conventional view because we are assuming that the light is emitted at $t = 0$ for all observers. Conventionally, the instant of emission of a light pulse is the same for all observers, and only the instant of detection of light differs between observers depending on their position and velocity.

According to the new theory proposed in this paper, however, (absolute) motion of an observer not only changes the time of light detection but also the (apparent) time of light emission! For observers at different positions and moving with different velocities, the times of emission of the same light pulse are different!

At $t = 0$ the source emits a short light pulse (for observers at rest) and the moving observer is just passing through point O. The light for the moving observer is emitted earlier, at $t = -t_1$.

At $t = -t_1$ light is emitted for the moving observer from point S', with the center of the wavefronts moving with the same velocity as the velocity of the observer ($V_{abs}$ to the right). At $t = -t_1$ the moving observer is at point $O''$, where:

$$
\text{distance } O''O = V_{abs} t_1 = x \quad . \quad . \quad (15)
$$
Therefore, by the time the observer arrives point O, the light for the moving observer will have travelled a distance \( SP \), in the reference frame of the observer.

\[
distance SP = c t_1 \quad . . \quad (16)
\]

This means that during the time interval that the observer moves distance \( O^\prime O \), light moves distance \( SP \).

The time of light emission for the moving observer is determined as follows.

\[
t_1 = \frac{D}{c} - \frac{D'}{c} \quad . . \quad (17)
\]

But \( D' \) and \( \Delta \) are determined from equations (1) and (2) re-written below.

\[
\frac{D'}{c} = \frac{\Delta}{V_{abs}} \quad . . \quad (18)
\]

and

\[
\Delta = D \cos \theta - \sqrt{(D')^2 - (D \sin \theta)^2} \quad . . \quad (19)
\]

The velocity of light depends on mirror velocity

This is not as such a separate assumption but a consequence of the other assumptions.

According to AST, the velocity of light is \( c \pm 2V \), where \( V \) is a component of the mirror velocity perpendicular to its surface, relative to the observer.

The new model described so far can give a consistent explanation of many light speed experiments such as the Michelson-Morley experiments, stellar aberration, moving source, moving mirror and moving observer experiments, the Sagnac effect, the Silvertooth and the Marinov experiments. Next I will present application of AST to the Sagnac effect, which is the main purpose of this paper.
The Sagnac effect

Although the Sagnac effect involves an accelerating observer/detector, it can be analyzed with sufficient accuracy by assuming an inertial observer/detector as follows.

For simplicity, we assume that the light source and the observer/detector are very close to the point on the mirror where light strikes the mirror. Also we assume that ABCD is a square.

The observer is moving to the right with velocity \( V \) where:

\[
V = \omega R
\]

The center of the light wave fronts moves with the same velocity as the observer. This means that the center of the light wave fronts is at a fixed point relative to the observer. *The speed of light is always constant relative to the center of the wave fronts.*

We can assume the observer to be at rest at point A, but the mirrors moving with velocity \( V \) to the left. Therefore, we also assume that the center of the light wave fronts also is fixed at A.

The counterclockwise light beam moves with velocity from point A to point B. However, after reflection from mirror B, the light acquires the velocity of the mirror, which is \( V \) to the left. According to the ballistic hypothesis, the velocity of the reflected light is the vector sum of \( c \) and \( 2V \), as shown below.
The resultant velocity of the CCW light reflected from mirror B will be:

\[ \sqrt{c^2 + (2V)^2 + 2c(2V)\cos\theta} \quad . . . \quad (20) \]

where \( \theta \) is 45° in this case.

Therefore, the resultant velocity will be:

\[ c' = \sqrt{c^2 + (2V)^2 + 2c(2V)\cos45°} = \sqrt{c^2 + (2V)^2 + c(2V)\sqrt{2}} \quad . . . \quad (21) \]

The CCW light beam will move with this speed the path BCD (velocity of mirror C will have no effect on the speed of the CW and CCW light beams).

The velocity of the CCW light is affected by the mirror D is moving to the left with velocity \( V \) and therefore will be equal to \( c \) after reflection.

The clockwise propagating light beam will move with velocity \( c \) between points A and D.

However, the velocity of the CW will be affected by the velocity of mirror D, as shown below.

The resultant velocity of CW light reflected from the mirror D will be:

\[ \sqrt{c^2 + (2V)^2 - 2c(2V)\cos\theta} \]

where \( \theta \) is 45° in this case.
Therefore, the resultant velocity will be:

\[ \sqrt{c^2 + (2V)^2 - 2c(2V)c \cos 45^0} = \sqrt{c^2 + (2V)^2 - c(2V)\sqrt{2}} \quad \ldots \quad (22) \]

The CW light beam moves with this velocity along the path DCB. After reflection from mirror B, the velocity of the CW light beam will be equal to \( c \).

We can see that both the CW and CCW light beams move with velocity \( c \) along the paths AB and AD. The velocities of the two light beams differ along the path BCD, with path length of 2L, and this is what causes the fringe shift of the Sagnac effect.

