A NOVEL SPACETIME MAP OF THE UNIVERSE

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To the memory of Albert Einstein.

His intuitions have made possible this work.

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ABSTRACT

A new cosmological model is proposed, which is based on two important intuitions of Albert Einstein. The model is admittedly conjectural, since it is based on gravitational repulsion between matter and antimatter, which is theoretically foreseen by some theorists, but denied by the first experiment. The values currently considered correct for some important system parameters (mass budget, Hubble constant, universe age) are assumed. Universe dimensions, expansion velocity and total mass are computed. The highly questioned inflation hypothesis is abandoned, and the matter-antimatter unbalance strongly reduced. As a consequence, the *dark side* of the cosmos assumes new, much smaller, dimensions. Another result of major importance is the time allowed for the formation of the first structures, which is much larger than predicted by traditional Big Bang models. A problem of horizon does not seem to exist: all parts of the universe are visible by every observer; this was true also in the past, with the exception of a relatively short initial period, and will be true also in the future. As concerns CMBR anisotropy, the structure seed dimension given by the model looks in reasonable agreement with the experimental data provided by the scientific satellites.

The first part of the paper shortly reviews the current situation of cosmology. In the second part the new model is discussed and evaluated from a quantitative viewpoint. The third part describes the novel spacetime map, including the derivation from measured redshift of comoving distance and time after Big Bang, and the discussion of the horizon problem.

COSMOLOGY TODAY

Experimental results

At the end of the 19th century most scientists were convinced that almost everything was understood. As a matter of fact, the theories set up by Newton, Maxwell, Boltzmann and other major scientists were so powerful to justify this position. But an experiment[1] performed by Michelson in 1881, then again with improved precision by Michelson and Morley in 1887, caused a major scientific crisis. It was thought, by similarity with the acoustic waves propagation, that an elastic medium existed also for the propagation of the electromagnetic waves; this medium was called “ether”. On the basis of this model, the Earth rotation was expected to produce an “ether wind”, and the light propagation velocity was expected to be slightly different in the direction of the wind and orthogonally to that direction. Interferometric techniques were used to detect the very small expected difference, but the result of the experiment was negative: the light propagation velocity was independent of the propagation direction, and the existence of the luminiferous ether was radically questioned.

The failure of the Michelson-Morley experiment was the start of new reflections by scientists like Lorentz, Poincaré, Heaviside. In 1904 Albert Einstein published his Special Relativity Theory[2][3], which brilliantly concluded this phase of theoretical developments. The publication of Einstein’s General Relativity Theory[4] followed in 1916. The first experimental confirmation of the General Relativity Theory was given in 1919 by Arthur Eddington, who succeeded in measuring, thanks to a total solar eclipse, the deviation of the light emitted by a star visible close to the Sun corona.

In1933 Edwin Hubble discovered that the spectrum of the light received from far galaxies is shifted towards the red, the shift amount being proportional to the galaxy distance. This means that galaxies are moving away from the Earth, with a velocity which is proportional to the galaxy distance. The phenomenon is measured by the so-called Hubble constant in [Km/sec/Megaparsec].

Extrapolating back in time the present behavior of the universe, Georges Lemaître proposed first the idea that the universe comes from a primordial explosion. This evolutionary model was for many years in competition with a stationary model of the universe, supported, among others, by the British scientist Fred Hoyle, who named the primordial explosion “Big Bang”; the intention of Hoyle was to kill with his humour the evolutionary model, but the name he suggested became the official name of a very successful model.

According to the cosmological principle, the macrostructure of the universe should be more or less the same everywhere; the very primitive structures existing shortly after the Big Bang are not present today, in any part of the universe. However, we are not able to observe the distant regions of the universe as they are today; as a matter of fact, we observe a celestial body thanks to the light, or, more generally, the electromagnetic radiation it has emitted in the past; since the light velocity is limited, we cannot see the celestial body as it is today, but only as it was when light was emitted. If the distance traveled by light to reach our planet is 1 [Gly], we see the celestial body as it was 1 [Gy] ago. But this limitation produces a wonderful gift: the primitive structures do not exist any more, but we can observe them thanks to the limited value of the light velocity! This is the reason why the Cosmic Microwave Background Radiation (CMBR) is called, by similitude with paleontology, the fossil radiation; we cannot observe the very primitive structure, but we can see its fossil remains, the microwave radiation which was emitted about 13,8 [Gy] ago and still propagates throughout the universe. The same applies to all past structures of the universe: they are like extinct species, that we cannot observe alive, but only through their fossil remains, i.e. the radiations they emitted in the past, which still travel the universe, and will continue to travel it in the future.

The definitive proof that the Big Bang model is correct was obtained at the beginning of the ‘60s by Arno A. Penzias and R. W. Wilson[5], who detected for the first time the CMBR and measured its level. Penzias and Wilson were not involved in scientific research, but worked as electronic engineers in the Bell Laboratories, with the responsibility to develop the low-noise receiving system for the Telstar experimental satellite; their receiving system was so low-noise and so well identified that they were able to understand that an excess noise was received from the sky. At a short distance from them a scientific team, headed by R. H. Dicke[6][7], was looking for the CMBR, but was not able to find it, because their receiving system was not adequate. We talk about serendipity when a scientist is looking for something, but he discovers instead something else, completely unforeseen; the discovery of Penzias and Wilson is not even a case of serendipity, because they were not looking for any scientific knowledge. Their discovery, however, was so important that in 1978 they were awarded the Nobel prize for physics.

After the Penzias-Wilson discovery, the CMBR has constantly been a subject of experimental research. Several scientific satellites have been launched to measure with increasing accuracy the CMBR, like the COBE[8], the Wilkinson Microwave Anisotropy Probe (WMAP)[9], and Planck[10][11]. Stratospheric balloons have also been used to this purpose[12][13]. All the measurements have confirmed that the early universe was almost exactly homogeneous and isotropic; the most recent result, due to the Planck spacecraft, is that the universe is homogeneous and isotropic to one part in 100.000. Of course homogeneity and isotropy are not absolute characteristics in the present universe, which is characterized by the presence of structures, i.e. by local unbalances; going back in time, however, a de-structuration process takes place, so that in the very early phase of universe life only seeds of the future structures were existing.

Major experimental research efforts have been devoted in the ‘90s to the determination of the acceleration of the universe expansion. The expectations were that, due to gravitational attraction, the expansion velocity of the universe decreases; this means that, looking to the far regions of the universe, i.e. in a remote past time, one should measure a significantly higher velocity of expansion. Due to these expectations, published papers talk about the measurement of a “deceleration parameter”; but the surprising result was the universe expansion is accelerating!

Two parallel research programs, named respectively “High-z Supernova Project”[14][15] and “Supernova Research Project”[16], reached the same conclusion: the universe is accelerating.
As a result, it is generally accepted today that the universe is composed by:
- about 4% of baryonic mass;
- about 26% of dark mass;
- about 70% of dark energy (which is needed to support the accelerated expansion of the universe).

It is interesting now to compare this situation with the scientific belief at the end of the 19th century; scientists were then convinced to have understood almost everything, whereas the adjective “dark” is associated today with 96% of the reality.

Einstein contributions

The most important theoretical contributions to modern cosmology are due to Albert Einstein; more than that, it can be affirmed that Einstein has founded modern cosmology. The key point of his General Relativity Theory (GRT) is that space-time is curved by matter/energy. Although Einstein defined the gravitational field equation, he never solved it; after the GRT publication, two solutions were found, by Friedmann and Schwarzschild, the first showing, among others, the possibility of the Big Bang, the second leading to the conception of the Black Hole.

Shortly after the publication of the GRT, in 1917 Einstein published the paper "Cosmological Considerations on the General Theory of Relativity" [17]. In this paper Einstein introduced two completely new concepts, which are described in the following.

To avoid the gravitational collapse of the universe, Einstein arbitrarily introduced in his field equation an arbitrary constant term that he named "cosmological constant", accounting for some form of repulsive gravity. Subsequently, he considered the cosmological constant as "the biggest mistake of my life". However, the recent discovery that the expansion of the universe is accelerating proves that Einstein was correct also in this case. Although most scientists prefer to talk about "dark energy", we will propose a cosmological model based on repulsive gravity, as suggested by Einstein. It is important here to remember that the Einstein preferred model was not an expanding universe, but a stationary universe; this seems however understandable, since Hubble discovered the universe expansion about 15 years after the publication of Einstein paper.

Since space is curved by matter, if we assume a uniform distribution of matter, we should have a constant curvature of space, therefore our three-dimensional space should be curved. In his cosmological paper Einstein suggested also this possibility, i.e. a universe which is unlimited, but of finite dimension. This is the second basic characteristic of the cosmological model proposed in this work. It is interesting to recall here the words used by Einstein himself to explain the idea of a curved 3-dimensional space: "… From our assumption as to the uniformity of distribution of the masses generating the field, it follows that the curvature of the required space must be constant. With this distribution of mass, therefore, the required finite continuum [….] will be a spherical space. We arrive at such a space, for example, in the following way. We start from a Euclidean space of four dimensions $\xi_1, \xi_2, \xi_3, \xi_4$, […..] In this space we consider the hyper-surface:

$$R^2 = \xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2$$

where R denotes a constant. The points of this hyper-surface form a three-dimensional continuum, a spherical space of radius of curvature R. [….] This view is logically consistent, and from the standpoint of the general theory of relativity lies nearest at hand [….] In order to arrive at this consistent view, we admittedly had to introduce an extension of the field equations of gravitation which is not justified by our actual knowledge of gravitation. It is to be emphasized, however, that a positive curvature of space is given by our results, even if the supplementary term is not introduced." [17]

Inflation

CMBR measurements have clarified that the early universe was homogeneous and isotropic. Immediately after, the building of structures started, but the absolute homogeneity and isotropy of the early ages guarantees that similar structures are present everywhere in the universe (cosmological principle). However a major problem arises here, since to reach homogeneity and isotropy some form of physical communication between the various parts of the universe is required, and we know that all forms of physical communication cannot exceed the speed of light.

In the current Big Bang model, expansion of the universe is assumed to start from a geometric point, which is a singularity; it is difficult, in this context, to create the conditions which guarantee homogeneity and isotropy in the early universe; a solution named *inflation* was therefore proposed at the beginning of the ‘80s.

The inflation is a very quick increase of the universe dimensions in a small fraction of the first second of life of the universe. The inflationary epoch lasts from 10^{-36} seconds to 10^{-32} seconds after the Big Bang; in this very short period of time the universe linear dimensions increase by a factor of at least 10^{26} to around 10 centimeters. This dimension may seem, and in fact is, very small, but, when the extremely short time duration is considered, one understands that a velocity much higher than the speed of light is required! The inflationary theory is based therefore on a major infringement of a basic postulate established by Einstein.

The inflation theory was developed in the late ‘70s and early ‘80s by several theoretical physicists, including Alexei Starobinsky at Landau Theoretical Physics Institute, Alan Guth[18][19] at Cornell University, and Andrei Linde at Lebedev Physical Institute. The theory explains the origin of the large-scale structure in the cosmos. Quantum fluctuations in the microscopic inflationary region, magnified to cosmic size, become the seeds for the growth of structure in the universe. Many physicists also believe that inflation explains why the universe appears to be the same in all directions (isotropic), why the CMB radiation is evenly distributed, why the universe is flat, and why no magnetic monopoles have been observed.

The detailed particle physics mechanism responsible for inflation is unknown. The basic inflationary paradigm is accepted by most physicists, as a number of inflation model predictions have been confirmed by observation. However, a substantial minority of scientists dissent from this position.

Many physicists, mathematicians and philosophers of science claim that the theory has produced untestable predictions and lacks serious empirical support. In 1999, John Earman and Jesús Mosterín[20] published a thorough critical review of inflationary cosmology, concluding: “We do not think that there are, as yet, good grounds for admitting any of the models of inflation into the standard core of cosmology.” Paul Steinhardt[21], one of the founding fathers of inflationary cosmology, has recently become one of its sharpest critics. Ijjas, Steinhardt and Loeb[22] claimed that the inflationary paradigm is in trouble in view of the data from the Planck satellite.

But the most severe criticisms come perhaps from Roger Penrose[23][24]. He points out that, in order to work, inflation requires extremely specific initial conditions of its own, so that the problem of initial conditions is not solved: “There is something fundamentally misconceived about trying to explain the uniformity of the early universe as resulting from a thermalization process. […..] For, if the thermalization is actually doing anything […..] then it represents a definite increasing of the entropy. Thus, the universe would have been even more special before the thermalization than after.” Penrose considered all possible configurations: some lead to inflation, some others lead to a uniform, flat universe directly, without inflation. Obtaining a flat universe is unlikely overall. Penrose’s shocking conclusion is that obtaining a flat universe without inflation is much more likely than with inflation, by a factor of 10 to the googol (10 to the 100) power!

Major problems in today cosmology

We feel three major problems in today cosmology:
- 96% of reality is labeled “dark”;
- the inflation process is highly questionable;
- although antimatter existence has been experimentally proven, it gets immediately annihilated when in contact with matter, and the unbalance between matter and antimatter is absolute.

In the following we propose a cosmological model which allows some interesting steps forward in all these respects. It is important to recognize that this model is based on two major Einstein intuitions, i.e. repulsive gravity and curved three-dimensional space. These two hypotheses offer a very simple explanation of the observed phenomena. In fact a hyper-spherical surface explains very well how the universe can be expanding in all directions in a uniform way. On the other hand, repulsive gravity can explain why the universe expansion is accelerating, as recently established by Supernovae observations.

DESCRIPTION AND ANALYSIS OF THE PROPOSED COSMOLOGICAL MODEL

Cosmological model description

Figure 1 shows the proposed cosmological model. Space has four dimensions. An antimatter white hole is located at the center of a curved 3-dimensional universe of matter, with curvature radius R_u. In other words, the universe occupies the surface of a hyper-sphere of radius R_u, therefore the volume of the universe is $2\pi^2 R_u^3$ [m^3].

The equation of the hyper-sphere surface in Cartesian coordinates is:

$$(x_1 - x_{10})^2 + (x_2 - x_{20})^2 + (x_3 - x_{30})^2 + (x_4 - x_{40})^2 = R_u^2 \qquad (1)$$

where $C(x_{10}, x_{20}, x_{30}, x_{40})$ is the center of the hyper-sphere.

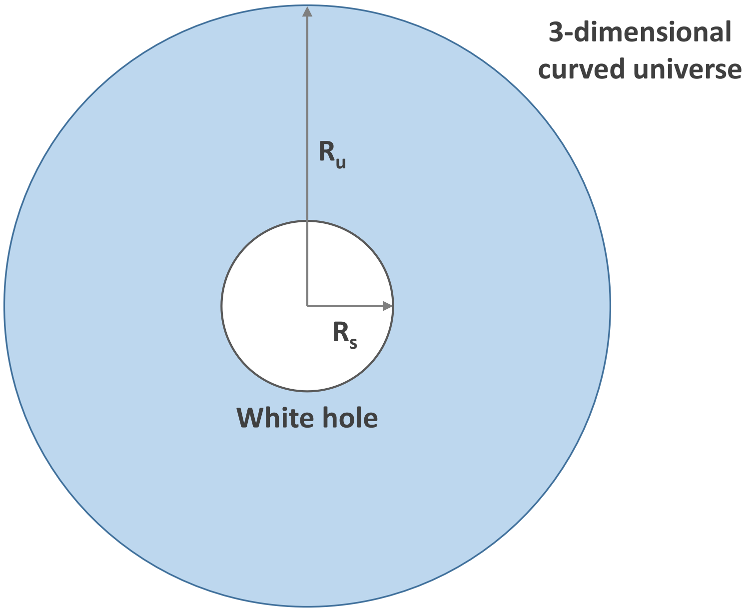

Fig. 1 – The proposed cosmological model

R_s = White hole Schwarzschild radius

R_u = Universe radius

The equation can also be written in polar coordinates as follows:

$$x_1 = R_u \cos\varphi_1$$
$$x_2 = R_u \sin\varphi_1 \cos\varphi_2$$
$$x_3 = R_u \sin\varphi_1 \sin\varphi_2 \cos\varphi_3$$
$$x_4 = R_u \sin\varphi_1 \sin\varphi_2 \sin\varphi_3 \quad (2)$$

where:

$$0 \leq \varphi_1 \leq \pi$$
$$0 \leq \varphi_2 \leq \pi$$
$$0 \leq \varphi_3 \leq 2\pi$$

The infinitesimal element of the hyper-sphere surface is:

$$\mathrm{d}S = R_u^3 \sin^2\varphi_1 \sin\varphi_2 \mathrm{d}\varphi_1 \mathrm{d}\varphi_2 \mathrm{d}\varphi_3 \quad (3)$$

It is assumed that the matter of the universe and the antimatter of the white hole are in relation through a gravitational force of repulsive nature, the modulus of which is given by the usual Newton law. Mass and electrical charge should behave in opposite ways: whereas electrical charges of different signs attract each other, antimatter should push matter away. This behavior is predicted by a theory[25], according to which the matter of the universe could be a CPT transformation of antimatter precipitating in the white hole. These assumptions give a beautiful explanation of the cosmological constant and of the accelerating universe expansion.

