Eliminate the Irrelevant to the Subject and Prove Equations and Inequalities related to Beal's Conjecture

Abstract

The subject of this article is exactly to analyze and prove Beal's Conjecture.

First, classify A, B and C according to their respective parity, and two types of $A^X+B^Y \neq C^Z$ are excluded, for they have nothing to do with the conjecture.

Next, several types of $A^X+B^Y=C^Z$ under the necessary constraints are exemplified, where A, B, and C have at least one common prime factor.

Secondly, divide $A^X+B^Y \neq C^Z$ under the necessary constraints into four inequalities under the known constraints, in order to make more detailed proofs, where A, B and C have not any common prime factor.

Then, expound the interrelation between an even number as the center of symmetry and a sum of two odd numbers in the symmetry, and draw four conclusions which can be used as basis for judging certain results in the processes of proofs for the four inequalities.

After that, two inequalities under the known constraints are proved by the mathematical induction. Then again, two other inequalities under the known constraints are proved by the reduction to absurdity.

Finally, after comparing $A^X+B^Y=C^Z$ and $A^X+B^Y \neq C^Z$ under necessary constraints, the conclusion is that the Beal's conjecture is true.

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1. Introduction

Beal's conjecture states that if $A^X+B^Y=C^Z$, where A, B, C, X, Y and Z are positive integers, and X, Y and Z are all greater than 2, then A, B and C must have a common prime factor.

The conjecture was discovered by Andrew Beal in 1993. Later, it was announced in December 1997 issue of the Notices of the American Mathematical Society, [1]. However, it remains a conjecture that has neither been proved nor disproved.

The conjecture shows that whoever wants to solve it, must both enumerate $A^X+B^Y=C^Z$ in which case A, B and C have at least one common prime factor, and prove $A^X+B^Y\neq C^Z$ in which case A, B and C have no any common prime factor.

Let us consider limits of values of A, B, C, X, Y, and Z within the indefinite equation $A^X+B^Y=C^Z$ as the necessary constraints, in order to describe briefly related indefinite equations and inequalities after this.

2. The Selection On Combinations of Values of A, B and C

First, we classify A, B and C according to their respective parity, and then, the following two types of $A^X+B^Y\neq C^Z$ are excluded:

1) A, B and C are all odd numbers.

2) A, B and C are two even numbers and an odd number.

After that, merely continue to have following two types which contain $A^X+B^Y=C^Z$ under the necessary constraints:

1) A, B and C are all positive even numbers.

2) A, B and C are two positive odd numbers and one positive even number.

3. Exemplifying $A^X+B^Y=C^Z$ Under the Necessary Constraints

For the indefinite equation $A^X+B^Y=C^Z$ satisfying either of the above-mentioned two constraints, in fact, there are many sets of solution with A, B and C as positive integers, as shown in the following examples.

If A, B and C are all positive even numbers, let A=B=C=2, X=Y $\geq$3 and Z=X+1, so $A^X+B^Y=C^Z$ is changed to $2^X+2^X=2^{X+1}$. Thus, $A^X+B^Y=C^Z$ in this case has one set of solution with A, B, and C as 2, 2 and 2, and A, B and C have one common prime factor 2.

In addition to this, let A=B=162, C=54, X=Y=3 and Z=4, so $A^X+B^Y=C^Z$ is changed to $162^3+162^3=54^4$. Thus, $A^X+B^Y=C^Z$ in this case has a set of solution with A, B, and C as 162, 162 and 54, and A, B, and C have two common prime factors 2 and 3.

If A, B and C are two positive odd numbers and one positive even number, let A=C=3, B=6, X=Y=3 and Z=5, so $A^X+B^Y=C^Z$ is changed to $3^3+6^3=3^5$. Thus, $A^X+B^Y=C^Z$ in this case has one set of solution with A, B, and C as 3, 6 and 3, and A, B and C have one common prime factor 3.

In addition to this, let A=B=7, C=98, X=6, Y=7 and Z=3, so $A^X+B^Y=C^Z$ is changed to $7^6+7^7=98^3$. Thus, $A^X+B^Y=C^Z$ in this case has one set of solution with A, B, and C as 7, 7 and 98, and A, B and C have one common prime factor 7.

It follows that there must be $A^X+B^Y=C^Z$ under the necessary constraints, but A, B and C have at least one common prime factor.

