A Proof of the Erdös-Straus Conjecture

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Abstract

In this article, we classify gradually positive integers ≥2, and express each and every class of positive integers into a sum of 3 unit fractions.

First, divide all positive integers ≥2 into 8 kinds, and then formulate each of 7 kinds of these 8 kinds into a sum of 3 unit fractions.

For the unsolved kind, divide it into 3 genera, and then formulate each of 2 genera of these 3 genera into a sum of 3 unit fractions.

For the unsolved genus, further divide it into 5 sorts, and formulate each of 3 sorts of these 5 sorts into a sum of 3 unit fractions.

For two unsolved sorts, let each of them be expressed as a sum of an unit fraction plus a true fraction, and that take out the unit fraction as one of 3 unit fractions which express the sort as the sum. After that, if the true fraction can be transformed identically into an unit fraction, then we follow the formula that Ernst G. Straus made to transform either of these two unit fractions into a sum of two each other's- distinct unit fractions, such that this part of the unsolved sort becomes a sum of 3 unit fractions.

If the true fraction can not be transformed identically into an unit fraction,

then we let it to equal the sum of an unit fraction plus another true fraction, and that take out the unit fraction as one of 3 unit fractions which express the sort as the sum. Next, prove that another proper fraction can be identically converted into an unit fraction.

Due to c≥0, above two cases exist surely when c is taken different values.

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1. Introduction

The Erdös-Straus conjecture relates to Egyptian fractions. In 1948, Paul Erdös conjectured that for any integer n≥2, there be $\frac{4}{\mathrm{n}}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ invariably, where X, Y and Z are positive integers; [1].

Later, Ernst G. Straus conjectured that X, Y and Z satisfy X≠Y, Y≠Z and Z≠X, because there are the convertible formulas $\frac{1}{2\mathrm{r}}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ and $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ where r≥1; [2].

Thus, the Erdös conjecture and the Straus conjecture are equivalent from each other, and they are called the Erdös-Straus conjecture collectively.

As a general rule, the Erdös-Straus conjecture states that for every integer n≥2, there are positive integers X, Y and Z, such that $\frac{4}{\mathrm{n}}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

Yet, it remains a conjecture that has neither is proved nor disproved; [3].

2. Divide integers≥2 into 8 kinds and formulate 7 kinds of these 8 kinds

First, divide integers≥2 into 8 kinds, i.e. 8k+1with k≥1; 8k+2, 8k+3, 8k+4, 8k+5, 8k+6, 8k+7 and 8k+8, where k≥0, and arrange them as follows:

K\n:	8k+1,	8k+2,	8k+3,	8k+4,	8k+5,	8k+6,	8k+7,	8k+8
0,	①,	2,	3,	4,	5,	6,	7,	8,
1,	9,	10,	11,	12,	13,	14,	15,	16,
2,	17,	18,	19,	20,	21,	22,	23,	24,
…,	…,	…,	…,	…,	…,	…,	…,	…,

Excepting n=8k+1, formulate each of other 7 kinds into $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$:

(1) When n=8k+2, there are $\frac{4}{8k+2}=\frac{1}{4k+1}+\frac{1}{4k+2}+\frac{1}{(4k+1)(4k+2)}$;

(2) When n=8k+3, there are $\frac{4}{8k+3}=\frac{1}{2k+2}+\frac{1}{(2k+1)(2k+2)}+\frac{1}{(2k+1)(8k+3)}$;

(3) When n=8k+4, there are $\frac{4}{8k+4}=\frac{1}{2k+3}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+1)(2k+2)}$;

(4) When n=8k+5, there are $\frac{4}{8k+5}=\frac{1}{2k+2}+\frac{1}{(8k+5)(2k+2)}+\frac{1}{(8k+5)(k+1)}$;

(5) When n=8k+6, there are $\frac{4}{8k+6}=\frac{1}{4k+3}+\frac{1}{4k+4}+\frac{1}{(4k+3)(4k+4)}$;

(6) When n=8k+7, there are $\frac{4}{8k+7}=\frac{1}{2k+3}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+2)(8k+7)}$;

(7) When n=8k+8, there are $\frac{4}{8k+8}=\frac{1}{2k+4}+\frac{1}{(2k+2)(2k+3)}+\frac{1}{(2k+3)(2k+4)}$.

