The Collatz-Compare Conjecture

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dedicated to my father

Abstract

Choose a positive integer,

if it is even divide it by 2, if it is odd multiply it by 3 and add 1, repeat. What is the final result? This problem is known as the 3n+1 problem and the Collatz Conjecture states that no matter the initial value you will always reach the cycle 1,4,2.

Demonstration

In order to solve it, we need to remember that 1/2+ 1/4+1/8...=1 and we need to invent a tool that can calculate infinite iterations: a programmable watch. It could be used to calculate the zeros of the Riemann ζ function or solving the odd perfect number problem, but now we are programming it specifically to solve the Collatz Conjecture.

So in this case the clock needs to have 2 hands, a short one and a long one and no numbers displayed yet. The long hand move clockwise by 1 full turn per unit of time and the short one anticlockwise by 1 full turn. The starting point for both is as in midnight in a regular watch. And at every unit of time the long hand will make a full clockwise turn around the entire display which is circular and when activated it will display consecutive numbers where the long hand is pointing at each time unit starting with a 1 at the initial midnight position. The short hand will move anticlockwise of a full circle every time the long hand reaches midnight. But this clock shrinks its hands movements by half whenever the long hand completes a full circle. Meaning that 2 is displayed at an angle equal to midnight, but 3 is displayed at an angle of half circle and 5 at 1/4 of a circle.

What is interesting about this clock is that every odd number has an unique angle (being exactly between two previous numbers with the exception of the starting 1) and every even number of the form odd 2^n share the same position with any other number of the form odd 2^n where m and n are positive integers and odd is an odd number; so we don't need to care too much about the part of the problem that states :"if it is even divide it by 2" and we just need to read the root value (which is the value of the only odd number at the specific angle we are considering), meanwhile the short hand indicates how much angle do the long hand moves and after infinite time the short hand will shrink to an angle of 0 because it will be at an angle of $\frac{1}{2^{\text{infinite}}}$

In the 3n+1 problem we can have 3 possible outcomes:

the initial value will tend to become $-$infinite, which can be easily excluded since we start with a positive integer and after every iteration we can only decrease its value by halfing it, if and only if the number is even, so the smallest number is $2/2=1$;

another possibility is that we reach +infinite, which can also be easily excluded because at the increase of the value of the number, the short hand shrinks to 0 and then the long hand can't move anymore and the only case where the function is still valid is when a number x agrees with the equation $3x+1=x2^n$ which has only one acceptable positive integer solution at $x=1$ and $n=2$;

the last possibility is a cycle or a number leading to a cycle. But the only possible cycle is the cycle 1,4,2. To demonstrate it, we could show that every number will lead to 1 or alternatively we can show that 1 is connected to every positive integer. If we look at the roots on our watch, we can show that the root 1 is connected to any other root and because there is only one possible path of steps that we can have while making the iterations (if it is odd we multiply by 3 and add 1 and if it is even divide it by 2, until it becomes an odd) it is sufficient. So if it exists a number divisible by 3 for a value in the root 1 after we subtracts 1 to it, we will get a number which has to lead to the root 1; and if we can demonstrate that the root 1 is connected to any and exactly all numbers with the special exception of 1 itself, we have solved and verified the conjecture because there is no number left for a different cycle because a number can't generate another cycle if it has to escape it to reach 1 and then reaching the cycle 1,4,2.

The demonstration is that we can write all numbers of the root 1 as $1\,(2^{-1+d})$ where d is any positive integer from 1 to infinite. With the exception of 1, which is not divisible by 3 when you subtract 1, every other number is alternatively divisible by 3 when subtracted 1 because they are alternatively in the form 1 mod 3 and 2 mod 3, since 1 is 1 mod 3 and multiplied by 2 becomes 2 mod 3 and a 2 mod 3 becomes a 1 mod 3 when doubled, but 1 is special because it will be a 0 mod 3 and equal to 0 when you subtract 1 to it. And the conjecture do not allow the starting number 0 and if it would, it is arguable to consider it an odd number.

So d represents every number in N (natural numbers) and when it is odd (with the exception of 1) it is connected to a unique odd number in the clock and this odd number will alternatively be 1mod3 every other number in the root line (all the numbers with the same root) and so on, meaning that 1 is connected by first degree with half of N and half of theese numbers are connected to another number so 1 is connected to half of half of N too and so on, meaning that the root 1 is connected to 1/2+1/4+1/8...of N which is equal to 1 entirety of N, so all numbers and exactly all numbers with the exception of 1 are connected to the root 1 which always leads to the cycle 1,4,2 because 1 is odd so $1\,(3)+1=4$ which is even, so becoming $4/2=2$, which is also even so becoming a $2/2=1$.

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