Of course the *push* given by the antimatter white hole would be counteracted by the *pull* due to the universe matter; without the push, in the long term the universe would collapse, and this is the very reason why Einstein introduced the cosmological constant in his field equation. As a matter of fact, our calculations show that the push due to white hole repulsive action prevails over the pull due to universe matter attraction, so that the universe continues to expand, as also confirmed by the Supernovae measurements.

Of course major efforts have been devoted to determining if gravity between matter and antimatter is attractive or repulsive. Since the gravitational force is much smaller than the electric force, the related experiments must make use of neutral particles, like antineutrons, atoms of positronium or atoms of anti-hydrogen[26]. Just before the conclusion of this work, the first result in this respect was obtained by the ALPHA experiment performed using the antimatter factory located at CERN[27]. It seems that also the antimatter falls down in the Earth gravitational field[28], and of course this is not a very comfortable result for our proposed model. We can still ask ourselves if the nature of gravity may change in the extreme conditions of a white hole, but for the time being it would be prudent to make a step back: should we accept, for instance, to talk about "vacuum energy" (a concept that we do not like)? We decided, however, to give for consideration the very promising results of our model.

The repulsive force due to the antimatter white hole

The antimatter located at the center of our cosmos is a white hole for the matter, which is pushed away. Due to the enormous mass value, an event horizon will exist, given by the formula:

$$R_s = \frac{2GM_{wh}}{c^2} \qquad (4)$$

where:
- R_s = Schwarzschild radius of the white hole, i.e. event horizon [m];
- M_{wh} = white hole mass [Kg];
- G = gravitational constant = $6{,}6743\cdot10^{-11}$[$m^3Kg^{-1}s^{-2}$];
- c = velocity of light = 299.792.458 [m/s].

The repulsive force, acting on the unit universe mass, due to the white hole mass will be:

$$F_{wh}(R_u) = -G\frac{M_{wh}}{R_u^2} \qquad (5)$$

and the potential of this force field will be:

$$P_{wh}(R_u) = G\frac{M_{wh}}{R_u} \qquad (6)$$

The attractive force due to the 3-dimensional universe of matter

It is generally accepted today that the universe is composed by:
- about 4% of baryonic mass;
- about 26% of dark mass;
- about 70% of dark energy.

In our cosmological model we explain the accelerating expansion of the universe (i.e. the cosmological constant, i.e. the dark energy) by the presence of a gigantic antimatter white hole, so that also the dark energy corresponds to a mass. In our bipolar (matter-antimatter) model, however, we call "universe" just what is composed by matter, which accounts for 30% of the total; we call instead "cosmos" the sum of the antimatter white hole and of the 3-dimensional curved universe made of matter; the mass of the cosmos is therefore 100% of what exists.

The mass of the universe will therefore be:

$$M_u = \frac{3}{7} \cdot M_{wh} \qquad (7)$$

Let now compute the attractive force due to the universe, which would in the long term cause the universe to collapse, if not counteracted by the repulsive gravity due to the white hole.

The universe density is given by:

$$\rho_u = \frac{M_u}{2\pi^2 R_u^3} \ [\mathrm{Kg/m^3}] \qquad (8)$$

and the mass of the infinitesimal element of hypersphere surface will be:

$$\mathrm{d}M_u = \frac{M_u}{2\pi^2 R_u^3} \cdot \mathrm{d}S = \frac{M_u}{2\pi^2} \cdot \sin^2\varphi_1 \sin\varphi_2 \mathrm{d}\varphi_1 \mathrm{d}\varphi_2 \mathrm{d}\varphi_3 \ [\mathrm{Kg}] \qquad (9)$$

Let now consider a unit mass element of our universe, which is subject to gravitational attraction by all other mass elements constituting the universe. Suppose that, just in order to simplify the calculations, the unit mass element is located in the point of Cartesian coordinates (R_u, 0, 0, 0). The distance between this element and the generic element of Cartesian coordinates given by (2) is:

$$D = R_u \cdot \sqrt{2 \cdot (1 - \cos\varphi_1)} \qquad (10)$$

The distance D depends only on the angle φ_1 thanks to the choice to put the unit mass element in (R_u, 0, 0, 0). As shown in figure 2, the generic element $\mathrm{d}M_u$ located in B attracts the unit mass element located in A (R_u, 0, 0, 0) with a force of intensity:

$$\mathrm{d}F = G\frac{\mathrm{d}M_u}{D^2} \qquad (11)$$

This force, however, is directed from A to B, whereas we are interested in the component of this force directed from A to O (=white hole), that we will call element of the universe force $\mathrm{d}F_u$. It is easily seen that:

$$\mathrm{O\hat{A}B} = \frac{\pi}{2} - \frac{\varphi_1}{2}$$

therefore:

$$\mathrm{d}F_u = \mathrm{d}F \cdot \cos \mathrm{O\hat{A}B} = \mathrm{d}F \cdot \sin\frac{\varphi_1}{2} \qquad (12)$$

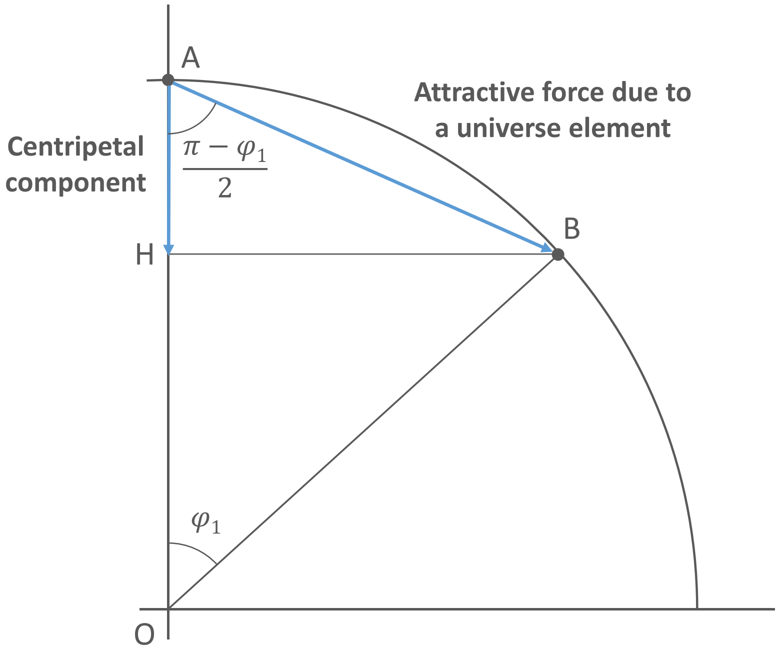

Fig. 2 – The centripetal component of the attractive force due to the universe

By substitution of (10) and (11) in (12) we obtain:

$$\mathrm{d}F_u = G \cdot \frac{\mathrm{d}M_u}{2R_u^2(1 - \cos\varphi_1)} \cdot \sin\frac{\varphi_1}{2} \qquad (13)$$

Substituting (9) in (13) we obtain:

$$\mathrm{d}F_u = \frac{1}{4\pi^2} \cdot \frac{GM_u}{R_u^2} \cdot (1 + \cos\varphi_1) \cdot \sin\frac{\varphi_1}{2} \cdot \sin\varphi_2 \cdot \mathrm{d}\varphi_1\mathrm{d}\varphi_2\mathrm{d}\varphi_3 \qquad (14)$$

To obtain the attractive force due to all the universe we must now integrate (14) with respect to φ_1, φ_2 and φ_3, obtaining the triple integral:

$$F_u = \frac{1}{4\pi^2} \cdot \frac{GM_u}{R_u^2} \int_0^{\pi}\int_0^{\pi}\int_0^{2\pi} (1 + \cos\varphi_1) \cdot \sin\frac{\varphi_1}{2} \cdot \sin\varphi_2 \cdot \mathrm{d}\varphi_1\mathrm{d}\varphi_2\mathrm{d}\varphi_3 \qquad (15)$$

This triple integral can be split in the product of three simple integrals as follows:

$$F_u = \frac{1}{4\pi^2} \cdot \frac{GM_u}{R_u^2} \int_0^{\pi} (1 + \cos\varphi_1) \cdot \sin\frac{\varphi_1}{2} \cdot \mathrm{d}\varphi_1 \int_0^{\pi} \sin\varphi_2 \, \mathrm{d}\varphi_2 \int_0^{2\pi} \mathrm{d}\varphi_3 \qquad (16)$$

Recalling the cosine duplication formula, we obtain:

$$\int_0^{\pi} (1 + \cos\varphi_1) \cdot \sin\frac{\varphi_1}{2} \cdot \mathrm{d}\varphi_1 = \frac{4}{3}$$

therefore:

$$F_u = \frac{1}{4\pi^2} \cdot \frac{GM_u}{R_u^2} \cdot \left(\frac{4}{3}\right) \cdot (2) \cdot (2\pi) = \frac{4}{3\pi} \cdot \frac{GM_u}{R_u^2} \qquad (17)$$

Recalling (5) and (7) we finally obtain:

$$F_u = \frac{4}{7\pi} \cdot \frac{GM_{wh}}{R_u^2} = -\frac{4}{7\pi} F_{wh} \qquad (18)$$

Total gravitational potential

The unit mass element of the universe will be subject to the total force:

$$F_t = F_{wh} + F_u = -\left(1 - \frac{4}{7\pi}\right) \cdot \frac{GM_{wh}}{R_u^2} \qquad (19)$$

The total gravitational potential will therefore be:

$$U(R_u) = -\int_{R_u}^{\infty} F_t \, \mathrm{d}R_u = \left(1 - \frac{4}{7\pi}\right) \cdot \frac{GM_{wh}}{R_u} \qquad (20)$$

Introducing now a Mass Ratio parameter (MR) defined as:

$$MR = \frac{total\ quantity\ of\ matter}{total\ quantity\ of\ antimatter} \qquad (21)$$

formula (19) will generalize in:

$$F_t = F_{wh} + F_u = -\left(1 - \frac{4}{3\pi} \cdot MR\right) \cdot \frac{GM_{wh}}{R_u^2} \qquad (19bis)$$

and we get the interesting result that the total force will be repulsive for any:

$$MR < \frac{3\pi}{4} \qquad (22)$$

FLRW metrics and radial expansion velocity

We can now compute the increase of the radial expansion velocity of the universe from R_s (white hole Schwarzschild radius) to R_u (universe radius).

The relativistic expression of the kinetic energy is:

$$E = mc^2 \left(\frac{1}{\sqrt{1 - \frac{v_u^2}{c^2}}} - 1 \right) \qquad (23)$$

The increase of kinetic energy equals the decrease of potential energy, according to equation:

$$\Delta E = mc^2 \left(\frac{1}{\sqrt{1 - \frac{v_u^2}{c^2}}} - 1 \right) - 0 = \Delta U = U(R_s) - U(R_u) \qquad (24)$$

Since the mass value equals 1, we obtain:

$$\frac{c^2}{\sqrt{1 - \frac{v_u^2}{c^2}}} = U(R_s) - U(R_u) + c^2 \qquad (25)$$

$$\sqrt{1 - \frac{v_u^2}{c^2}} \cdot \left[\left(1 - \frac{4}{7\pi}\right) \cdot \frac{GM_{wh}}{R_s} \cdot \left(1 - \frac{R_s}{R_u}\right) + c^2 \right] = c^2 \qquad (26)$$

Squaring both members and recalling formula (4), we obtain, by easy developments:

$$v_u = c \cdot \sqrt{1 - \frac{1}{\left[K\left(1 - \frac{R_s}{R_u}\right) + 1\right]^2}} \qquad (27)$$

where:

$$K = \frac{1}{2} \cdot \left(1 - \frac{4}{7\pi}\right) \qquad (28)$$

As R_u tends to infinity, the radial expansion velocity tends to:

$$v_u = c \cdot \sqrt{1 - \frac{1}{(K+1)^2}} \qquad (27bis)$$

which does not depend on R_s; in other words, the limit value of the radial expansion velocity does not depend on the total mass of the cosmos, but only on the Mass Ratio; if the Mass Ratio equals 3/7, we get K = 0,409054318, and the limit value of the radial expansion velocity is 211.206.244 [m/s].

It is interesting to notice here that, for simple symmetry reasons, in our cosmological model the dynamic evolution of the universe becomes a unidimensional problem, which can be studied along any universe radius. The Friedmann-Lemaître-Robertson-Walker (FLRW) metric reduces to the determination of the universe radius, and the expansion of space is given by the ratio of two values of the radius, related to two different moments in time.

For similarity, the specific expansion velocity of the universe (i.e. the Hubble parameter) e-quals the specific radial expansion velocity (i.e. the radial velocity normalized to the universe radius). The value of the Hubble parameter corresponding to the universe radius R_u is therefore found using the formula:

$$H(R_u) = \frac{v_u}{R_u} \qquad (29)$$

In the long term, the expansion velocity tends to saturate, whereas the universe radius continues to increase; as a consequence, the Hubble parameter decreases as the universe age increases; this allows to determine the universe age, provided we know with sufficient precision the present value of the Hubble parameter.

The present value of the Hubble parameter (that we will call H_0) is estimated to be within 67÷75 [Km/s/Megaparsec]. The UCLA cosmological calculator[29] assumes for H_0 the value of 69,6 [Km/sec/Mpc], whereas we will assume the value of 71 [Km/sec/Mpc]. The age of the universe will be determined accordingly, as we will show in the following.

Age of the universe

The time needed to cover the distance dR_u at the speed v_u will be:

$$dt\ [sec] = \frac{dR_u\ [Km]}{v_u\ \left[\frac{Km}{sec}\right]}$$

Be now:

S = number of seconds in one year = 31.557.600

c = speed of light = 299.792,458 [Km/sec]

$$dt\ [years] = \frac{dR_u\ [light - years]\ \cdot S\ \cdot c}{v_u\ \left[\frac{Km}{sec}\right]}$$

$$dt\ [years] = \frac{dt\ [sec]}{S}$$

By simple developments, recalling formula (27), we will get:

$$Age = \int_{R_s}^{R_{ut}} \frac{dR_u}{\sqrt{1 - \frac{1}{\left[K\left(1 - \frac{R_s}{R_u}\right) + 1\right]^2}}} \qquad (30)$$

where:

Age = age of the universe in years, when the universe radius is R_{ut};

R_s , R_u and R_{ut} are given in light-years.