4. On $A^X+B^Y\neq C^Z$ and Divide It into Four Inequalities

According to the requirement of the conjecture, if we can prove $A^X+B^Y\neq C^Z$ under the necessary constraints, where A, B and C have not any common prime factor, then the conjecture must be true.

In which case A, B and C are all even numbers, they have at least one common prime factor 2, so A, B and C without common prime factor can only be two odd numbers and one even number.

If A, B, and C have not a common prime factor, then any two of them have not a common prime factor either, because if two have a common prime factor, you can extract the common prime factor, yet another does not have it. so this will lead up to $A^X+B^Y\neq C^Z$ according to the unique factorization theorem of natural number.

Without doubt, following two inequalities, taken together, are sufficient to replace $A^X+B^Y\neq C^Z$ under the necessary constraints, where A, B and C are two odd numbers and one even number without common prime factor.

1) $A^X+B^Y\neq(2W)^Z$, i.e. $A^X+B^Y\neq 2^ZW^Z$;

2) $A^X+(2W)^Y\neq C^Z$, i.e. $A^X+2^YW^Y\neq C^Z$.

In above two inequalities, A, B and C are odd numbers; X, Y and $Z\geq 3$; $W\geq 1$; and three terms of each inequality have not a common prime factor.

Continue to divide $A^X+B^Y\neq 2^Z W^Z$ into the following two inequalities:

(1) $A^X+B^Y\neq 2^Z$;

(2) $A^X+B^Y\neq 2^Z O^Z$.

Continue to divide $A^X+2^Y W^Y\neq C^Z$ into the following two inequalities:

(3) $A^X+2^Y\neq C^Z$;

(4) $A^X+2^Y O^Y\neq C^Z$.

In the above-listed four inequalities, A, B, C and O are positive odd numbers; $X\geq 3$, $Y\geq 3$ and $Z\geq 3$; and three terms of each inequality have not a common prime factor.

Moreover, regard aforesaid constraints as the known constraints, in order to describe briefly related inequalities and indefinite equations after this.

As thus, proving $A^X+B^Y\neq C^Z$ under the necessary constraints are changed to prove the above-listed four inequalities under the known conditions.

5. Main Bases of Proving the First Two Inequalities in Order

Before the proof begins, it is necessary to state some basic concepts, in order to consider them as the main basis for judging certain results in the processes of proving the first two inequalities in order.

First of all, on positive half line of the number axis, if any even point is taken as a center of symmetry, then odd points on the left side of the center and odd points concerned on the right side are one-to-one symmetric , [2].

Like that, in the sequence of natural numbers, if any even number is taken as a center of symmetry, then odd numbers less than the even number and part odd numbers more than the even number are one-to-one symmetric.

Take any of $2^{H-1}W^{V}$ as a center of symmetry, then two distances between the center and two odd points/odd numbers on two sides of the center are two equilong line segments/same differences, where H, W and V are integers, and $W\geq1$, $H\geq3$ and $V\geq1$.

Accordingly, we can draw following four conclusions from the interrelation between an even number as the center of symmetry and a sum of two odd numbers in the symmetry, whether they are integers within the sequence of natural numbers or integers which mark integer's points on positive half line of the number axis.

Conclusion 1˙ The sum of two each other's−symmetric odd numbers is equal to the double of the even number as the center of the symmetry.

Conclusion 2˙ The sum of two asymmetric odd numbers does not equal the double of the even number as the center of the symmetry.

Conclusion 3˙ If the sum of two odd numbers is equal to the double of an even number, then these two odd numbers are symmetric with the even number as the center of the symmetry.

Conclusion 4˙ If the sum of two odd numbers does not equal the double of an even number, then these two odd numbers are not symmetric with the even number as the center of the symmetry.

Besides, any odd number can be represented as one of O^V, where O is an odd number, and $V \geq 1$. Also, when V=1 or 2, you can write O^V as $O^{1\sim2}$.

In following paragraphs, the author is going to prove each of aforementioned four inequalities, one by one.

6. Proving $A^X+B^Y \neq 2^Z$ Under the Known Constraints

Consider each of 2^{Z-1} as a center of symmetry about related odd numbers to prove $A^X+B^Y \neq 2^Z$ under the known constraints by the mathematical induction.

(1) When Z-1=2, 3, 4, 5 and 6, bilateral symmetric odd numbers on two sides of each center of symmetry are successively listed below.