By this token, above 7 kinds of integers are suitable to the conjecture.

3. Divide the unsolved kind into 3 genera and formulate 2 genera of these 3 genera

For the unsolved kind when n=8k+1 with k≥1, let us divide it by 3 and get 3 genera, as listed below:

1. the remainder is 0, when k=1+3t, where t≥0;

2. the remainder is 2, when k=2+3t, where t≥0;

3. the remainder is 1, when k=3+3t, where t≥0.

These 3 genera of odd numbers and remainders of them divided by 3 are listed below:

k:	1, 2, 3,	4, 5, 6,	7, 8, 9,	10, 11, 12,	13, 14, 15, ...
8k+1:	9, 17, 25,	33, 41, 49,	57, 65, 73,	81, 89, 97,	105, 113, 121, ...
the remainder:	0, 2, 1,	0, 2, 1,	0, 2, 1,	0, 2, 1,	0, 2, 1, ...

Excepting the genus (3), we formulate other 2 genera as follows:

(8) Where the remainder of $\frac{8k+1}{3}$ is equal to 0, there be

$$\frac{4}{8k+1}=\frac{1}{\frac{8k+1}{3}}+\frac{1}{8k+2}+\frac{1}{(8k+1)(8k+2)}$$

Due to k=1+3t and t≥0, then there be $\frac{8k+1}{3}=8t+3$, so we confirm that $\frac{8k+1}{3}$ in the above equation is an integer.

(9) Where the remainder of $\frac{8k+1}{3}$ is equal to 2, there be

$$\frac{4}{8k+1}=\frac{1}{\frac{8k+2}{3}}+\frac{1}{8k+1}+\frac{1}{\frac{(8k+1)(8k+2)}{3}}.$$

Due to k=2+3t and t≥0, there be $\frac{8k+2}{3}=8t+6$, so we confirm that $\frac{8k+2}{3}$ and $\frac{(8k+1)(8k+2)}{3}$ in the above equation are two integers.

4. Divide the unsolved genus into 5 sorts and formulate 3 sorts of these 5 sorts

For the unsolved genus $\frac{8k+1}{3}$ where the remainder is equal to 1, and k=3+3t and t≥0, as listed above 8k+1=25, 49, 73, 97, 121 etc. So divide them into 5 sorts: 25+120c, 49+120c, 73+120c, 97+120c and 121+120c where c≥0, as listed below.

C\n:	25+120c,	49+120c,	73+120c,	97+120c,	121+120c,
0,	25,	49,	73,	97,	121,
1,	145,	169,	193,	217,	241,
2,	265,	289,	313,	337,	361,
…,	…,	…,	…,	…,	…,

Excepting n=49+120c and n=121+120c, formulate other 3 sorts, they are:

(10) When n=25+120c, there are $\frac{4}{25+120c}=\frac{1}{25+120c}+\frac{1}{50+240c}+\frac{1}{10+48c}$;

(11) When n=73+120c, there be $\frac{4}{73+12oc}=\frac{1}{(73+120c)(10+15c)}+\frac{1}{20+30c}+$ $\frac{1}{(73+120c)(4+6c)}$;

(12) When n=97+120c, there be $\frac{4}{97+12oc}=\frac{1}{25+30c}+\frac{1}{(97+120c)(50+60c)}+\frac{1}{(97+120c)(10+12c)}$.

For each of preceding 12 equations which express $\frac{4}{\text{n}}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$, please each reader self to make a check respectively.