The integral (29) can be computed in closed form as follows:

$$Age = \int_{R_s}^{R_{ut}} \frac{K \cdot \left(1 - \frac{R_s}{R_u}\right) + 1}{\sqrt{K^2 \cdot \left(1 - \frac{R_s}{R_u}\right)^2 + 2K \cdot \left(1 - \frac{R_s}{R_u}\right)}} \; dR_u$$

$$Age = \int_{R_s}^{R_{ut}} \frac{(K+1) \cdot R_u - KR_s}{\sqrt{(K^2 + 2K) \cdot R_u^2 - 2K(K+1)R_sR_u + K^2R_s^2}} \; dR_u$$

Putting now:

$$A = K^2 + 2K$$

$$B = -2K(K+1)R_s \qquad (31)$$

$$C = K^2R_s^2$$

$$X(R_u) = AR_u^2 + BR_u + \mathrm{C}$$

we will get:

$$Age = (K+1)\int_{R_s}^{R_{ut}} \frac{R_u}{\sqrt{X(R_u)}} \; dR_u - KR_s \int_{R_s}^{R_{ut}} \frac{dR_u}{\sqrt{X(R_u)}} \qquad (32)$$

Recalling now the well known integrals:

$$\int \frac{dx}{\sqrt{X}} = \frac{1}{\sqrt{a}} \, ln\left(\sqrt{X} + \frac{2ax + b}{2\sqrt{a}}\right) \qquad (33)$$

$$\int \frac{xdx}{\sqrt{X}} = \frac{\sqrt{X}}{a} - \frac{b}{2a}\int \frac{dx}{\sqrt{X}} \qquad (34)$$

where $X = ax^2 + bx + c$, we obtain by simple developments:

$$Age = \frac{K+1}{K(K+2)} \cdot \sqrt{X(R_{ut})} + \frac{R_s}{(K+2)\sqrt{A}} \cdot ln\left[\frac{2AR_{ut} + \sqrt{4AX(R_{ut})}}{2KR_s} - (K+1)\right] \qquad (35)$$

This formula may be simplified for $R_{ut} \gg R_s$ obtaining:

$$Age \cong \frac{K+1}{\sqrt{K(K+2)}} \cdot R_{ut} + \frac{R_s}{(K+2)\sqrt{K(K+2)}} \cdot ln\left[(K+2)\frac{2R_{ut}}{R_s}\right] \qquad (35bis)$$

A further simplification is obtained as $R_{ut} \to \infty$:

$$Age \cong \frac{K+1}{\sqrt{K(K+2)}} \cdot R_{ut} \cong 1{,}4194 \cdot R_{ut} \qquad (35ter)$$

The last result is in perfect agreement with the fact that, in the limit, the expansion rate of the universe tends to be constant. Formula (35ter) underestimates the present age of the universe by about 0,15%.

We are now interested in estimating the present dimension of the universe, i.e. R_{ut}. If the value of R_{ut} is very large, we may approximate v_u by the limit value 211.206,244 [Km/s]. Since the present value of H is 71 [Km/s/Mpc] = 21,7687 [Km/s/Mly], then we will obtain:

$$R_{ut} \cong \frac{211.206{,}244}{21{,}7687} \text{ [Mly]} = 9702{,}29 \text{ [Mly]}$$

A more accurate calculation of R_{ut} gives the results summarized in Table 1. It is interesting to notice that the present radius and age of the universe do not vary significantly within a wide range of white hole dimensions. The criteria which dictate an appropriate decision about the white hole dimension are discussed in the next paragraph.

R_s [Mly] \ H_0 [Km/s/Mpc]	67	69,6	71	75
0,001	10270/14480	9900/13949	9702/13771,32	9180/12930
0,01	10270/14480	9900/13949	9702/13771,36	9180/12930
0,1	10270/14480	9900/13950	9702/13771,77	9180/12930
1	10270/14480	9900/13953	9702/13774,97	9180/12930
10	10280/14510	9900/13970	9700/13795,5	9180/12950
100	10250/14619	9860/14070	9673/13903,7	9155/13070

Table 1 - Radius/age of the universe versus R_s and H_0

The radius is given in [Mly] and the age in [My]

Adiabatic expansion of the universe and white hole mass

According to the Big Bang model, the early universe was very hot, and as a consequence protons and electrons were separated, and the resulting ionized plasma was not transparent to electromagnetic radiation. As the universe expanded, the plasma cooled down until protons recombined with electrons forming hydrogen atoms; photons not interacting with electrically neutral atoms began to freely travel through the universe. This recombination event, producing a medium transparent to radiation, happened at the universe age of about 379000 years, when the universe temperature was about 3000 [K]. Since the universe is a isolated system, the expansion is adiabatic, and the universe temperature varies inversely with the FLRW scale factor, which in our model is the universe radius; also the peak of the black-body radiation spectrum varies inversely with the expansion of space. Since the color temperature of the CMB photons is today about 2,726 [K], it follows that the expansion of the universe has been:

$$Exp = \frac{3000}{2{,}726} = 1100 \qquad (36)$$

Since the present radius of the universe is about 9700 [Mly], it follows that the universe radius at the time of recombination is about:

$$R_s \cong \frac{9700}{1100} = 8{,}81818 \text{ [Mly]}$$

The model allows to evaluate the increment of R_u in the time of 379000 years; due to the relatively small radial velocity (we are at the very beginning of the expansion) the increment is only 3,35 [Kly]; the white hole Schwarzschild radius is therefore obtained as follows:

$$R_s \cong 8{,}81818 - 0{,}00335 = 8{,}81483 \text{ [Mly]} \qquad (37)$$

The maximum red-shift is measured with the CMB photons, emitted 379000 years after the Big Bang, when the universe radius was about 8,81818 [Mly]; this red-shift equals the space expansion minus one, therefore its value is 1099.

The white hole mass can now be computed using formula (4):

$$M_{wh} = \frac{c^2 \cdot R_s}{2G} = 5{,}614929 \cdot 10^{49} \text{ [Kg]} \qquad (38)$$

and the total mass of the universe is as a consequence:

$$M_u = \frac{30}{70} \cdot M_{wh} = 2{,}406398 \cdot 10^{49} \text{ [Kg]} \qquad (39)$$

The order of magnitude of the mass is in reasonable agreement with some other estimates.

Dynamic evolution of the universe

The results of our calculations have been summarized in some figures.

Fig. 3 shows the increase of the universe radius during the universe life; it is an almost linear trend, with the exception of the early universe. The radius is today 9700 [Mly].

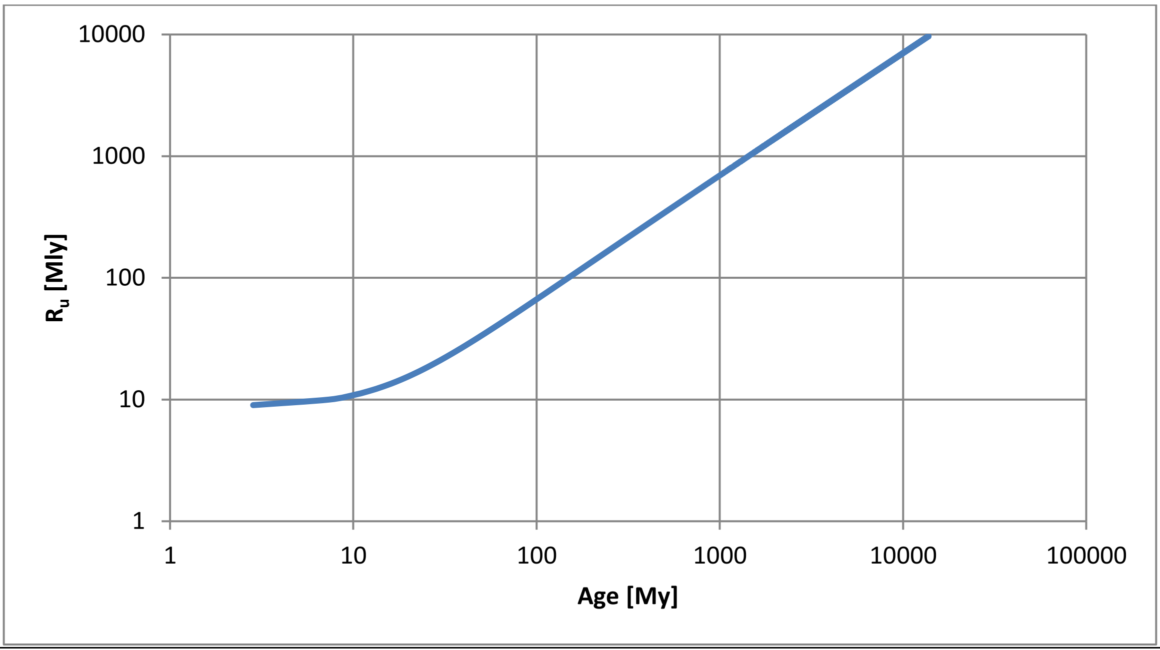

Fig. 3 – Universe radius versus universe age (or TABB)

Fig. 4 shows the increase of the radial expansion velocity, which tends however to the limit value of 211.206,244 [Km/s] as time goes to infinity.

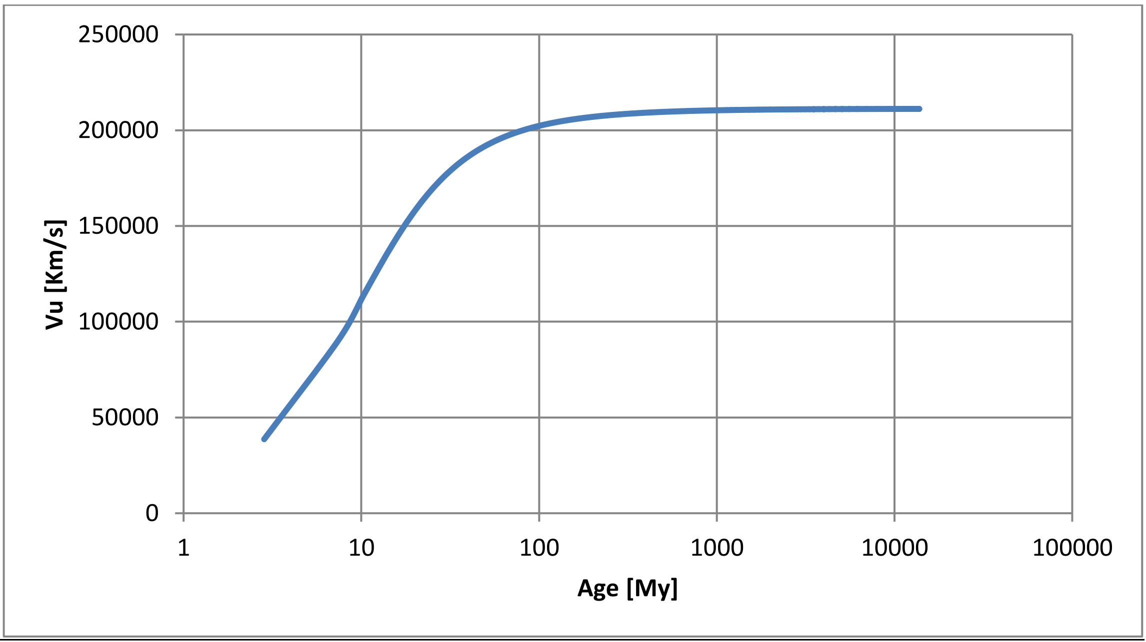

Fig. 4 – Radial expansion velocity versus universe age

Fig. 5 shows the behavior of the Hubble parameter: it increases very rapidly in the first phase of the universe life, to reach a maximum value of about 35330,72 [Km/sec/Megaparsec] after 13,956 Million years; today, at the universe age of 13792,76 [My], the value of the Hubble parameter is 71 [Km/s/Mp], but it will tend to zero as time tends to infinity. It is important to note that the value of universe age provided by our model is in excellent agreement with the one determined by the Planck program [30], i.e. 13797 [My] ± 23 [My].

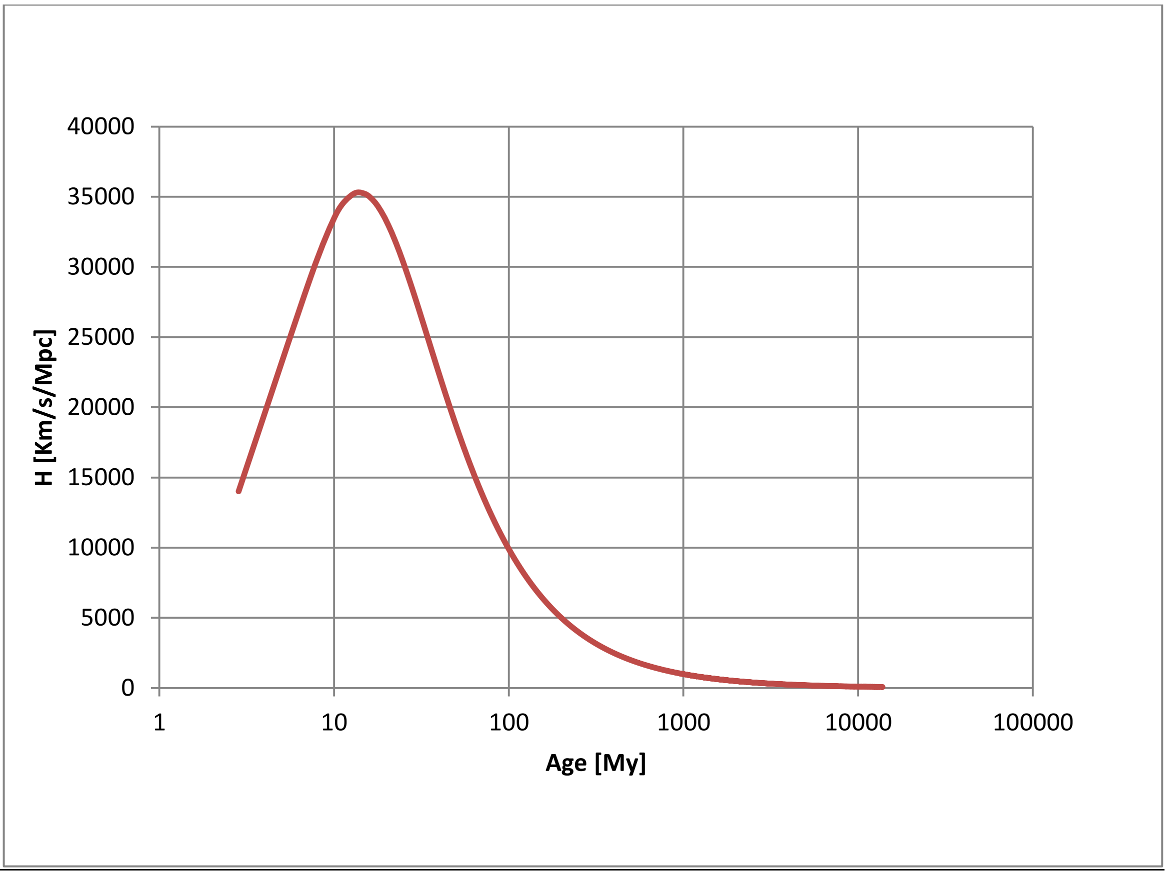

Fig. 5 – Hubble parameter versus universe age

Fig. 6 shows the acceleration of the expansion of the universe: it decreases rapidly, today is very small, and will tend to zero as time tends to infinity.

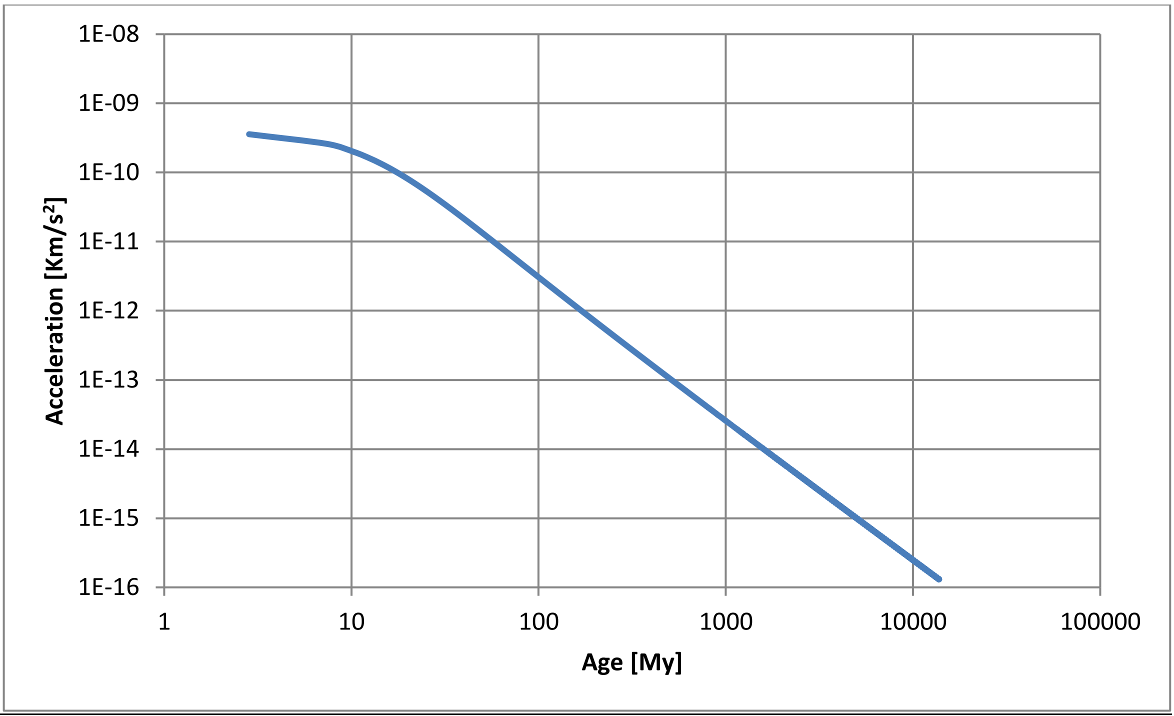

Fig. 6 – Radial acceleration versus universe age

The value of the acceleration was not computed in closed form, but determined as the ratio between an increment of velocity and an increment of age; the result is however very accurate, since very small increments were considered.