1^6, 3, (2^2), 5, 7, (2^3), 9, 11, 13, 15, (2^4), 17, 19, 21, 23, 25, 3^3, 29, 31, (2^5), 33, 35, 37, 39, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63, (2^6), 65, 67, 69, 71, 73, 75, 77, 79, 3^4, 83, 85, 87, 89, 91, 93, 95, 97, 99, 101, 103, 105, 107, 109, 111, 113, 115, 117, 119, 121, 123, 5^3, 127

As listed above, it can be seen that there are no two of O^V with $V \geq 3$ on two positions of each pair of bilateral symmetric odd numbers with 2^{Z-1} as each center of symmetry, where Z-1=2, 3, 4, 5 and 6.

So there are $A^X+B^Y \neq 2^3$, $A^X+B^Y \neq 2^4$, $A^X+B^Y \neq 2^5$, $A^X+B^Y \neq 2^6$ and $A^X+B^Y \neq 2^7$ under the known constraints, according to the preceding Conclusion 2.

(2) When Z-1=K with $K \geq 6$, we suppose that there only are $A^X+B^Y \neq 2^{K+1}$

under the known constraints.

(3) When Z-1=K+1, it needs us to prove that there only are $A^X+B^Y \neq 2^{K+2}$ under the known constraints.

Proof: Suppose that A^X and B^Y are two bilateral symmetric odd numbers with 2^K as the center of symmetry, then there are $A^X+B^Y=2^{K+1}$, according to the preceding Conclusion 1.

While, there only are $A^X+B^Y \neq 2^{K+1}$ under the known constraints in line with second step of the mathematical induction. Namely there are no two of O^V with $V \geq 3$ on two positions of each pair of bilateral symmetric odd numbers with 2^K as the center of symmetry. In this case, we tentatively regard A^X as one of O^V with $V \geq 3$, and regard B^Y as one of $O^{1\sim 2}$, i.e. let $X \geq 3$ and Y=1 or 2.

Taken one with another, if there are $A^X+B^Y=2^{K+1}$, then A^X and B^Y must be two bilateral symmetric odd numbers with 2^K as the center of symmetry, and at least one of Y and X is equal to 1 or 2.

If you change the above-mentioned constraints, even a little, then it will inevitably lead to $A^X+B^Y \neq 2^{K+1}$. Vice versa, there are surely $A^X+B^Y=2^{K+1}$ under the known constraints except for Y, and Y=1 or 2.

Now that there are $A^X+B^Y=2^{K+1}$, then there are also $A^X+(A^X+2B^Y)=2^{K+2}$ under the known constraints except for Y, and Y=1 or 2, so A^X and A^X+2B^Y are two bilateral symmetric odd numbers with 2^{K+1} as the center of symmetry, according to the preceding Conclusion 3.

But then, since there are $A^X+B^Y \neq 2^{K+1}$ under the known constraints, thus there are $A^X+(A^X+2B^Y)\neq 2^{K+2}$ under the known constraints, then A^X and A^X+2B^Y can only be two asymmetric odd numbers with 2^{K+1} as the center of symmetry, according to the preceding Conclusion 4.

In any case, the sum of A^X+2B^Y is an odd number, so let A^X+2B^Y equal O^E, where O is fixedly an odd number, it has nothing to do with Y, and $E\geq 1$.

After the substitution, on the one hand, there are $A^X+(A^X+2B^Y)=A^X+O^E=2^{K+2}$ under the known constraints except for Y, and Y=1 or 2, then A^X and O^E are two bilateral symmetric odd numbers with 2^{K+1} as the center of symmetry, according to the preceding Conclusion 3.

On the other hand, there are $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ under the known constraints, yet A^X and O^E are not two symmetric odd numbers with 2^{K+1} as the center of symmetry, according to the preceding Conclusion 4.

In which case A^X and O^E are asymmetric, whichever positive integer that E equals, it can satisfy $A^X+O^E\neq 2^{K+2}$, according to the preceding Conclusion 2.

Since there are $Y\geq 3$ in $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ and Y=1 or 2 in $A^X+(A^X+2B^Y)=A^X+O^E=2^{K+2}$, so A^X+2B^Y within $A^X+(A^X+2B^Y)=A^X+O^E\neq 2^{K+2}$ are greater than A^X+2B^Y within $A^X+(A^X+2B^Y)=A^X+O^E=2^{K+2}$. That is to say, O^E within $A^X+O^E\neq 2^{K+2}$ is greater than O^E within $A^X+O^E=2^{K+2}$.