5. Prove the sort $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

For a proof of the sort $\frac{4}{49+120c}$, it means that when c is equal to each of positive integers plus 0, there be always $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

Since the fraction $\frac{4}{49+120c}$ can be substituted by each of infinitely many a sum of an unit fraction plus a proper fraction. In addition, if c is given every value of it, then each such sum contains infinitely many a sum of an unit fractional number plus a proper fractional number, and that there is no a repetition in all fractions and all fractional numbers, as listed below:

$$\frac{4}{49+120c}$$

$$=\frac{1}{13+30\text{c}}+\frac{3}{(13+30c)(49+120c)}$$

$$=\frac{1}{14+30c}+\frac{7}{(14+30c)(49+120c)}$$

$$=\frac{1}{15+30c}+\frac{11}{(15+30c)(49+120c)}$$

…

$$=\frac{1}{13+\alpha+30c}+\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$$, where α and c ≥ 0.

…

It is obvious that $\frac{1}{13+\alpha+30\mathrm{c}}$ in the above equation is an unit fraction.

If $\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ in the above equation can be expressed as an unit fraction $\frac{1}{W}$ where W is a positive integer that is greater than 1, then we regard c in the equation $\frac{4}{49+120c}=\frac{1}{13+\alpha+30c}+\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ as c_1。In this way, there be $\frac{4}{49+120c_1}=\frac{1}{13+\alpha+30c_1}+\frac{1}{W}$ and let $\frac{1}{13+\alpha+30c_1}$ or $\frac{1}{W}$ to equal the sum of two identical unit fractions, then follow the formula $\frac{1}{2\mathrm{r}}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ or $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ to transform these two identical unit fractions into two each other's-distinct unit fractions, such that $\frac{4}{49+120c_1}$ is equal to $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

For example, when α=1 and c_1=0, $\frac{1}{49+120c_1}=\frac{4}{49}$, $\frac{1}{13+\alpha+30c_1}=\frac{1}{14}$,and

$$\frac{4\alpha+3}{(13+\alpha+30c_1)(49+120c_1)}=\frac{1}{2\times 49}=\frac{1}{2(2\times 49)}+\frac{1}{2(2\times 49)}=\frac{1}{2\times 49+1}+\frac{1}{2\times 49(2\times 49+1)},$$

then we get $\frac{4}{49}=\frac{1}{14}+\frac{1}{2\times 49+1}+\frac{1}{2\times 49(2\times 49+1)}$.

For another example, when α=1 and c_1=7, $\frac{4}{49+120c_1}=\frac{4}{889}$, $\frac{1}{13+\alpha+30c_1}=$

$\frac{1}{2\times 224}+\frac{1}{2\times 224}=\frac{1}{224+1}+\frac{1}{224(224+1)}$ and $\frac{1}{(13+\alpha+30c_1)(49+120c_1)}=\frac{1}{224\times 127}$,

then we get $\frac{4}{889}=\frac{1}{224+1}+\frac{1}{224(224+1)}+\frac{1}{224\times 127}$.

If $\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ cannot be expressed as an unit fraction $\frac{1}{W}$, then we regard c in the equation $\frac{4}{49+120c}=\frac{1}{13+\alpha+30c}+\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ as c_2, such that c_1 with c_2 express conjointly all positive integers plus 0.

In this way, there be $\frac{4}{49+120c_2}=\frac{1}{13+\alpha+30c_2}+\frac{4\alpha+3}{(13+\alpha+30c_2)(49+120c_2)}$.

Also, there be $\frac{4\alpha+3}{(13+\alpha+30c_2)(49+120c_2)}=\frac{1}{(13+\alpha+30c_2)(49+120c_2)}+$

$\frac{4\alpha+2}{(13+\alpha+30c_2)(49+120c_2)}$.

Of course, $\frac{1}{(13+\alpha+30c_2)(49+120c_2)}$ in above equation be an unit fraction too.

Thus it can be seen, we only need to prove that $\frac{4\alpha+2}{(13+\alpha+30c_2)(49+120c_2)}$ can be identically converted to an unit fraction, *ut infra*.