Let consider now the Hubble parameter given by formula (29); derivating with respect to time we obtain:

$$\frac{dH}{dt} = \frac{a_u \cdot R_u - v_u^2}{R_u^2} = \frac{a_u}{R_u} - H^2$$

Introducing now the "deceleration" parameter q, defined as:

$$q = -\frac{a_u \cdot R_u}{v_u^2} \qquad (40)$$

we will obtain the following important formula:

$$\frac{dH}{dt} = -(q+1) \cdot H^2 \qquad (41)$$

which connects the values of H and q.

When $H = 35330{,}72$ (maximum value), it must be $dH/dt = 0$, therefore $q = -1$.

As a matter of fact, at the universe age of 13,956 [My], when the value of H is maximum, we have:

$$R_u = 12{,}5217 \text{ [Mly]}; \qquad v_u = 135525{,}5 \left[\frac{\text{Km}}{\text{s}}\right]; \qquad a_u = 1{,}55043 \cdot 10^{-10} \left[\frac{\text{Km}}{\text{s}^2}\right]$$

Combining these values by formula (40) we obtain for q the value – 1.

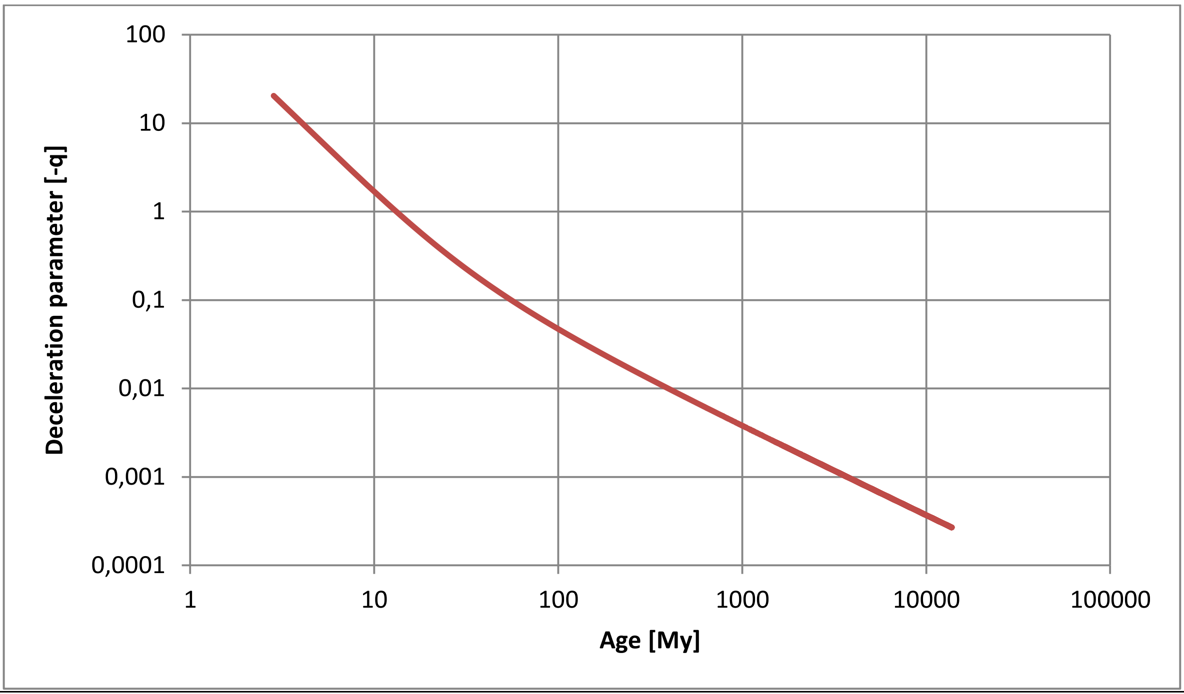

Fig. 7 – “Deceleration” parameter versus universe age

SPACETIME MAP OF THE UNIVERSE

A 4-dimensional model for our observations

In the previous chapters we have proposed and discussed a 4-dimensional model of the cosmos (four *space* dimensions). The total mass of the cosmos is subdivided in white hole anti-matter (70%) and universe matter (30%). Due to repulsive gravity, the universe is expanding with a small radial acceleration, and has reached at present a radius of 9,700 [Gly]. During its 13,792 [Gy] life, the universe has evolved from elementary particles to atoms, therefore molecules, gas clouds, stars, planets, until a comfortable environment was created to host life and intelligent beings, who are able to observe the universe and understand its structure and its history.

As already clarified, thanks to the limited value of the light velocity, we can observe today all past structures of the universe, which do not exist anymore.

We will now use the same 4-dimensional model which was proposed for the distribution of mass in the universe to understand the meaning of our observations. Again, if we put our observation point (the Earth) in point $(R_u, 0, 0, 0)$, the distance to a generic point of the universe will depend only on the colatitude φ_1, and not on the angles φ_2 and φ_3 (see formula (10)). This means that for each distance (or colatitude) we will have ∞^2 directions of arrival of the radiation; in other words, the observation point can be reached by the radiation through ∞^2 different paths, each defined by a different pair of $(\varphi_2,\ \varphi_3)$ values.

For symmetry reasons the light will follow similar paths, whichever the direction from which it arrives; the light path can be evaluated in the simplest way if we assume $\varphi_2 = \varphi_3 = 0$. This is the polar equation of the $(R_u,\ \varphi_1)$ plane. In this case the generic point coordinates become $(R_u \cos\varphi_1, R_u \sin\varphi_1,\ 0,\ 0)$, and we may call φ_1 the separation angle between the celestial body and the observation point. Formula (10) gives the chord-distance between the observer and the celestial body, but what matters here is the arc-distance, since the light is forced by gravitation to travel through the universe, following a curved path; the arc-distance is generally expressed by the formula $R_u \cdot \varphi_1$, but it is important here to distinguish three different values of distance:

- the *physical distance*, which equals $R_{ue} \cdot \varphi_1$, where R_{ue} is the radius of the universe when radiation is emitted by the celestial body;
- the *co-moving distance*, which equals $R_{ur} \cdot \varphi_1$, where R_{ur} is the universe radius when radiation is received by the observer;
- the *light-covered distance*, which equals $c \cdot \Delta T_{e-r}$, where ΔT_{e-r} is the time employed by the light emitted by the celestial body to reach the observer; the light-covered distance is intermediate between the physical distance and the luminosity distance, and equals the length of the quasi-spiral arc connecting the celestial body and the observer in the $(R_u,\ \varphi_1)$ plane.

Co-moving distance

Between the Earth and an observed celestial body there is a distance in space, but also, due to the finite velocity of propagation of the light, a distance in time.

The distance in space existing at the time of light reception between the observation point (the Earth in our case) and the observed celestial body is a great circle arc on the hyper-sphere surface (which contains the universe), with curvature radius equal to the hyper-sphere radius at that time. This distance will change in time proportionally to the universe radius (FLRW metrics), and for this reason is called the *co-moving distance*. The distance existing at the time of light emission is called, instead, *physical distance*. Of course co-moving distance and physical distance are in the same proportion as the universe radius at the reception time and the one at emission time.

Be now R_{ut} the universe radius at light reception time (where *t* stands for *today*) and R_{ue} the radius at light emission time. We will now compute the co-moving distance as a function of R_{ut} and R_{ue}.

Be:

$$dt = \frac{dR_u}{v_u} = \text{time needed to increase the universe radius from } R_u \text{ to } R_u + dR_u$$

$$\frac{dR_u}{v_u} \cdot c = \text{distance covered by the light in the time } dt$$

$$\frac{dR_u}{v_u} \cdot c \cdot \frac{R_{ut}}{R_u} = \text{distance magnified by the universe expansion}$$

The co-moving distance will be given by the integral:

$$D_c = \int_{R_{ue}}^{R_{ut}} \frac{R_{ut}}{R_u} \cdot \frac{1}{\frac{v_u}{c}} \cdot dR_u \qquad (42)$$

$$D_c = R_{ut} \cdot \int_{R_{ue}}^{R_{ut}} \frac{1}{\sqrt{1 - \frac{1}{\left[K\left(1 - \frac{R_s}{R_u}\right) + 1\right]^2}}} \cdot \frac{dR_u}{R_u}$$

$$D_c = R_{ut} \cdot \int_{R_{ue}}^{R_{ut}} \frac{(K+1) \cdot R_u - KR_s}{\sqrt{(K^2 + 2K)R_u^2 - 2K(K+1)R_sR_u + K^2R_s^2}} \cdot \frac{dR_u}{R_u}$$

Using now the same definitions already given by (31), we will obtain:

$$D_c = R_{ut} \cdot \left[(K+1)\int_{R_{ue}}^{R_{ut}} \frac{dR_u}{\sqrt{X(R_u)}} - KR_s \cdot \int_{R_{ue}}^{R_{ut}} \frac{dR_u}{R_u\sqrt{X(R_u)}}\right]$$

Recalling now the integral (33) and the other well known integral:

$$\int \frac{dx}{x\sqrt{X}} = -\frac{1}{\sqrt{c}} \cdot ln\left(\frac{\sqrt{X} + \sqrt{c}}{x} + \frac{b}{2\sqrt{c}}\right)$$

where $X(x) = ax^2 + bx + c$,

we will obtain:

$$D_c = R_{ut} \cdot \left[\frac{K+1}{\sqrt{A}} \cdot ln\left(\sqrt{X(R_u)} + \frac{2AR_u + B}{2\sqrt{A}}\right) + \frac{KR_s}{\sqrt{C}} \cdot ln\left(\frac{\sqrt{X(R_u)} + \sqrt{C}}{x} + \frac{B}{2\sqrt{C}}\right)\right]_{R_{ue}}^{R_{ut}}$$

As:

$$\frac{KR_s}{\sqrt{C}} = 1 \qquad \frac{B}{2\sqrt{C}} = -(K+1)$$

we will finally obtain:

$$D_c = R_{ut} \cdot \left[\frac{K+1}{\sqrt{A}} ln\left(\sqrt{X(R_u)} + \frac{2AR_u + B}{2\sqrt{A}}\right) + ln\left(\frac{\sqrt{X(R_u)} + \sqrt{C}}{x} - (K+1)\right)\right]_{R_{ue}}^{R_{ut}} \quad (43)$$

Fig. 8 shows how the co-moving distance depends on the universe radius: the distance is zero if the universe radius at the light emission time equals the present radius of the universe, i.e. if the light is emitted and immediately received; the distance reaches values larger than 100 [Gly] if light was emitted close to the Big Bang.

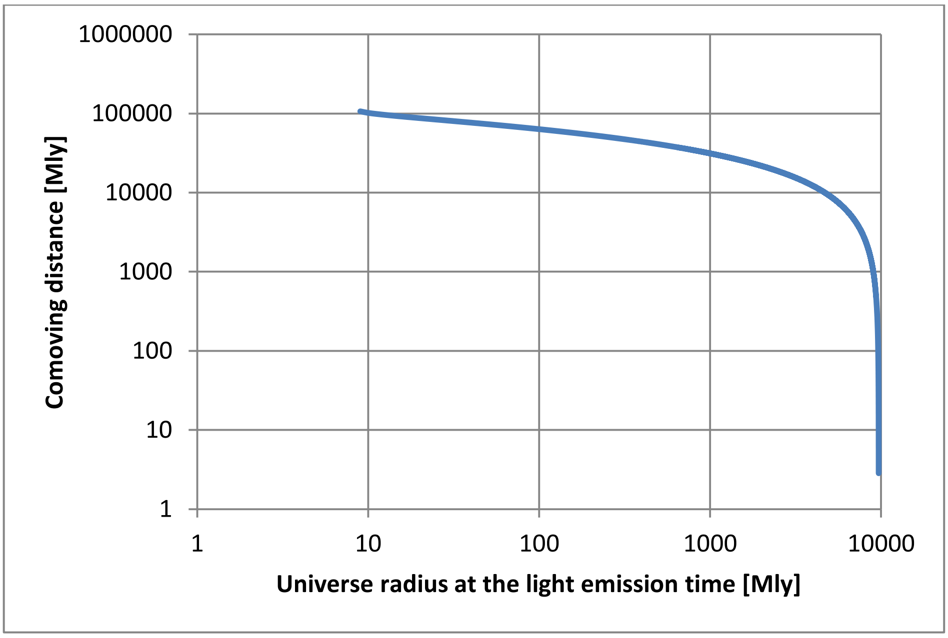

Fig. 8 – Co-moving distance versus R_{ue}

Lookback time and Time after Big Bang

As already explained, the distance in space goes together with a distance in time, due to the finite value of the light velocity. The distance in time may be computed by a simple modification of formula (30). We define:

$$\int_{R_s}^{R_{ue}} \frac{dR_u}{\sqrt{1 - \frac{1}{\left[K\left(1 - \frac{R_s}{R_u}\right) + 1\right]^2}}} = \text{Time After Big Bang (TABB)} \qquad (44)$$

$$\int_{R_{ue}}^{R_{ut}} \frac{dR_u}{\sqrt{1 - \frac{1}{\left[K\left(1 - \frac{R_s}{R_u}\right) + 1\right]^2}}} = \text{Look} - \text{Back Time (LBT)} \qquad (45)$$

$$Age = \text{TABB} + \text{LBT} \qquad (46)$$

Fig. 9 shows how the LBT depends on the universe radius at the light emission time.

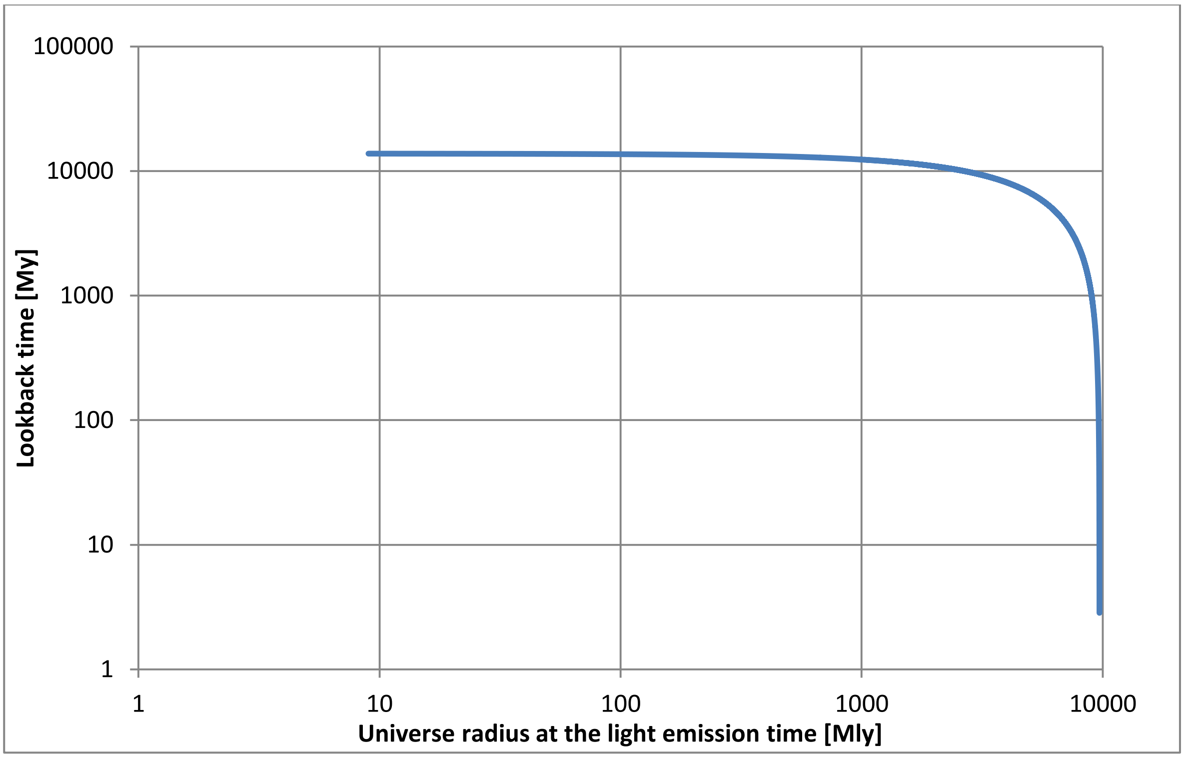

Fig. 9 – Look-back time versus R_{ue}

Expansion of space and Cosmological redshift

Due to repulsive gravity, universe space expands, with the universe radius defining completely the universe dimensions at any given time. The space expansion is given by the formula:

$$Exp = \frac{R_{ut}}{R_u} \qquad (47)$$

where R_{ut} is the present value of the radius, i.e. 9700 [Mly]. As R_u varies between 8,81483 Mly and 9700 [Mly], the space expansion ratio varies between 1100,418 and 1.

