Looping of Collatz conjecture number series
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Abstract
Number series of Collatz conjecture reaches to value 1 finally if the series of number has no looping. This has been mentioned on viXra:2204.0151 (*1).
There the possibility of looping has been estimated very small. But it has not been proven that the possibility is zero. Here focusing to the looping, investigation is done.

1. Introduction
On *1, the possibility of looping was tried to estimate statistically. Here at first the mechanism of Collatz operation is analyzed, then it is confirmed that there is no possibility of looping algebraically.

2. Collatz conjecture procedure
Procedure of collatz problem has following operations and conjecture.

   It starts with positive odd number integer \( n_1 = 2n + 1 \).
   
   \[
   \begin{align*}
   \text{Compute } n_2 &= 3 \times n_1 + 1 = 3 \times 2n + 4. \quad (1) \\
   n_1 &\to \frac{n_2}{2^m} \quad (2)
   \end{align*}
   \]

   Repetition of above operations makes \( n_1 \) changed and finally \( n_1 \) becomes value 1 even if it is started from any positive odd integer \( n_1 \).

   Above (2) means multiple positive odd integer \( n_1 \)'s which have following value after (1), becomes same positive odd integer \( n_t \) after (2).

   \[
   n_2 = 2^m n_t
   \]

   \( n_t: \text{positive odd integer} \quad m: \text{depend on } n_2 \)

   When \( m=1, \ n_t = 3n+2 \). \quad (4)
   When \( m=2, \ n_t = \frac{3}{2}n+1 \). \quad (5)

   Therefore

   when \( m=1 \), the one iteration increase the number \( n_1 \). Its increment rate is \( (3n + 2)/(2n + 1) \). \quad (6)
   when \( m=2 \), the one iteration decrease the number \( n_1 \). Its decrement rate is \( (\frac{3}{2}n + 2)/(2n + 1) \). \quad (7)
This means only when \( m=1 \), next \( n_1 \) value becomes larger than the value of one iteration before.

Regarding to large \( n \) region where looping existence is suspected yet, value 1,2 in (6) (7) can be neglected. Then (6) (7) become

\[
\begin{align*}
\frac{3}{3} & \quad \frac{3}{4} \\
\Rightarrow & \\
\frac{3}{4} & \\
\end{align*}
\]

(8) (9)

3. Looping

As introduced on *1, the fact that Collatz conjecture has looping means that it has two equal numbers in the numbers chain as following sample.

\[
m_1,m_2,m_3,\ldots,m_{n-1},m_n, \quad m_n = m_1
\]

As a simpler and more possible looping sample, following case is considered.

\[
m_1 \quad m_n
\]

\[
\begin{array}{c}
\Downarrow \\
r \\
\Downarrow \\
s \\
\end{array}
\]

\[
m_j
\]

Fig. 1

From \( m_1 \) to \( m_j \) \( r \) times iterations are steps each decrease by \( \frac{3}{4} \). From \( m_j \) to \( m_n \) \( s \) times iterations are steps each increase by \( \frac{3}{2} \). Therefore

\[
m_1\left(\frac{3}{4}\right)^r = m_j \quad \text{then}
\]

\[
m_1 = \left(\frac{3}{4}\right)^r m_j
\]

(11)

\[
m_n = \left(\frac{3}{2}\right)^s m_j
\]

(12)

On looping condition (10),

\[
\left(\frac{3}{4}\right)^r m_j = \left(\frac{3}{2}\right)^s m_j \quad \text{then}
\]

(13)

\[
\left(\frac{3}{4}\right)^r = \left(\frac{3}{2}\right)^s
\]

(14)

\[
\begin{align*}
\text{rlog}_{\frac{3}{4}} = \text{slog}_{\frac{3}{2}} \\
0.124938737r = 0.176091259s
\end{align*}
\]

(15) (16)
As same way, more general sample is considered.

\[ m_1 \cdots m_n \]

Fig. 2

In this case,

\[ m_1 \left( \frac{3}{4} \right)^r \left( \frac{3}{2} \right)^s = m_n \]

On looping condition (10),

\[ m_1 \left( \frac{3}{4} \right)^r \left( \frac{3}{2} \right)^s = m_n = m_1 \]

\[ \left( \frac{3}{4} \right)^r \left( \frac{3}{2} \right)^s = 1 \]

Therefore this case also get same result as (16).

Following consideration is for whether (10) can be accomplished or not on above investigation.

Increment and decrement rate combination with which looping is more possible, is used. (6)(7) \hspace{1cm} (a)

Increment and decrement rate is calculated closer but approximately. (8)(9) \hspace{1cm} (b)

Most possible r and s ratio for (10) is calculated. (16) \hspace{1cm} (c)

Even under above condition, \( m^1 \) could not be exactly equal to \( m^n \) even considering approximated error. The reason of this is (14) has no integer solution with r and s because \( \frac{4}{3} \) is a repeating decimal and \( \frac{3}{2} \) is a finite decimal.

(Regarding to sample calculation, refer to 5. appendix) \hspace{1cm} (d)

Therefore on other condition than (a) or for all two numbers on other condition than (c), the two numbers in the chain have not even closer value each other.

So (10) is not accomplished.

On this consideration, Collatz conjecture number series has no looping.

4. Conclusion

It has been mentioned on *1 that Collatz conjecture is correct statistically if there is no looping. If the proof of no looping here is true, it makes possible sufficient repetition of Collatz conjecture operation. Therefore the statistical truth becomes general truth.
5. Appendix

When \( r \) and \( s \) are two digits, more appropriate these values for (16) are \( r=17, s=12 \).

In this case result is
\[
\binom{4}{3}^{17} = 133.0321704 \\
\binom{3}{2}^{12} = 129.7463379.
\]

When \( r \) and \( s \) are three digits, more appropriate these values for (16) are \( r=176, s=125 \)

In this case result is
\[
\binom{4}{3}^{176} = 9.75435 \times 10^{21} \\
\binom{3}{2}^{125} = 1.02661 \times 10^{22}.
\]

Reference
(*1)  https://vixra.org/abs/2204.0151