Proof· First, let us compare the size of the numerator 4α+2 and the denominator (13+α+30c_2)(49+120c_2).

Due to c_2 $\in$positive integers plus 0, then $13+\alpha+30c_2$ can always be greater than $4\alpha+2$. And then, we just take $13+\alpha+30c_2$ as the denominator temporarily, while reserve $49+120c_2$ for later.

In the fraction $\frac{4\alpha+2}{13+\alpha+30c_2}$, since the numerator $4\alpha+2$ is an even number, in addition, the reserved $49+120c_2$ is an odd number, so the denominator $13+\alpha+30c_2$ must be an even numbers. Only in this case, it can reduce the fraction to become possibly an unit fraction, so α in the denominator $13+\alpha+30c_2$ is a positive odd numbers, accordingly α of $4\alpha+2$ is too.

After α is assigned to odd numbers 1, 3, 5 and otherwise, the numerator and the denominator of the fraction $\frac{4\alpha+2}{13+\alpha+30c_2}$ divided by 2, then the fraction $\frac{4\alpha+2}{13+\alpha+30c_2}$ is turned into the fraction $\frac{3+4k}{7+k+15c_2}$ where $c_2\in$positive integers plus 0, $\alpha=2k+1$ and $k\geq 1$. If let $k=0$, then the numerator is 3 and the denominator is $7+15c_2$, since $15c_2$ be integral multiples of 3, yet 7 is not, then $\frac{3}{7+15c_2}$ cannot become an unit fraction, so we abandon $k=0$, and get derivable $\alpha\neq 1$.

After assigning values of k from small to large to the fraction $\frac{3+4k}{7+k+15c_2}$, there be $\frac{3+4k}{7+k+15c_2}=\frac{7}{8+15c_2}$, $\frac{11}{9+15c_2}$, $\frac{15}{10+15c_2}$, …

Such being the case, letting the numerator and the denominator of the fraction $\frac{3+4k}{7+k+15c_2}$ divided by 3+4k, then we get an indeterminate unit fraction, and its denominator is $\frac{7+k+15c_2}{3+4k}$, and its numerator is 1.

Thus, we are necessary to prove that the fraction $\frac{7+k+15c_2}{3+4k}$ as the denominator contains a number of positive integers or infinitely many positive integers, where k≥1 and c_2 ∈positive integers plus 0.

After k is assigned values from small to large, $\frac{7+k+15c_2}{3+4k}$ be equal to

$\frac{8+15c_2}{7}$, $\frac{9+15c_2}{11}$, $\frac{10+15c_2}{15}$, ...

As listed above, it can be seen that each positive odd number as the denominator can match infinite more numerators if c_2≥1, but $\frac{7+k+15c_2}{3+4k}$ as positive integers are merely a part therein due to c_2 ∈positive integers plus 0, such as $\frac{7+k+15c_2}{3+4k}=\frac{7+2+15\times 6}{3+4\times 2}=\frac{99}{11}=9$, where k=2 and c_2=6.

After k is given a value and c_2 is given a kind of values, it can too enable $\frac{7+k+15c_2}{3+4k}$ to become at least one kind of positive integers, and vice versa.

For example, when k=1, there be $\frac{7+k+15c_2}{3+4k}=\frac{8+15c_2}{7}=14+15s$ where c_2=6+7s, and s is equal to each of positive integers plus 0.

For another example, when k=8, there be $\frac{7+k+15c_2}{3+4k}=\frac{15+15c_2}{35}=3+3s$ where c_2=6+7s, and s is equal to each of positive integers plus 0.

From two equations of above two examples, since s is equal to each of positive integers plus 0, so $\frac{7+k+15c_2}{3+4k}$ contains infinite more positive integers, then let us use the symbol μ to represent each and every such positive integer.

Of course, $\frac{3+4k}{7+k+15c_2}$ expresses also infinite more unit fractions of $\frac{1}{\mu}$.