As a consequence of space expansion, superluminal velocities are possible, and wavelengths increase, such that red-shifts larger than one may be experienced. Due to its origin, this type of red-shift is called “cosmological”. Let now compute the cosmological red-shift corresponding to a given space expansion.

Be:

$$\Delta s = c \cdot \Delta t \cdot \frac{R_{ut}}{R_u} = \text{space traveled by light in time } \Delta t \text{ and magnified}$$

$$\Delta t = \frac{\Delta R_u}{v_u} = \text{time needed to increase the universe radius by } \Delta R_u$$

$$\Delta z = \frac{\Delta s \cdot H}{c} = \frac{R_{ut}}{R_u^2}\, \Delta R_u = \text{redshift increment due to elementary radial expansion}$$

$$z = \int_{R_{ue}}^{R_{ut}} \frac{R_{ut}}{R_u^2}\, dR_u = \frac{R_{ut}}{R_{ue}} - 1 = Exp - 1 \qquad (48)$$

The cosmological red-shift will therefore vary between 0 (very close star) and 1099,418 (Big Bang).

When we observe a celestial body, we measure its cosmological red-shift, from which we can derive the R_{ue} value (formula 48). From the knowledge of R_{ue} we will derive the distance in space (=Co-moving distance – formula 43), and the distance in time (=LBT – formula 45).

From Cosmological red-shift to Co-moving distance and Look-back time

Figs. 10 and 11 give respectively the Co-Moving Distance (CMD) and the Time After Big Bang (TABB) versus the red-shift for today’s universe. Similar curves can be easily computed for other epochs of the universe life. In the next paragraphs we will first compare our results with those obtained using the UCLA Cosmological Calculator[29], then we will show how difficult was for the two Supernovae Research Projects to reach the conclusion that the universe expansion is accelerating by comparing red-shifts and luminosity distances of far Supernovae with those of close Supernovae.

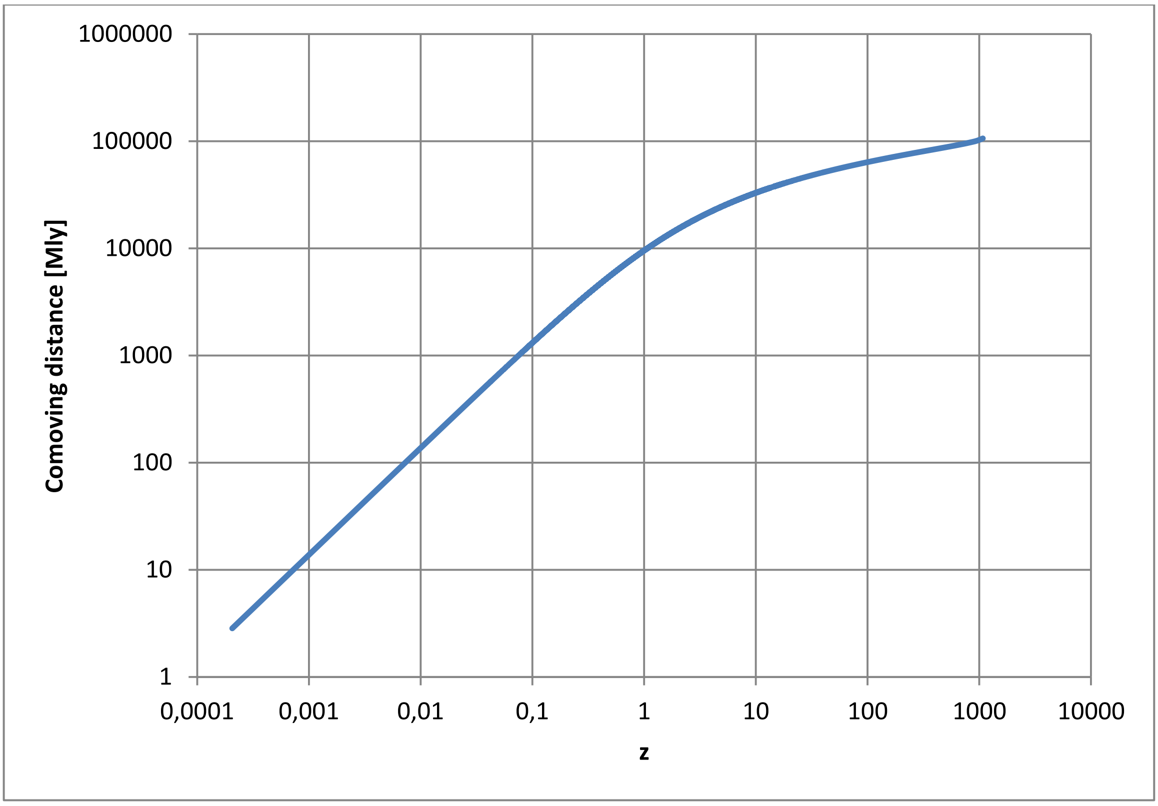

Fig. 10 – Co-moving distance versus red-shift

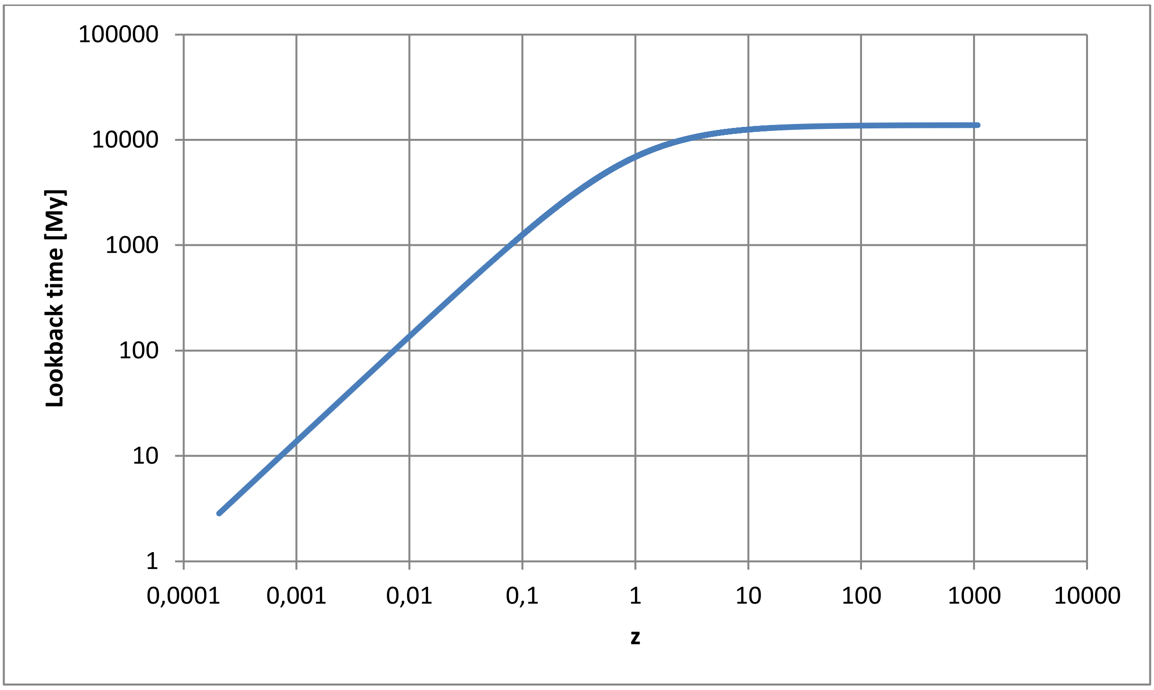

Fig. 11 – Look-back time versus red-shift

Time available for structures creation

CMD and TABB provided by our model have been compared with those given by the UCLA Cosmological Calculator [29].

There are some differences in the underlying hypotheses: we assume for the Hubble parameter a value of 71 [Km/s/Mpc] against a value of 69,6 assumed by UCLA; the MR is 30/70 in our case, against 28,6/71,4 ~ 28/70 for UCLA; the resulting universe age is 13,722 [Gy] for UCLA and 13792,76 [Gy] for our model. Another very important difference is that in the UCLA model the Big Bang starts from a geometric point, whereas in our case it starts from a dimension of 8,81483 [Mly].

Nevertheless the results provided by the two models are rather close for small to medium values of red-shift (see Fig 12). For z > 1, instead, the results are very different: in particular, the TABB for a given value of red-shift is much higher in the case of our model; this could be an important advantage, since much more time would be available for the construction of the first structures (stars, galaxies, quasars). Our curves start from red-shift 1100,418, whereas the UCLA model allows red-shift going to infinity, due to the Big Bang starting from a geometric point.

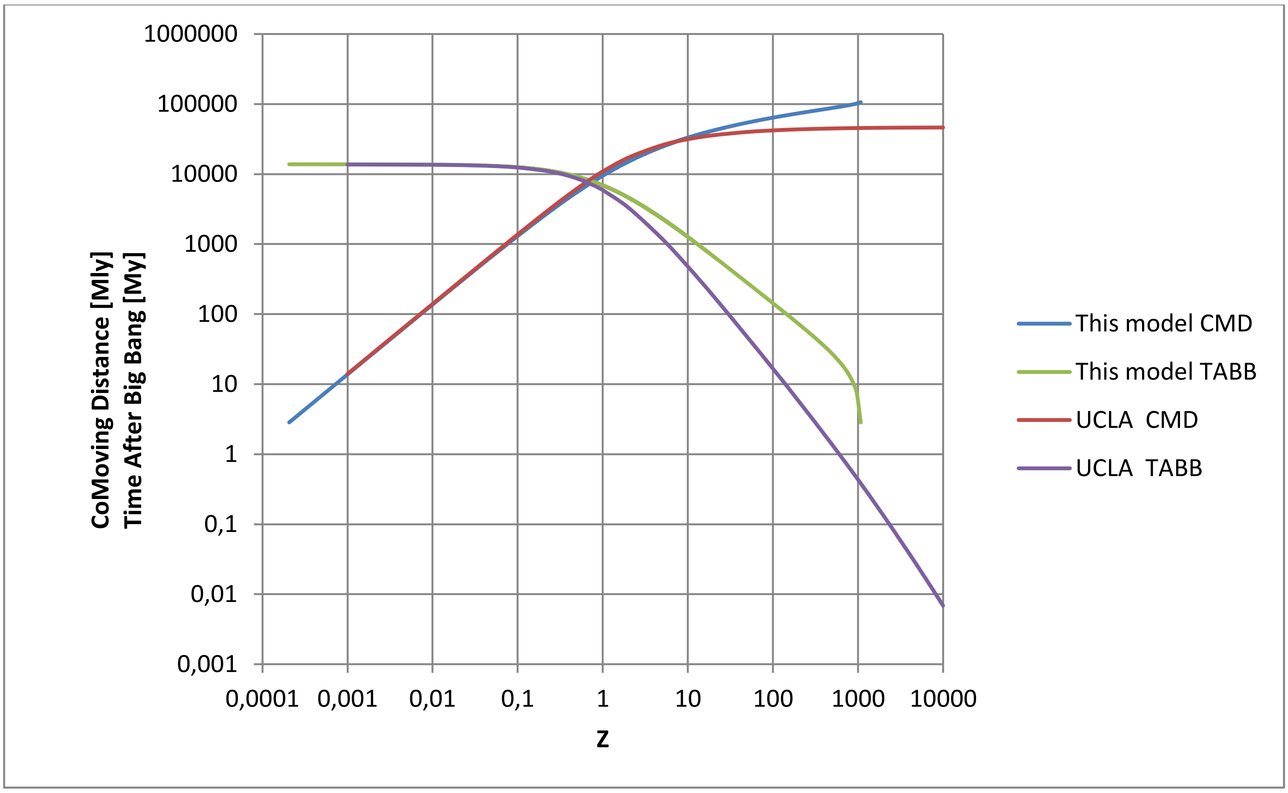

Fig. 12 – Proposed model versus UCLA Cosmological Calculator

Light geodesics

The ratio between the co-moving distance and the universe radius at a given time gives the angular separation between the light emitting star and our planet:

$$\alpha = \frac{D_c}{R_u} \cdot \frac{180}{\pi} \quad [\text{deg}] \qquad (49)$$

This allows to obtain a polar representation of all the stars observed from our planet today, in a $(R_u,\ \alpha)$ plane; the α angle is the same as the colatitude φ_1. Similar representations can be obtained for other epochs of the universe life. Fig. 13 shows the results if the R_u scale is linear; the trend of the curves gets confused in the proximity of the white hole, where big variations of the angular separation are masked by the scale inadequacy. A better visibility of what happens in the proximity of the white hole is obtained if a logarithmic scale is adopted for R_u (see fig. 14); conversely, the curves get closer for big values of R_u.

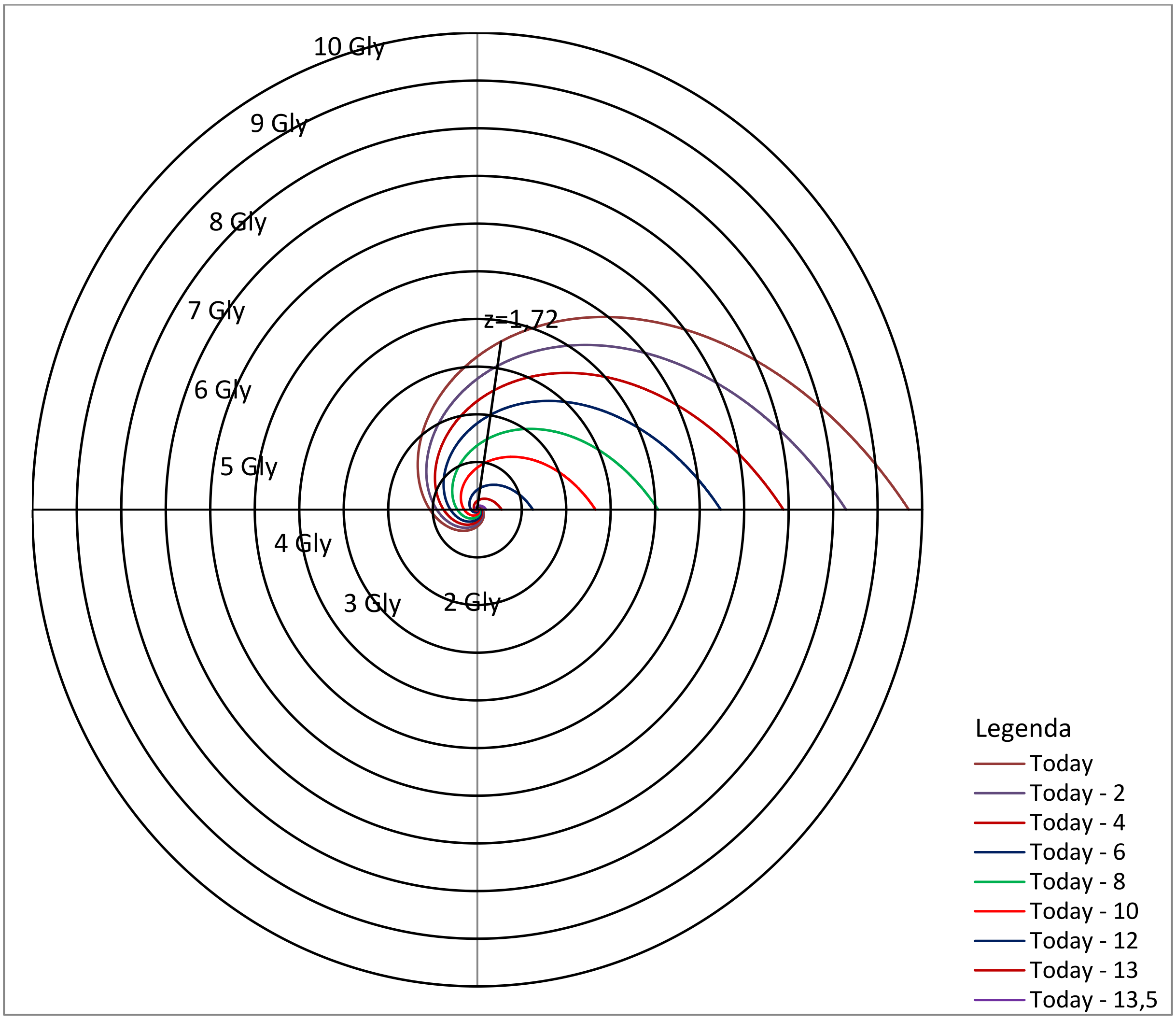

Fig. 13 – Light geodesics in linear scale

The curves which we have obtained are spiral-like, as expected. In fact a spiral is obtained by combination of a circular uniform motion and a linear radial uniform motion. Our curves are not perfect spirals, because the circular motion (light travelling the universe) is uniform, but the linear motion (governing the universe expansion) is not uniform, since the expansion velocity is ever increasing; however, as time tends to infinity, the radial expansion velocity of the universe tends to a constant value, and the curves tend to perfect spirals.