The difference in arrival times of the CW and CCW light beams at the observer will be:

\[
\Delta \tau = \frac{2L}{\sqrt{c^2 + (2V)^2 - c(2V)\sqrt{2}}} - \frac{2L}{\sqrt{c^2 + (2V)^2 + c(2V)\sqrt{2}}} \quad \ldots \quad (23)
\]

\[
= \frac{2L}{c} \left( \sqrt{\frac{(2V)^2}{c^2} - \sqrt{2} \frac{2V}{c}} \right) - \frac{2L}{c} \left( \sqrt{\frac{(2V)^2}{c^2} + \sqrt{2} \frac{2V}{c}} \right)
\]

Using the Taylor series expansion:

\[
\frac{1}{\sqrt{1 + x}} = 1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \ldots
\]

\[
\Delta \tau \approx \frac{2L}{c} \left( 1 + \frac{\sqrt{2}}{2} \frac{2V}{c} + \frac{3}{8} \frac{2(2V)^2}{c^2} \right) - \frac{2L}{c} \left( 1 - \frac{\sqrt{2}}{2} \frac{2V}{c} + \frac{3}{8} \frac{2(2V)^2}{c^2} \right)
\]

\[
\Delta \tau = \frac{4\sqrt{2} LV}{c^2} \quad \ldots \quad (24)
\]

Let us check this result against the well-known Sagnac effect formula for the time difference between the two counter-propagating light beams.

\[
\Delta \tau = \frac{4A\omega}{c^2} \quad \ldots \quad (25)
\]

where A is the area of the closed path, which is a square in this case.
\[ A = L^2 \quad \text{and} \quad \omega = \frac{V}{R} \]

where \( R \) is the radius of the circle.

But

\[ R = \frac{L}{\sqrt{2}} \]

Therefore,

\[ \omega = \frac{V}{R} = \frac{V}{\frac{L}{\sqrt{2}}} = \frac{\sqrt{2}}{L} \]

Substituting the above values:

\[ \Delta \tau = \frac{4A\omega}{c^2} = \frac{4L^2\frac{\sqrt{2}}{L} \frac{V}{L}}{c^2} = \frac{4\sqrt{2}LV}{c^2} \quad \ldots \quad (26) \]

which is the same as equation (24).

In the above analysis of Sagnac effect, we have assumed that the light beams enclose a square area, i.e. \( L_1 = L_2 = L \). Next we analyze the general case of rectangular area where \( L_1 \neq L_2 \).

We first determine the velocities of the mirrors relative to the observer. We need to determine only the relative velocities of mirrors \( M_1 \) and \( M_2 \) (relative to the observer) because \( M_3 \) will not affect the velocity of the reflected light, as its velocity is parallel to its own surface (Fig.11).

The velocity of \( M_1 \) relative to the observer is \( v' \) and the velocity of \( M_3 \) relative to the observer is \( v'' \). The vector diagrams for \( v' \) and \( v'' \) are shown. The velocity of light reflected from mirror \( M_1 \) and \( M_3 \) is the vector sum of the velocity of incident light and twice the perpendicular components of \( v' \) and \( v'' \), which are \( v_1'' \) and \( v_2'' \), respectively.
The velocity of the counter-clockwise propagating light beam, i.e. the velocity of light after reflection from M1, will be:

\[ v' = \sqrt{v^2 + v^2 - 2vv \cos 2\theta} = \sqrt{v^2 + v^2 - 2v^2 \cos 2\theta} = \sqrt{2} \ v \sqrt{1 - \cos 2\theta} \]

\[ \frac{\sin x}{v} = \frac{\sin 2\theta}{v'} \]

\[ \Rightarrow \ x = \sin^{-1}\left(\frac{v}{v'} \sin 2\theta\right) \]

\[ y = x - \sin(90 - 2\theta) \]

\[ v_1' = 2v' \cos y \]

The velocity of the counter-clockwise propagating light beam, i.e. the velocity of light after reflection from M1, will be:

\[ c_{ccw} = \sqrt{c^2 + v_1'^2 + 2cv_1' \cos \theta} \]

Note again that the ccw light beam has speed \( c \) before reflection from M1 and after reflection from M3. The ccw beam moves with velocity \( c_{ccw} \) only on its path between M1 and M3.

\[ v'' = \sqrt{v^2 + v^2 - 2vv \cos(180 - 2\theta)} = \sqrt{v^2 + v^2 - 2v^2 \cos(180 - 2\theta)} \]

\[ v'' = \sqrt{2v^2 - 2v^2 \cos(180 - 2\theta)} = \sqrt{2} \ v \sqrt{1 - \cos(180 - 2\theta)} \]
The velocity of the clockwise propagating light beam, i.e. the velocity of light after reflection from M3, will be:

\[ \sqrt{c^2 + v''^2 + 2c v'' \cos(180 - \theta)} \]

Again that the cw light beam has speed \( c \) before reflection from M3 and after reflection from M1. The ccw beam moves with velocity \( c_{ccw} \) only on its path between M3 and M1.

\[
\Delta t = t_{ccw} - t_{cw}
\]

\[
\Rightarrow \Delta t = \frac{L_1 + L_2}{c_{ccw}} - \frac{L_1 + L_2}{c_{cw}}
\]

The classical time difference is:

\[
\Delta t = \frac{4A\omega}{c^2}
\]

where

\[ A = L_1 L_2 \quad \text{and} \quad \omega = \frac{V}{R} \]

\[ L_1 = R \quad \text{and} \quad L_2 = R \sqrt{3} \]
Using Excel, I found that the prediction of AST is approximately equal to the classical prediction, with a typical discrepancy of 8%. Note that the above analysis itself is an approximate one and a more precise analysis may reduce the discrepancy.
Conclusion

The contradiction between the Michelson-Morley experiment and the Sagnac effect is perhaps the most challenging problem of the speed of light. There is no model of the speed of light so far that provides consistent explanations for these experiments. I have been working on a new theory called Apparent Source Theory (AST) for years. Even though I knew from the beginning the potential of AST to explain the Sagnac effect, the actual quantitative analysis which gives the simple Sagnac effect formula we know remained a daunting task. The new analysis presented in this paper is a significant progress in the development of AST. However, there are still a few remaining challenges to be overcome.

Glory be to Almighty God Jesus Christ and His Mother Our Lady Saint Virgin Mary.

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