After that, we multiply the denominator of $\frac{1}{\mu}$ by $49+120c_2$ reserved to get the unit fraction $\frac{1}{\mu(49+120c_2)}$.

Or rather, there doubtlessly be $\frac{4\alpha+2}{(13+\alpha+30c_2)(49+120c_2)}=\frac{1}{\mu(49+120c_2)}$.

To sum up, we have proved $\frac{4}{49+120c_2}=\frac{1}{\mu(49+120c_2)}+\frac{1}{13+\alpha+30c_2}+\frac{1}{(13+\alpha+30c_2)(49+120c_2)}$ in the case where $\frac{4\alpha+3}{(13+\alpha+30c)(49+120c)}$ cannot be expressed as an unit fraction $\frac{1}{W}$; in addition, μ expresses every positive integer of $\frac{7+k+15c_2}{3+4k}$, $k\geq1$, $\alpha=2k+1$, and $c_2\in$positive integers plus 0.

Enable $\frac{4}{49+120c}$ to become the sum of two terms which consist of positive integers, there are only two cases, namely two terms are unit fractions and only one term is an unit fraction.

Since c_1 with c_2 in these two cases express conjointly c, namely, c1 with c2 express conjointly all positive integers plus 0, and that we have proved that $\frac{4}{49+120c_1}$ and $\frac{4}{49+120c_2}$ are expressed as the sum of 3 unit fractions, therefore, we have proved $\frac{4}{49+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$, where c≥0.

6. Prove the sort $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$

The proof in this section is exactly similar to that in the section 5. Namely, for a proof of the sort $\frac{4}{121+120c}$, it means that when c is equal to each of positive integers plus 0, there always be $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

Since the fraction $\frac{4}{121+120c}$ can be substituted by each of infinitely many a sum of an unit fraction plus a proper fraction. In addition, if c is given every value of it, then each such sum contains infinitely many a sum of an unit fractional number plus a proper fractional number, and that there is no a repetition in all fractions and all fractional numbers, as listed below:

$$\frac{4}{121+120c}$$

$$=\frac{1}{31+30c}+\frac{3}{(31+30c)(121+120c)}$$

$$=\frac{1}{32+30c}+\frac{7}{(32+30c)(121+120c)}$$

$$=\frac{1}{33+30c}+\frac{11}{(33+30c)(121+120c)}$$

…

$$=\frac{1}{31+\alpha+30c}+\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$$, where α and c ≥ 0.

…

It is obvious that $\frac{1}{31+\alpha+30c}$ in the above equation is an unit fraction.

If $\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ can be expressed as an unit fraction $\frac{1}{V}$ where V is a positive integer that is greater than 1, then we regard c in the equation $\frac{4}{121+120c}=\frac{1}{31+\alpha+30c}+\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ as c_1. In this way, there be $\frac{4}{121+120c_1}=\frac{1}{31+\alpha+30c_1}+\frac{1}{V}$ and let $\frac{1}{31+\alpha+30c_1}$ or $\frac{1}{V}$ to equal the sum of two identical unit fractions, then follow the formula $\frac{1}{2r}+\frac{1}{2r}=\frac{1}{r+1}+\frac{1}{r(r+1)}$ or $\frac{1}{2r+1}+\frac{1}{2r+1}=\frac{1}{r+1}+\frac{1}{(r+1)(2r+1)}$ to transform these two identical unit fractions into two each other's-distinct unit fractions, such that $\frac{4}{121+120c_1}$ be equal to $\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$.

For example, when α=2 and c_1=0, $\frac{4}{121+120c_1}=\frac{4}{121}$, $\frac{1}{31+\alpha+30c_1}=\frac{1}{33}$ and

$$\frac{4\alpha+3}{(31+\alpha+30c_1)(121+120c_1)}=\frac{1}{3\times121}=\frac{1}{2(3\times121)}+\frac{1}{2(3\times121)}=\frac{1}{3\times121+1}+\frac{1}{3\times121(3\times121+1)}$$

, then we get $\frac{4}{121}=\frac{1}{33}+\frac{1}{3\times121+1}+\frac{1}{3\times121(3\times121+1)}$.

