Fractal Belief Jensen–Shannon Divergence

Yingcheng Huang\textsuperscript{a}, Fuyuan Xiao\textsuperscript{b}

\textsuperscript{a}School of Big Data and Software Engineering, Chongqing University, Chongqing 401331, China

Abstract

In this paper, a novel belief divergence measurement method, fractal belief Jensen–Shannon (FBJS) divergence is proposed to better measure conflicts between evidences. The proposed FBJS divergence is the first belief divergence that combines the belief divergence theory and the concept of fractal.

Keywords: Dempster–Shafer evidence theory; Belief divergence; Fractal.

1. The proposed method

Definition 1 (FBJS divergence).

Let $m_1$ and $m_2$ be two BPAs in the frame of discernment $\Theta$. The fractal belief Jensen–Shannon divergence (FBJS divergence) is defined as

$$
\text{FBJS}(m_1 || m_2) = \frac{1}{2} \cdot 2^{N-1} \sum_{i=1}^{2^{N-1}} \sum_{j=1}^{2^{N-1}} \tau(A_i, A_j) m_{F_1}(A_i) \log_2 \left[ \frac{2m_{F_1}(A_i)}{m_{F_1}(A_i) + m_{F_2}(A_j)} \right]
+ \frac{1}{2} \cdot 2^{N-1} \sum_{i=1}^{2^{N-1}} \sum_{j=1}^{2^{N-1}} \tau(A_i, A_j) m_{F_2}(A_j) \log_2 \left[ \frac{2m_{F_2}(A_j)}{m_{F_1}(A_i) + m_{F_2}(A_j)} \right],
$$

(1)

$m_{F_j}$ is defined as

$$
m_{F_j}(A_i) = \sum_{A_i \subseteq G_i} \frac{m_j(G_i)}{2^{|G_i|} - 1}, \quad j = 1, 2.
$$

(2)

where $G_i$ is any subset of $A_i$. $m_{F_j}$ represents that BPAs are transferred for one time.

$\tau(A_i, A_j)$ is defined as

$$
\tau(A_i, A_j) = \frac{|A_i \cap A_j|}{|A_i \cup A_j|}
$$

(3)

where $A_i$ are hypotheses of $m$. $A_i \cap A_j$ is the intersection of $A_i$ and $A_j$, $A_i \cup A_j$ is the union of $A_i$ and $A_j$. $|A_i|$ represents the cardinality of $A_i$. 

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