Since A^X within $A^X+O^E\neq 2^{K+2}$ and A^X within $A^X+O^E=2^{K+2}$ are one and the same; in addition, O in $A^X+O^E\neq 2^{K+2}$ and O in $A^X+O^E=2^{K+2}$ are one and the same, therefore, E in $A^X+O^E\neq 2^{K+2}$ be greater than E in $A^X+O^E=2^{K+2}$.

In $A^X+B^Y=2^{K+1}$ and $A^X+O^E=2^{K+2}$, except that same symbols represent integers in same range, both B and O represent all odd numbers$\geq$1.

Now that there are $A^X+B^Y=2^{K+1}$ under the known constraints except for Y, and Y=1 or 2, so in the same way, there are $A^X+O^E=2^{K+2}$ under the known constraints except for E, and E=1 or 2.

In general, in such an equality that consists of three terms, at least one of the three terms must have an exponent that is equal to 1 or 2. If the exponent of every term is greater than or equal to 3, then it is turned into an inequality.

In addition, as has been proved that E in $A^X+O^E\neq2^{K+2}$ is greater than E in $A^X+O^E=2^{K+2}$, so we get that E in $A^X+O^E\neq2^{K+2}$ is greater than or equal to 3, therefore, there are $A^X+O^E\neq2^{K+2}$ under the known constraints.

Or rather, E in $A^X+O^E=2^{K+2}$ can only be equal to 1 or 2, since we have supposed X$\geq$3 and K$\geq$6 before this. Yet, for O^E within $A^X+O^E\neq2^{K+2}$ under the known constraints, after you consider 2^{K+1} as the center of symmetry, if A^X and O^E lie not on two symmetric positions, then O^E can be any odd number out of the symmetry with A^X; if A^X and O^E lie on two symmetric positions, then it allows only E$\geq$3 under the prerequisites of X$\geq$3 and K$\geq$6. If not, it can lead up to $A^X+O^E=2^{K+2}$.

For the inequality $A^X+O^E\neq2^{K+2}$, substitute B for O, since both B and O can be every positive odd number; in addition, substitute Y for E, where E$\geq$3, also Y$\geq$3, then, we get $A^X+B^Y\neq2^{K+2}$ under the known constraints.

In this proof, if B^Y is one of O^V with V$\geq$3, then A^X is surely one of $O^{1\sim2}$, or A^X and B^Y are two of $O^{1\sim2}$. And yet, conclusions concluded finally from these two cases are only one and the same with $A^X+B^Y\neq 2^{K+2}$ under the known constraints.

So much for, the author has proved that when Z-1=K+1 with K≥6, there only are $A^X+B^Y\neq 2^{K+2}$ under the known constraints.

By the preceding way, we can continue to prove that when Z-1=K+2, K+3…up to every integer more than or equal to K+2, there are $A^X+B^Y\neq 2^{K+3}$, $A^X+B^Y\neq 2^{K+4}$ … up to general $A^X+B^Y\neq 2^Z$ under the known constraints.

7. Proving $A^X+B^Y\neq 2^Z O^Z$ Under the Known Constraints

Consider each of $2^{Z-1}O^Z$ as a center of symmetry about related odd numbers to prove successively $A^X+B^Y\neq 2^Z O^Z$ under the known constraints by the mathematical induction, and we emphasize that O is an odd number ≥3.

(1) When O=1, $2^{Z-1}O^Z$ i.e. 2^{Z-1}. As has been proved, there only are $A^X+B^Y\neq 2^Z$ under the known constraints, in chapter 6 above.

(2) When O=J and J is an odd number ≥1, $2^{Z-1}O^Z$ i.e. $2^{Z-1}J^Z$, we suppose that there only are $A^X+B^Y\neq 2^Z J^Z$ under the known constraints.

(3) When O=S and S=J+2, $2^{Z-1}O^Z$ i.e. $2^{Z-1}S^Z$, it needs us to prove that there only are $A^X+B^Y\neq 2^Z S^Z$ under the known constraints.

***Proof*·** Under the prerequisite of X≥3, suppose that A^X and B^Y are two bilateral symmetric odd numbers with $2^{Z-1}J^Z$ as the center of symmetry, then there are $A^X+B^Y=2^Z J^Z$, according to the preceding Conclusion 1.