For another example, when α=1 and c=5, $\frac{4}{121+120c}=\frac{4}{721}$, $\frac{1}{31+\alpha+30c}=\frac{1}{182}$ and

$$\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}=\frac{1}{182\times103}=\frac{1}{2\times18746}+\frac{1}{2\times18746}=\frac{1}{18746+1}+\frac{1}{18746(18746+1)}$$

, then we get $\frac{4}{721}=\frac{1}{182}+\frac{1}{18746+1}+\frac{1}{18746(18746+1)}$.

If $\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ cannot be expressed as an unit fraction $\frac{1}{V}$, then we regard c in the equation $\frac{4}{121+120c}=\frac{1}{31+\alpha+30c}+\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ as c_2, such that c_1 with c_2 express conjointly all positive integers plus 0.

In this way, there be $\frac{4}{121+120c_2}=\frac{1}{31+\alpha+30c_2}+\frac{4\alpha+3}{(31+\alpha+30c_2)(121+120c_2)}$.

Also, there be $\frac{4\alpha+3}{(31+\alpha+30c_2)(121+120c_2)}=\frac{1}{(31+\alpha+30c_2)(121+120c_2)}+$

$\frac{4\alpha+2}{(31+\alpha+30c_2)(121+120c_2)}$

Of course, $\frac{1}{(31+\alpha+30c_2)(121+120c_2)}$ in above equation be an unit fraction too.

Thus it can be seen, we only need to prove that $\frac{4\alpha+2}{(31+\alpha+30c_2)(121+120c_2)}$ can be identically converted to an unit fraction, *ut infra*.

Proof· First, let us compare the size of the numerator 4α+2 and the denominator $(31+\alpha+30c_2)(121+120c_2)$.

Due to c_2 ∈positive integers plus 0, then $31+\alpha+30c_2$ can always be greater than 4α+2. And then, we just take $31+\alpha+30c_2$ as the denominator temporarily, while reserve $121+120c_2$ for later.

In the fraction $\frac{4\alpha+2}{31+\alpha+30c_2}$, since the numerator 4α+2 is an even number, in addition, the reserved $121+120c_2$ is an odd number, so the denominator

$31+\alpha+30c_2$ must be an even numbers. Only in this case, it can reduce the fraction to become possibly an unit fraction, so α in the denominator $31+\alpha+30c_2$ is a positive odd number, accordingly α of $4\alpha+2$ is too.

After α is assigned to odd numbers 1, 3, 5 and otherwise, the numerator and the denominator of the fraction $\frac{4\alpha+2}{31+\alpha+30c_2}$ divided by 2, then the fraction $\frac{4\alpha+2}{31+\alpha+30c_2}$ is turned to the fraction $\frac{3+4k}{16+k+15c_2}$, where $c_2 \in$ positive integers plus 0, $k \geq 1$, and $\alpha=2k+1$. If let $k=0$, then the denominator is $16+15c_2$ and the numerator is 3, since $15c_2$ be integral multiples of 3 yet 16 is not, so $\frac{3}{16+15c_2}$ cannot become an unit fraction, so we abandon $k=0$, and get derivable $\alpha \neq 1$.

After assigning k values from small to large to the fraction $\frac{3+4k}{16+k+15c_2}$, there are $\frac{3+4k}{16+k+15c_2}=\frac{7}{17+15c_2}$, $\frac{11}{18+15c_2}$, $\frac{15}{19+15c_2}$ ...

Such being the case, letting the numerator and the denominator of the fraction $\frac{3+4k}{16+k+15c_2}$ divided by $3+4k$, then we get an indeterminate unit fraction, and its denominator is $\frac{16+k+15c_2}{3+4k}$, and its numerator is 1.