The length of the curve corresponding to today's universe equals the distance covered by the light during the universe life, i.e. 13,792 billion light-years, which must be compared to a universe radius of 9,700 billion light-years.

The red-shift corresponding to a given angular separation keeps almost constant throughout universe life for small values of the angle, whereas significant to large variations appear beyond a separation of about 60 degrees.

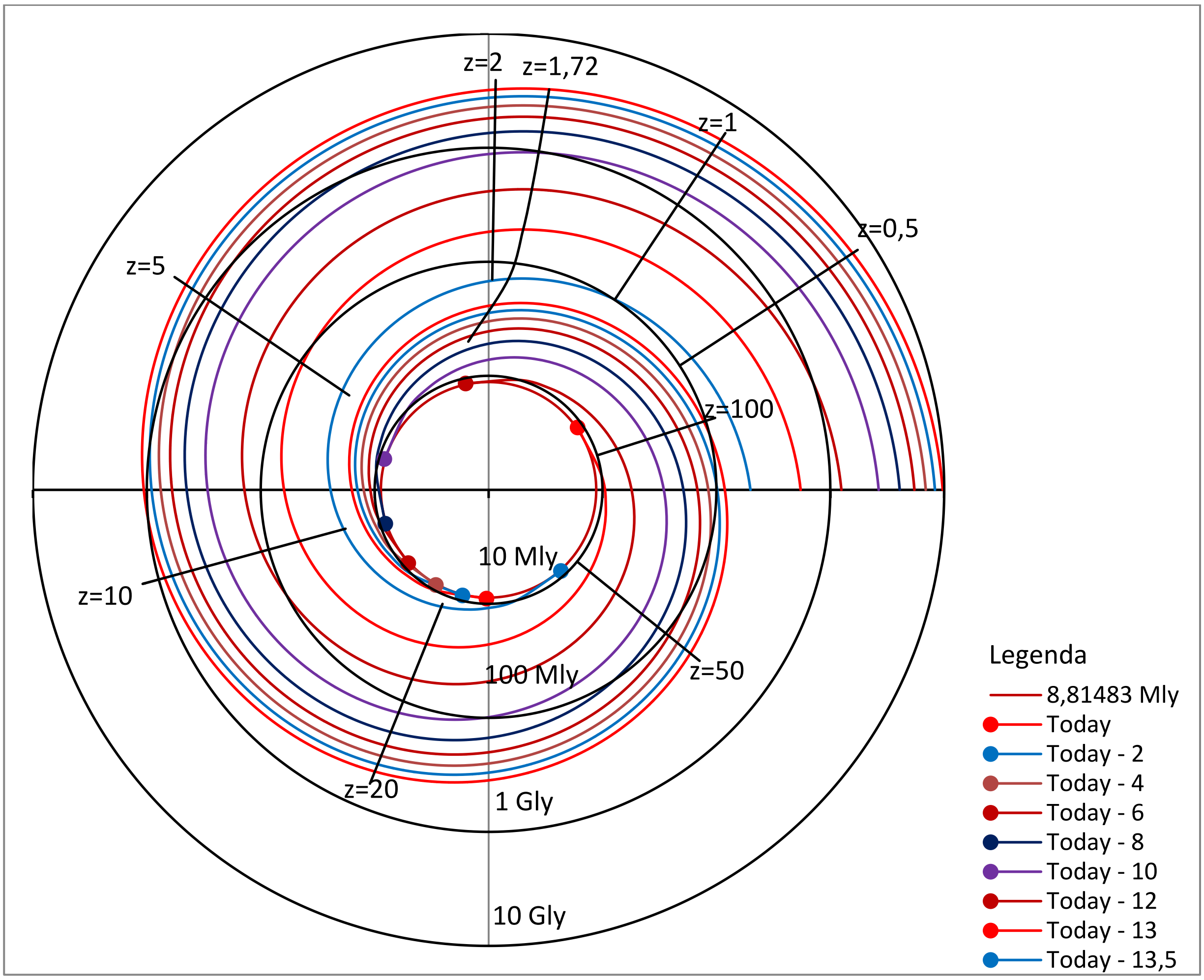

Fig. 14 – Light geodesics in logarithmic scale

The quasi-spiral curves are geodesics, i.e. minimal-length paths for the light to connect two points in the given gravitational field. The observation point and the light source move on two different radii separated by an angle $\alpha = \varphi_1$ (the colatitude); points located on the observation radius and on the source radius cannot be connected by the light in an arbitrary way, but a biunivocal correspondence exists between the elements of the two ensembles. At the beginning of universe life, the geodesic keeps close to the white hole event horizon, but after a few billion years the geodesic rolls out, so that very large regions of the universe do not seem observable for a long time to come; one could even question if some far regions of the universe will ever be observable. This has originated the very complex problem of the *horizon*, which will be discussed in a subsequent paragraph.

Fig. 14 also shows the constant-redshift lines. These are straight lines passing through the hyper-sphere center, if we are sufficiently far from the Big Bang and/or the red-shift is not too large. This means that, if the angular separation between the Earth and the observed celestial body does not change, the celestial body red-shift will not change in time.

It is interesting to observe that the maximum CMD may be well in excess of $2\pi R_{ut}$; this means that in the long term it is possible to receive light more than once from the same celestial body. For instance (see Fig. 14), from celestial bodies showing an angular separation of 32,98° from the Earth we receive two images, one with red-shift 0,5 (Look-back Time 4,5905 Gy), and a second one with red-shift 120,25 (angular separation 360°+32,98°, Look-back Time 13,673 Gy). If, instead, the angular separation is about zero, we observe the celestial body as it is today (red-shift ~ 0) and as it was 13,617 [Gy] ago (red-shift 80,9).

Detecting the acceleration of the universe expansion

Recent measurements have demonstrated that the expansion of the universe is accelerating. This has been possible only recently because the expansion velocity was significantly different only in a sufficiently remote past, i.e. at large distances from the observation point. Our cosmological model allows to easily evaluate the increase of the co-moving distance due to universe acceleration, with respect to the constant velocity expansion.

Let us assume that the radial expansion velocity has been constantly equal to the present value during all the life of the universe, i.e. $c/v_u = \text{constant} = 1{,}4198102$; equation (42) becomes therefore:

$$D_{c,0} = 1{,}4198102 \cdot \int_{R_{ue}}^{R_{ut}} \frac{R_{ut}}{R_u} \cdot dR_u \qquad (50)$$

where the pedix "0" in the distance indicates that we are considering here the case of zero-acceleration. We will obtain:

$$D_{c,0} = 1{,}4198102 \cdot R_{ut} \cdot ln \frac{R_{ut}}{R_u}$$

Changing the logarithm base from e to 10, and recalling equation (48), we finally obtain:

$$D_{c,0} = 31711{,}57 \cdot log(1+z) \quad [\text{Mly}] \qquad (51)$$

If we assume a logarithmic scale for the (1+z) axis, this is the equation of a straight line.

If the universe expansion is accelerating, this means that the expansion velocity in the past was smaller than today, therefore the distances traveled by the light will be higher, and the co-moving distance will be higher than given by equation (51). The difference ΔD_c between the value given by equation (43) (accelerating universe) and the value given by equation (51) (constant velocity expansion) is very small for R_u close to 9700 [Mly], and becomes significant only for distances large in space and time. Fig. 15 shows this difference. Fig. 16 shows the same difference in logarithmic scale. The difference in percent is shown in fig. 17: it is very small for red-shifts up to 0,5, then it increases slowly and reaches a value of 10,8% close to the Big Bang, for z = 1077. We may therefore understand how difficult was the task of detecting a positive acceleration; in fact the difference in percent is absolutely negligible for small red-shift values, and is still small for far Supernovae, with red-shifts included between 0,3 and 1,0; therefore it is not surprising that the acceleration of the universe expansion was discovered only by the observation of far Supernovae; on the contrary, it is surprising that the acceleration was detected in a reliable way, despite the very low value.

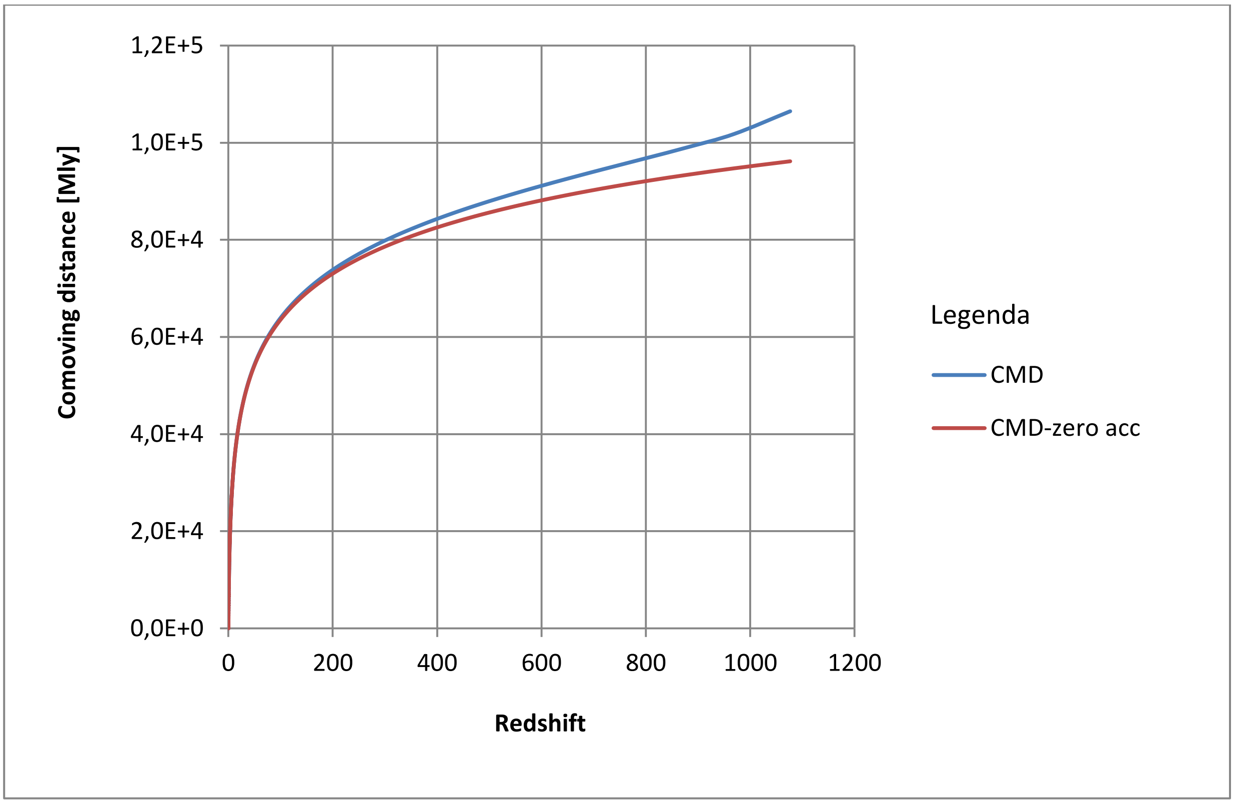

Fig. 15 – Co-moving distance at constant velocity and with positive acceleration

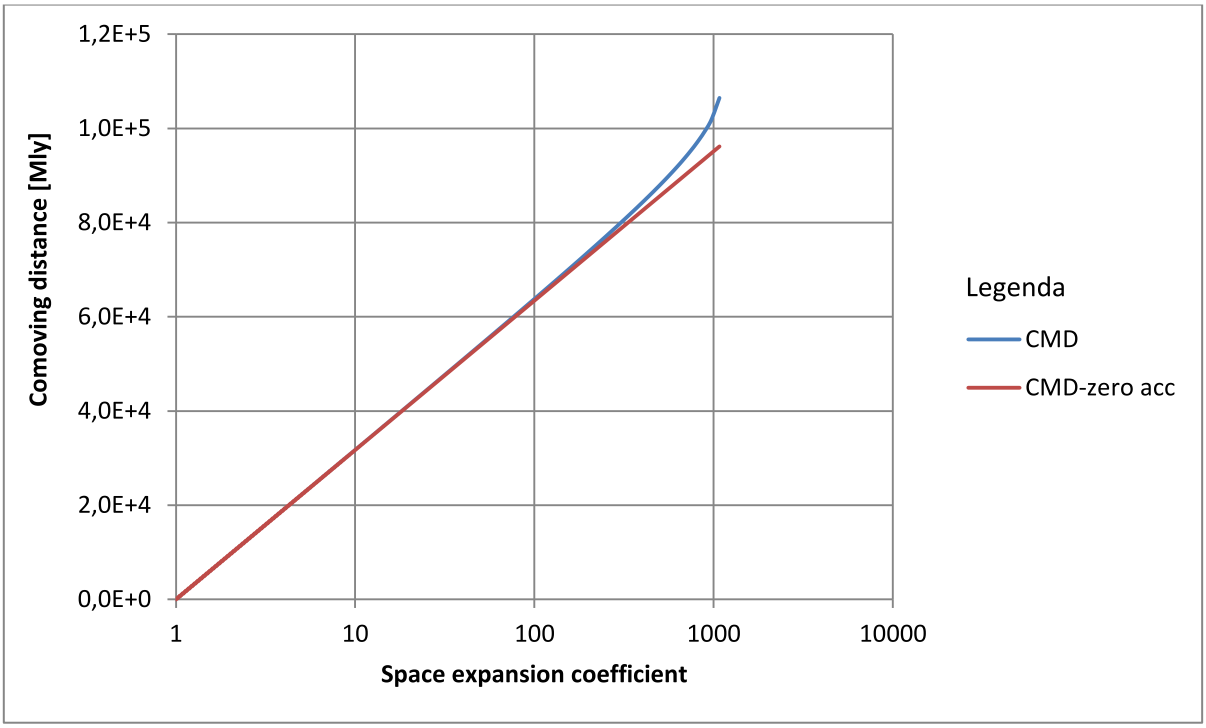

Fig. 16 – Co-moving distance versus space expansion coefficient (log scale)