Thus, we are necessary to prove that the denominator $\frac{16+k+15c_2}{3+4k}$ contains infinitely many positive integers in the case where $k \geq 1$ and $c_2 \in$ positive integers plus 0.

After k is assigned values from small to large, $\frac{16+k+15c_2}{3+4k}$ be equal to

$\frac{17+15c_2}{7}, \frac{18+15c_2}{11}, \frac{19+15c_2}{15}, \ldots$

As listed above, it can be seen that each positive odd number as the denominator can match infinite more numerators if $c_2 \geq 1$, but $\frac{16+k+15c_2}{3+4k}$ as positive integers are merely a part therein due to $c_2 \in$ positive integers plus 0, such as $\frac{16+k+15c_2}{3+4k} = \frac{16+1+15\times 4}{3+4\times 1} = \frac{77}{7} = 11$, where k=1 and c_2=4.

After k is given a value and c_2 is given a kind of values, it can too enable $\frac{16+k+15c_2}{3+4k}$ to become at least one kind of positive integers,and vice versa.

For example, when k=2, there be $\frac{16+k+15c_2}{3+4k} = \frac{18+15c_2}{11} = 3+15s$, where c_2=1+11s, and s is equal to each of positive integers plus 0.

For another example, when k=4, there be $\frac{16+k+15c_2}{3+4k} = \frac{95+15\times 38s}{19} = 5+30s$, where c_2=5+38s, and s is equal to each of positive integers plus 0.

From two equations of above two examples, since s is equal to each of positive integers plus 0, so $\frac{16+k+15c_2}{3+4k}$ contains infinite more positive integers, and let us use the symbol λ to represent each and every such positive integer.

Of course, $\frac{3+4k}{16+k+15c_2}$ expresses also infinite more unit fractions of $\frac{1}{\lambda}$.

After that, we multiply the denominator of $\frac{1}{\lambda}$ by $121+120c_2$ reserved to get the unit fraction $\frac{1}{\lambda(121+120c_2)}$.

Or rather, there doubtlessly be $\frac{4\alpha+2}{(31+\alpha+30c_2)(121+120c_2)}=\frac{1}{\lambda(121+120c_2)}$.

To sum up, we have proved $\frac{4}{121+120c_2}=\frac{1}{\lambda(121+120c_2)}+\frac{1}{31+\alpha+30c_2}+\frac{1}{(31+\alpha+30c_2)(121+120c_2)}$ in the case where $\frac{4\alpha+3}{(31+\alpha+30c)(121+120c)}$ cannot be expressed as an unit fraction $\frac{1}{V}$; in addition, λ expresses every positive integer of $\frac{16+k+15c_2}{3+4k}$, k≥1, α=2k+1, and c_2∈positive integers plus 0.

Enable $\frac{4}{121+120c}$ to become the sum of two terms which consist of positive integers, there are only two cases, namely two terms are unit fractions and only one term is an unit fraction.

Since c_1 with c_2 in these two cases express conjointly c, namely, c1 with c2 express conjointly all positive integers plus 0, and that we have proved that $\frac{4}{121+120c_1}$ and $\frac{4}{121+120c_2}$ can be expressed as the sum of 3 unit fractions, therefore, we have proved $\frac{4}{121+120c}=\frac{1}{X}+\frac{1}{Y}+\frac{1}{Z}$ where c≥0.

The proof was thus brought to a close. As a consequence, the Erdös-Straus conjecture is tenable.

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References

[1] J. W. Sander, *On* $\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$ *and Iwaniec' Half Dimensional Sieve*; Journal of Number Theory, Volume 46, Issue 2, February 1994, Pages 123-136;

https://www.sciencedirect.com/science/article/pii/S0022314X84710080?via%3Dihub

[2] Wolfram Math world, the web's most extensive mathematics resource;

https://mathworld.wolfram.com/Erdos-StrausConjecture.html

[3] Konstantine Zelator, *An ancient Egyptian problem: the diophantine equation* $\frac{4}{n}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$, *n>or=2*; arXiv: 0912.2458.