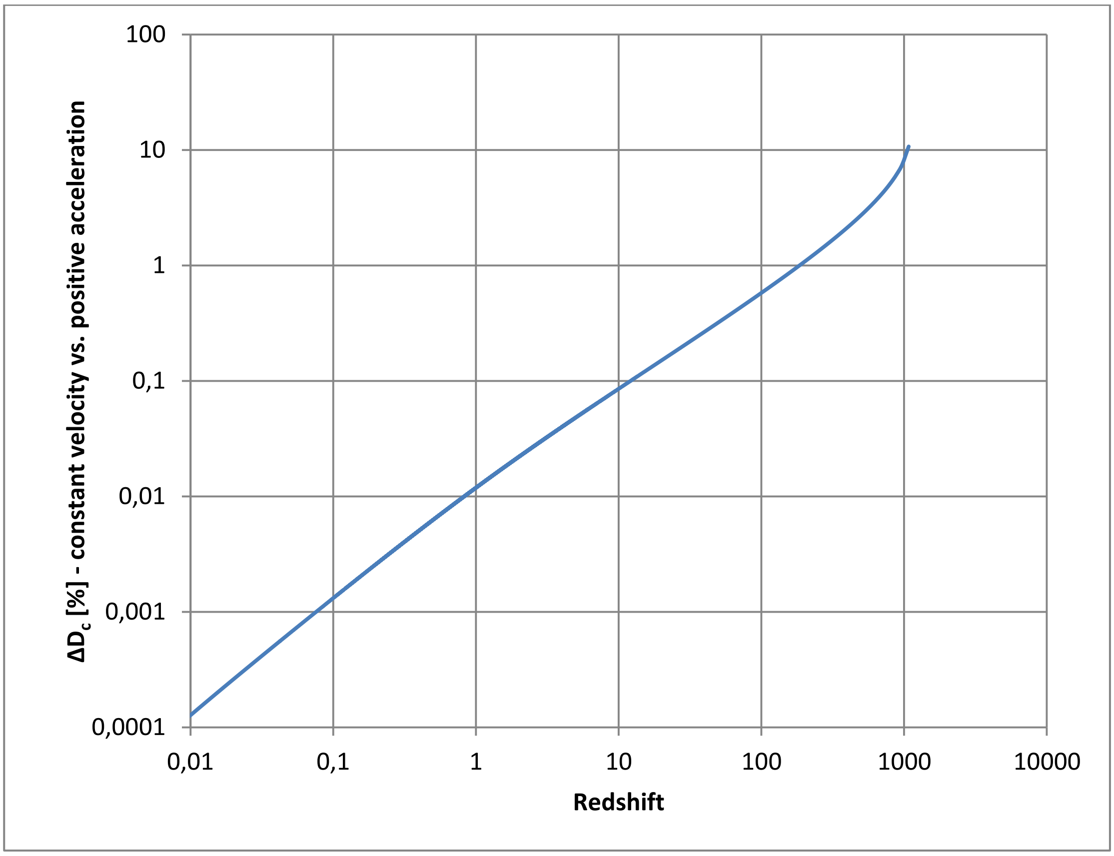

Fig. 17 – Percent variation of the CMD from constant velocity to positive acceleration

Co-moving and luminosity distances for several Supernovae

Table 2 compares the measured luminosity distance with the co-moving distance predicted by our cosmological model, for the close Supernovae used for a precise assessment of the Hubble parameter (Calán-Tololo set). The two values are rather close, as expected for small red-shift values. Table 3 compares the measured luminosity distance with the co-moving distance predicted by our cosmological model, for the far Supernovae used to detect the acceleration of the universe expansion. In this case the luminosity distance is significantly higher than the co-moving distance, by a factor which increases with the red-shift, and reaches 2÷2,5, as expected. The two tables also show, for comparison, the co-moving distance value computed using the UCLA Cosmological Calculator[29].

As concerns the accuracy of the luminosity distance measurements, it is important to recall here a prudent statement in the paper by Riess et al.[14]: "How reliable is this conclusion? Although the statistical inference is strong, here we explore systematic uncertainties in our results with special attention to those that can lead to overestimates of the SNe Ia distances." (page 18).

Supernova	Redshift z*	μ**	D_L [Mly] Measured	D_c [Mly] This model	D_c [Mly] UCLA
1992al	0,014	34,13	218,5	191,6	196
1992bo	0,018	34,88	308,6	245,5	252
1992bc	0,02	34,77	293,4	273	280
1992P	0,026	35,59	428	353,5	363
1992ag	0,026	35,53	416,3	353,5	363
1992bg	0,036	36,49	647,8	487	502
1992bl	0,043	36,53	659,8	580	598
1992bh	0,045	36,87	771,7	606,5	626
1993ag	0,05	37,11	861,8	672	695
1990af	0,05	36,67	703,8	672	695
1993O	0,052	37,31	945	698	722
1992bs	0,063	37,63	1095	841,6	873
1992bp	0,079	37,96	1274,8	1048	1091
1992br	0,088	38,09	1353,4	1162	1213
1992aq	0,101	38,33	1511,6	1325	1388

* Source: Perlmutter
**Source: Riess

Table 2 – Luminosity and co-moving distances for close Supernovae (Calán/Tololo)

Table 3 – Luminosity and co-moving distances versus red-shift for remote Supernovae

(Riess – Perlmutter)

Supernova	Red-shift z	μ*	D_L [Gly] Measured	D_c [Gly] This model	D_c [Gly] UCLA
1996J**	0,3	41,38/40,90	5,8	3,613	3,93
1996K**	0,38	41,63/42,21	8,17	4,436	4,879
1996U**	0,43	42,55/42,34	9,85	4,9266	5,451
1996E**	0,43	41,74/42,03	7,25	4,9266	5,451
1997cc**	0,44	41,95/42,26	8,17	5,022	5,564
1995K**	0,48	42,45/42,49	9,8	5,399	6,007
1997cj**	0,5	42,40/42,70	10,32	5,5845	6,226
1996I**	0,57	42,76/42,83	11,8	6,213	6,969
1996H**	0,62	42,98/43,01	12,97	6,645	7,482
1997ap***	0,83	43,67	17,5	8,323	9,481
1997ck**	0,97	44,39/44,30	24,6	9,3387	10,683

* First value by MLCS method
Second value by Template Fitting
** Source: Riess
*** Source: Perlmutter

The horizon

The co-moving distance has been computed starting from the observation point, and is related to the past history of the universe: it can be defined as the distance existing today between the observer and a star, the light of which was emitted in the past and is being received today. But if we change the viewpoint, and consider the light starting today from a star in the direction of the observer, what will happen? Are we sure that this light will be able to reach the stated observation point at some time in the future, despite the universe expansion? The answer is certainly positive if the angular separation between the star and the observation point is small, but could be negative if the separation angle is very large; in this case, how can we determine the break-even separation? These questions originate the concept of *horizon*, which is rather ambiguous and may easily give rise to misunderstandings and misconceptions.

A comprehensive discussion of the possible definitions of *horizon*, and of the related misconceptions is given by T. H. Davis and C. H. Lineweaver [31].
The first possibility of confusion originates from the fact that the recession velocities of very far objects can exceed the velocity of light. In fact, if H is the Hubble parameter, the recession velocity at distance D is $v_{rec} = H \cdot D$, which exceeds the velocity of light for $D_{HS} = c/H$.

This is physically possible because it is due to the expansion of space, whereas all physical objects (i.e. the parts of the universe) move with speed always lower than the speed of light; for instance, in our cosmological model the physical objects move radially, and the distances between them may increase even at superluminal velocity. The distance D_{HS} beyond which the recession velocity becomes superluminal is the radius of the Hubble Sphere, which is not an horizon, since it is possible to see beyond it.

Davis and Lineweaver define two different horizons:

- the *particle horizon*, which is the distance travelled by light in the past, to reach the observation point;
- the *event horizon*, which is the distance light will have to travel in the future, to reach the observation point.

If we consider now the CMBR received today on the Earth, the particle horizon will equal the age of the universe, but the corresponding co-moving distance will be much larger, due to space expansion. With our cosmological model we obtain:

Age of the universe = 13,792 [Gy]
CMBR particle horizon =13,792 [Gly]
CMBR comoving distance = 109,155 [Gly]
Aspect ratio = $109{,}155/13{,}792 = 7{,}91437$

However the fundamental questions we asked previously all pertain to the future, i.e. to the event horizon, then we will concentrate our efforts on the analysis of the related problems. Whereas Davis and Lineweaver always define the horizon as a distance, in our cosmological model it is possible to define it as a distance or as a separation angle, which equals the colatitude of the observed star with respect to the observation point, assumed to be the origin of our reference system.

We will start considering the horizon from the distance viewpoint. As time passes, the distance between the observer and the light emitted by a celestial body changes, due to two opposite reasons: the propagation of light, which causes a decrease of the distance, and the expansion of the universe, which causes a distance increase. When these variations are equal, the distance keeps unchanged; if the distance covered by light is constantly in excess of the distance increase due to universe expansion, the light emitted by the celestial body will finally reach the observer, so that we can say that the celestial body is observable; if, viceversa, the distance covered by light is constantly smaller than the distance increase due to universe expansion, the light emitted by the celestial body will never reach the observer, so that we can say that the celestial body is unobservable. But the increase of the distance due to universe expansion is proportional to the distance itself; it would seem, therefore, that a break-even distance exists, such that celestial bodies located within this distance from the observer will be observable, whereas the ones located beyond this distance will be unobservable. Have we already found the correct definition of the *event horizon*? We will show shortly that this constraint to guarantee that the star light reaches the observer is much more stringent than necessary, and must be considered only a sufficient constraint. We will however develop the calculations concerning this bound, and obtain interesting results.

Be now:
- α = star-observer separation angle;
- $R_u(T_i) = R_{u,i} =$ universe radius at time T_i;
- $R_u(T_{i+1}) = R_{u,i+1} =$ universe radius at time $T_{i+1} = T_i + \Delta T_i$;
- ΔT_i (variable quantity) = time difference between T_{i+1} and T_i;
- ΔR_u (constant quantity) = variation of the universe radius in the time interval ΔT_i;
- $v_u(T_i) = v_{u,i} = \Delta R_u / \Delta T_i$ = radial expansion velocity of the universe at time T_i;
- $v_u(T_{i+1}) = v_{u,i+1} = \Delta R_u / \Delta T_{i+1}$ = radial expansion velocity of the universe at time T_{i+1};
- c = velocity of light;
- $c \cdot \Delta T_i$ = distance covered by light in the time interval ΔT_i;
- $\alpha \cdot R_{u,i}$ = physical distance between the star and the observer at the light emission time T_i;
- $\alpha \cdot R_{u,i+1}$ = physical distance between the star and the observer at time T_{i+1};
- $\alpha \cdot \Delta R_u$ = distance increase due to the expansion of the universe in the time interval ΔT_i.

At time T_i the physical distance between the star and the observer is $\alpha \cdot R_{u,i}$, and light is emitted by the star. During the time interval ΔT_i the light covers the distance $c \cdot \Delta T_i$, and the universe expands with radial velocity $v_{u,i}$, so that at time T_{i+1} the star-observer distance is:

$$D(T_{i+1}) = D_{i+1} = \alpha \cdot R_{u,i} + \alpha \cdot \Delta R_u - \mathrm{c} \cdot \Delta T_i \qquad (52)$$

The distance D keeps unchanged (break-even condition) if $\alpha \cdot \Delta R_u - c \cdot \Delta T_i = 0$, i.e. if:

$$\alpha = c \cdot \frac{\Delta T_i}{\Delta R_u} = \frac{c}{v_{u,i}}$$

and this gives us the break-even value of the separation angle:

$$\alpha_{BE,i}[\text{radians}] = \frac{c}{v_{u,i}} \qquad (53)$$

Multiplying by the universe radius we obtain the break-even distance:

$$D_{BE,i} = {}^{c \cdot R_{u,i}}/_{v_{u,i}} \qquad (54)$$

We still hesitate to call this value "horizon", and this prudence will shortly appear well justified. But, for the time being, we want to better define this break-even curve.

At present the universe radius is 9,700 billion light-years and the radial expansion velocity is 211.149.672 m/s. The present value of the break-even separation angle is therefore:

$$^{299.792.458}/_{211.149.672} = 1{,}41981\ [\text{rad}] = 81{,}3491°$$

When R_u goes to infinity, the radial expansion velocity tends asymptotically to $0{,}704508 \cdot c$, which corresponds to a break-even separation angle:

$$^{1}/_{0{,}704508} = 1{,}41943\ [\text{rad}] = 81{,}3273°$$

Going back in time, both radius and expansion velocity decrease, reaching the minimum values at the Big Bang. In our cosmological model, at the beginning the radius of the universe equals the Schwarzschild radius of the white hole R_s, and the radial velocity is zero; as a consequence, the value of the break-even angle is very large, and a horizon problem does not seem to exist. As time passes, however, the radial velocity increases, and the break-even separation angle gradually decreases: when the increase of the distance between two antipodal points equals the light-covered distance, the break-even angle is exactly π. Immediately after that moment, the increase of distance between two antipodal points exceeds the light-covered distance, and the break-even angle becomes smaller than π; in these conditions, a horizon problem would seem to exist. The universe radius for which the break-even angle is π can be found imposing $v_u/c = 1/\pi$ in equation (27). The linear equation so derived can be easily solved obtaining:

$$(R_u)_{\alpha=\pi} = 1{,}15491 \cdot R_s \qquad (55)$$

This result is independent of the white hole dimension. With our assumptions the Schwarzschild radius is 8,81483 [Mly], so $(R_u)_{\alpha=\pi} = 10{,}180334$ [Mly], a value which was reached after about 7,9806 [My] of universe life.

The break-even curve is shown in figs. 13 and 14. It can be easily verified that the geodesic lines cross the break-even curve ($\alpha = \alpha_{BE}$) for red-shift z ∼ 1,8 (see Table 4). This result seems to match very well a statement by Davis and Lineweaver[31]: "……Most observationally viable cosmological models have event horizons and in the ΛCDM model of fig. 1 galaxies with red-shift ∼ 1,8 are currently crossing our event horizon. These are the most distant objects from which we will ever be able to receive information about the present day. …." (Davis and Lineweaver, page 4).

Table 4 – Red-shift value when light geodesic crosses the Break-Even curve

Epoch E [Gy] (Look-back time)	**$\alpha=\alpha_{BE}$ [deg]**	**Time after Big Bang [My]**	**R_u [Mly]**	**Red-shift evaluated at epoch E**	**Red-shift evaluated today**
0 (today)	81,38667	5084,4	3567,5	1,719	1,719
-2	81,39678	4347,8	3049	1,719	2,18
-4	81,41104	3611,9	2531	1,719	2,83
-6	81,4327	2875,8	2013	1,719	3,82
-8	81,4693	2139,4	1495	1,72	5,48
-10	81,5452	1402,6	977	1,72	8,93
-12	81,7951	664,5	459	1,724	20,13
-13	82,42	295,2	201	1,732	47,26
-13,5	84,597	108	72	1,751	133,7
-13,7	97,8	29	20,5	1,99	470
-13,75		No crossing			

But we can now ask ourselves: “Why reception from objects with z > 1,8, which was possible in the past, should become impossible in the future?”. For a deeper discussion of this problem, let us show first that $\alpha \leq \alpha_{BE}$ is just a sufficient condition of convergence; as a matter of fact, it is possible to receive information also when the separation angle is larger than $\alpha_{BE} = \alpha_{suf}$; in fact, be this the value of the separation angle at time T_i (we will call it α_i), let check which is the value of the separation angle at times T_{i+1} and T_{i-1}:

$$D(T_i) = D_i = \alpha_{i-1} \cdot R_{u,i-1} + \alpha_{i-1} \cdot \Delta R_u - \mathrm{c} \cdot \Delta T_{i-1} \qquad (56)$$

$$D(T_{i+1}) = D_{i+1} = \alpha_i \cdot R_{u,i} + \alpha_i \cdot \Delta R_u - \mathrm{c} \cdot \Delta T_i = \alpha_i \cdot R_{u,i} \qquad (57)$$

$$D(T_{i+2}) = D_{i+2} = \alpha_{i+1} \cdot R_{u,i+1} + \alpha_{i+1} \cdot \Delta R_u - \mathrm{c} \cdot \Delta T_{i+1} \qquad (58)$$

where:

$$\alpha_{i+1} = \frac{\alpha_i \cdot R_{u,i}}{R_{u,i+1}} = \alpha_i \cdot \frac{R_{u,i}}{R_{u,i} + \Delta R_u} \sim \alpha_i \cdot \left(1 - \frac{\Delta R_u}{R_{u,i}}\right)$$

and for simmetry:

$$\alpha_{i-1} \sim \alpha_i \cdot \left(1 + \frac{\Delta R_u}{R_{u,i}}\right) > \alpha_i = \alpha_{BE}$$

and substituting in (56):

$$D_i \sim \alpha_i \cdot \left(1 + \frac{\Delta R_u}{R_{u,i}}\right) \cdot \left(R_{u,i-1} + \Delta R_u\right) - \mathrm{c} \cdot \Delta T_{i-1} \qquad (56a)$$

$$D_i = \alpha_i \cdot R_{u,i} + \alpha_i \cdot \Delta R_u - \mathrm{c} \cdot \Delta T_{i-1}$$

and since:

$$\Delta T_{i-1} = \Delta T_i \cdot \frac{v_{u,i}}{v_{u,i-1}} > \Delta T_i$$

we obtain:

$$D_i < D_{i+1} = \alpha_i \cdot R_{u,i} = D_{BE,i}$$

We have shown that, for objects located beyond the break-even separation angle, although the distance increases, the separation angle decreases; this means that information can be received also from objects located beyond $\alpha_{BE,i}$. But we must still answer the questions: “Does a horizon exist? In the affirmative, which is the value of the horizon?”.

Since there are two conflicting causes of variation of the distance between the radiating object and the observer, the analysis of the evolution of the distance brought us to the determination of a break-even value for the distance; the value of the separation angle corresponding to this distance is not, however, a break-even value, since the separation angle decreases monotonically also when crossing this value. A strong danger of confusing horizon and break-even distance exists, as a consequence, if we analyze the problem from the distance viewpoint, but not necessarily if we analyze it from the viewpoint of the separation angle. As a matter of fact, the light propagation causes the separation angle to decrease, but the expansion of space does not produce any increase of the separation angle; therefore, the two actions are not conflicting and a break-even value will not exist for the separation angle. The separation angle will decrease monotonically, step by step, and the only question is: "The sum of potentially infinite steps will be finite or infinite? In case of finite sum, the value will be sufficient to cover the distance between the object and the observer or not?".

The reduction of the separation angle in the time needed to increase by 1 [Mly] the universe radius is:

$$\Delta\alpha_i = \frac{c \cdot \Delta T_i}{R_{u,i}} = \frac{c}{v_{u,i}} \cdot \frac{\Delta R_u}{R_{u,i}}$$

Today the universe radius is 9,700 billion light-years, and the radial expansion velocity is 211.149.672 [m/s]. The reduction of the separation angle is therefore:

$$\Delta\alpha_i = {}^{1{,}41981}/_{9.700}\ [\text{rad}] = 1{,}46372 \cdot 10^{-4} \cdot {}^{180}/_{\pi}\ [\text{deg}] = 8{,}3865 \cdot 10^{-3}\ [\text{deg}]$$

The present value of the radial velocity is rather close to the limit value for $i \to \infty$; assuming $v_{u,i} = \text{constant} = v_{u,\infty} = 0{,}704508 \cdot c$, we obtain a value of $\Delta\alpha_i$ which is slightly underestimated:

$$\Delta\alpha_\infty = {}^{1{,}41943}/_{9.700}\ [\text{rad}] = 1{,}46333 \cdot 10^{-4} \cdot {}^{180}/_{\pi}\ [\text{deg}] = 8{,}3843 \cdot 10^{-3}\ [\text{deg}]$$

To study the event horizon problem we must compute the total variation of the separation angle from today to infinity:

$$\Delta\alpha_{total} = \Delta\alpha_{9701} + \Delta\alpha_{9702} + \Delta\alpha_{9703} + \ldots..$$

$$\Delta\alpha_{total} = 1{,}41943 \cdot \sum_{9701}^{\infty} \frac{1}{i}$$

Since the sum of the harmonic series diverges, we find the important result that the light emitted by the star will always reach the observer at some time in the future, whichever the values of R_u and α.

Of course the divergence is guaranteed, *a fortiori*, also if the series starts at any time in the past; therefore it was always guaranteed, since the Big Bang, that signals emitted from any point in space and time will reach every observer, at some time in the future. This does not mean, however, that the observer was able to see the signals coming from all the parts of the universe in every moment of the past. There was in fact a time, in the early history of the universe, when the α corresponding to the maximum co-moving distance was smaller than π. Fig. 18 shows how the value of α depends on the age of the universe. The value of π is reached at the age of about 52,0091 [My], when the universe radius is about 34,80055 [Mly].

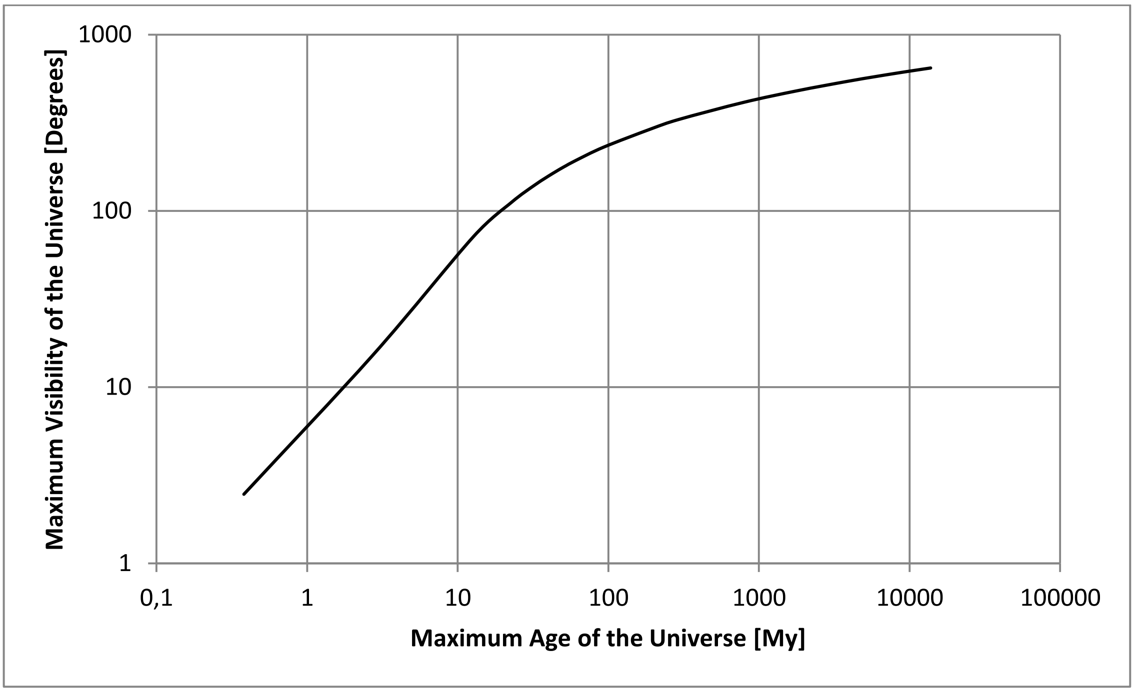

Fig. 18 – Maximum visibility of the universe versus maximum age

Figs. 19 and 20 show the light geodesics in the early universe, respectively in linear scale and in logarithmic scale. These figures show very clearly that in the very early times only part of the universe is visible, and that complete visibility is reached only 52,0091 [My] after the Big Bang (Look-back Time = 13,74075 [Gy]), when α reaches the value of π.

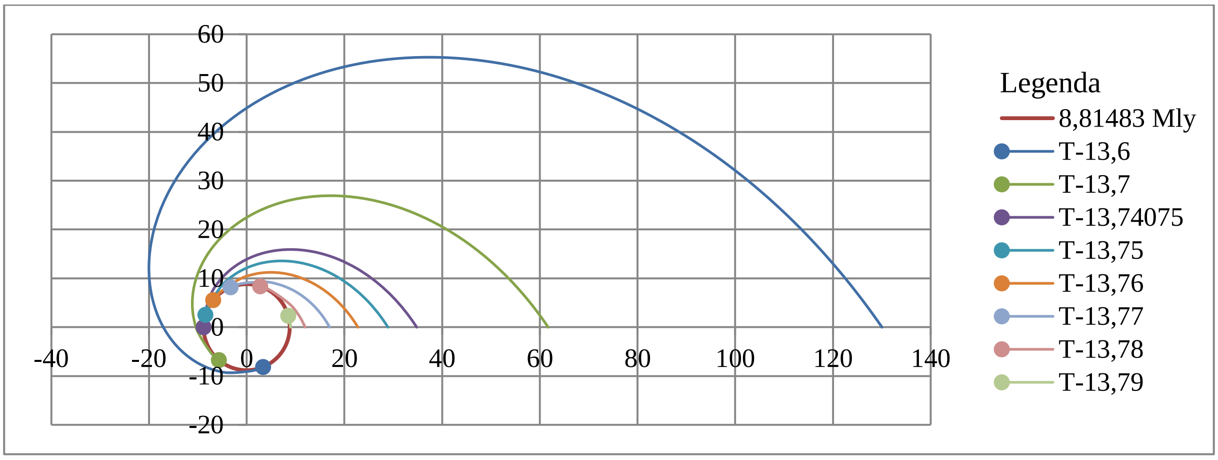

Fig. 19 – Light geodesics in the early universe (linear scale)

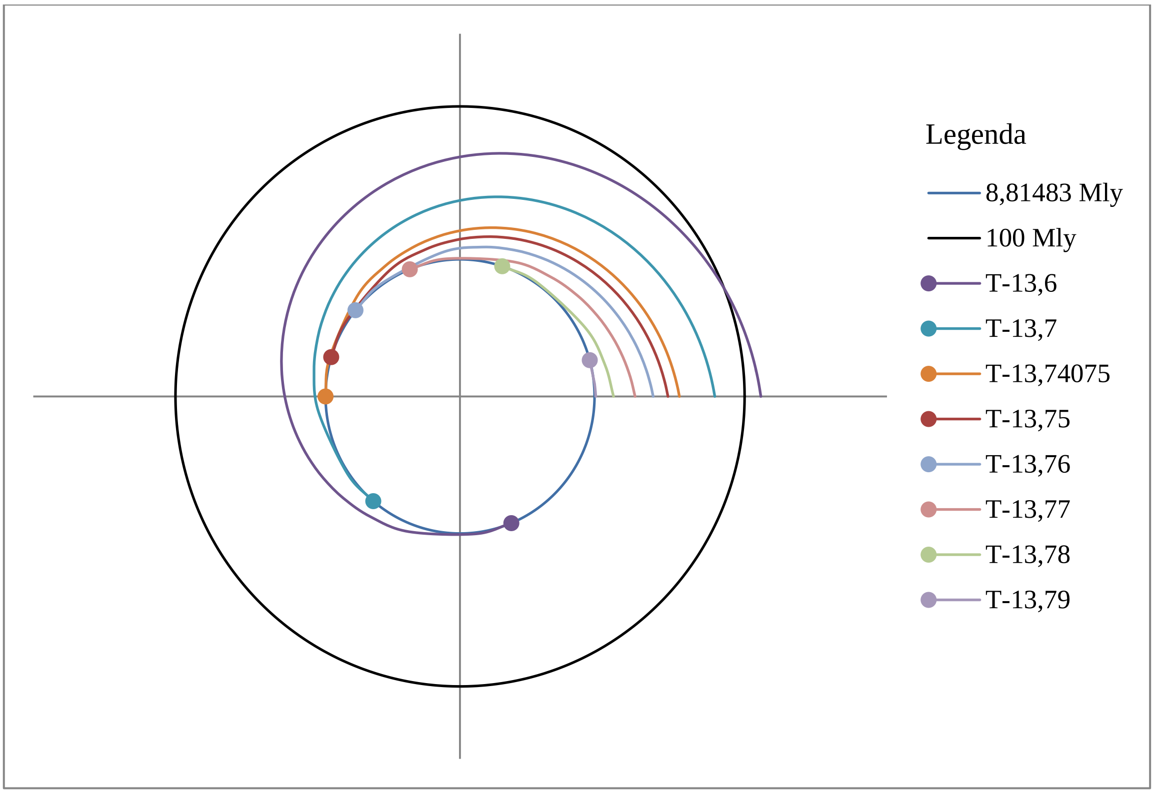

Fig. 20 – Light geodesics in the early universe (logarithmic scale)

CMBR anisotropy

Prior to the recombination time (TABB=380 Ky) the universe is still ionized, therefore it is not transparent to the electromagnetic radiation; however, whereas em waves cannot propagate, gravitational waves propagate (at the speed of light), therefore, due to gravitational attraction, the matter starts to agglomerate, so that the "seeds" of the future structures (stars, galaxies, etc.) are created. When the recombination process is completed, the universe becomes transparent and we may observe the CMBR; due to the gravitational action during the previous 380 [Ky], we must expect some degree of anisotropy in the observed CMBR. Our model allows to give an estimate of the expected CMBR anisotropy.

During a time period of 380 [Ky] the gravitational action of a given point covers a circular area of radius 380 [Kly]; during the same time period the universe radius increases by 3,35 [Kly], from 8,81483 [Mly] (Schwarzschild radius of the white hole = radius of the universe at the Big Bang), to 8, 81818 [Mly] (for TABB=380 Ky). The distance of 380 [Kly], seen from the center of the hyper-sphere of radius 8,81818 [Mly], corresponds to an angle of 2,4684°. Today the universe radius is 9700 [Mly] and the CMD between the Earth and the CMBR is 109155,115 [Mly], therefore the diameter of the expected seed, observed from the Earth, is:

$$2 \cdot 2{,}4684 \cdot \frac{9700}{109155{,}115} = 0{,}4387°$$

This value is in very good agreement with the CMBR observation results. In fact the angular power spectrum of the CMB temperature shows a clear peak [32] for a value of the spatial frequency ℓ equal to 200; this means a cycle of 360/200 = 1,8°, and a seed diameter which can be roughly estimated as 1/4 of this value, i.e. 0,45°; as a matter of fact, the formula which is currently employed to determine the most likely value of the seed diameter is:

$$\theta = \frac{100°}{\ell}$$

which gives a value of 0,5°.

CONCLUSIONS

The proposed cosmological model agrees very well with the present knowledge about the universe; in particular, the agreement looks practically perfect as concerns age of the universe, Hubble constant, mass budget, value of the red-shift at the break-even curve. Universe dimensions, expansion velocity and total mass are computed. The model also allows to determine the distance of a celestial body from the Earth in space (CMD) and time (LBT) starting from red-shift measurements (which are rather precise). A problem of horizon does not seem to exist: all parts of the universe are visible by every observer; this was true also in the past, with the exception of a relatively short initial period, and will continue to be true in the future.

The model reduces strongly the matter/antimatter unbalance and makes superfluous the inflation hypothesis; in addition, the dark side of reality reduces from 96% to 26%. This is true, however, only if the considerations are limited to our universe; the multiverse option remains on the table, and could be strongly supported by our model. Last but not least, the time available for the creation of the structures is much larger than allowed by traditional Big Bang models, and the dimension of the seeds of the structures (CMBR anisotropy) looks reasonable.

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INDEX

THE ROAD TO TRUTH

"Truth is not in a single dream, but in many dreams"

(from *The Book of One Thousand and One Nights*)

"God hath spoken once; twice have I heard this; that power belongeth unto God."

(King James Bible, Psalm 62, 11)

"[……..] Now it will be necessary for you to investigate everything,
both the eternal heart of the perfect truth,
and the opinions of the deadly humans, which do not deserve full confidence.
However you will also understand this: how it was necessary in the reality
the existence of the opinions, which exist in all possible ways with respect to everything."

(Parmenides, *About Nature*, Fragment 1, 28-32)

A <u>very</u> free translation gives what we may call the *Gospel according to Parmenides*:

"You shall love Truth with all your heart, all your soul, all your mind;
you shall love the opinion of your neighbour as your own opinion."

"At that season Jesus answered and said, I thank thee, O Father, Lord of heaven and earth, that thou didst hide these things from the wise and understanding, and didst reveal them unto babes: yea, Father, for so it was well-pleasing in thy sight."

(World English Bible, *The Gospel According to St. Matthew*, 11, 25-26)

"It is not pretentiousness to publish a book about Jesus at the age of thirty: it is pretentiousness hesitating to publish, because a theology book is not published when perfection is reached, but to make available to the others what we were given, with the hope to be surpassed by those who will do better than us. In this way, only in this way, I could publish all my books."

(Henry de Lubac to Bruno Forte, from *La sfida di Dio = The Challenge of God*, Mondadori 2001, page 135)

"We are like dwarfs perched on the shoulders of giants. We see more and farther than our predecessors, not because we have keener vision or greater height, but because we are lifted up and borne aloft on their gigantic stature."

(Bernard of Chartres, quoted by John of Salisbury in his *Metalogicon*)