And yet, there only are $A^X+B^Y\neq 2^Z J^Z$ under the known constraints in line with second step of the mathematical induction.

It is obvious that there are $A^X+B^Y=2^ZJ^Z$ under the known constraints except for Y, and Y=1 or 2. So there are $A^X+[B^Y+2^Z(S^Z-J^Z)]=(A^X+B^Y)+2^ZS^Z-2^ZJ^Z=2^ZS^Z$ under the known constraints except for Y, and Y=1 or 2, and that A^X and $B^Y+2^Z(S^Z-J^Z)$ are two bilateral symmetric odd numbers with $2^{Z-1}S^Z$ as the center of symmetry, according to the preceding Conclusion 3.

But then, there only are $A^X+B^Y\neq 2^ZJ^Z$ under the known constraints. Then, from this, we conclude $A^X+[B^Y+2^Z(S^Z-J^Z)]=(A^X+B^Y)+2^ZS^Z-2^ZJ^Z\neq 2^ZS^Z$ under the known constraints, so A^X and $B^Y+2^Z(S^Z-J^Z)$ are not two symmetric odd numbers with $2^{Z-1}S^Z$ as the center of symmetry, according to the preceding Conclusion 4.

In this case, let the odd number $B^Y+2^Z(S^Z-J^Z)$ be equal to D^E, where D is fixedly an odd number independent of the magnitude of Y, and $E\geq 1$.

After the substitution, on the one hand, there are $A^X+[B^Y+2^Z(S^Z-J^Z)]=A^X+D^E=2^ZS^Z$ under the known constraints except for Y, and Y=1 or 2, and that A^X and D^E are two bilateral symmetric odd numbers with $2^{Z-1}S^Z$ as the center of symmetry, according to the preceding Conclusion 3.

On the other hand, there are $A^X+[B^Y+2^Z(S^Z-J^Z)]=A^X+D^E\neq 2^ZS^Z$ under the known constraints, yet A^X and D^E are not two symmetric odd numbers with $2^{Z-1}S^Z$ as the center of symmetry, according to the preceding Conclusion 4.

In which case A^X and D^E are asymmetric, whichever positive integer that E equals, it can satisfy $A^X+D^E\neq 2^ZS^Z$, according to the preceding Conclusion 2.

Since there are $Y\geq 3$ in $A^X+[B^Y+2^Z(S^Z-J^Z)]=A^X+D^E\neq 2^HS^Z$ and Y=1 or 2 in

$A^X+[B^Y+2^Z(S^Z-J^Z)]=A^X+D^E=2^ZS^Z$, so $B^Y+2^Z(S^Z-J^Z)$ within $A^X+[B^Y+2^Z(S^Z-J^Z)]=A^X+D^E\neq 2^ZS^Z$ are greater than $B^Y+2^Z(S^Z-J^Z)$ within $A^X+[B^Y+2^Z(S^Z-J^Z)]=A^X+D^E=2^ZS^Z$. That is to say, D^E within $A^X+D^E\neq 2^ZS^Z$ is greater than D^E within $A^X+D^E=2^ZS^Z$.

Since A^X within $A^X+D^E\neq 2^ZS^Z$ and A^X within $A^X+D^E=2^ZS^Z$ are one and the same; in addition, D in $A^X+D^E\neq 2^ZS^Z$ and D in $A^X+D^E=2^ZS^Z$ are one and the same, therefore, E in $A^X+D^E\neq 2^ZS^Z$ is greater than E in $A^X+D^E=2^ZS^Z$.

In $A^X+B^Y=2^ZJ^Z$ and $A^X+D^E=2^Z(J+2)^Z$, except that same symbols represent integers within same range, J and J+2 can represent odd numbers ≥ 3 therein; in addition, both B and D represent all odd numbers≥ 1.

Since there are $A^X+B^Y=2^ZJ^Z$ under the known constraints except for Y, and Y=1 or 2, so in the same way, there are $A^X+D^E=2^Z(J+2)^Z$ under the known constraints except for E, and E=1 or 2, also known to $2^Z(J+2)^Z$ i.e. 2^ZS^Z.

In general, in such an equality that consists of three terms, at least one of the three terms must have an exponent that is equal to 1 or 2. If the exponent of every term is greater than or equal to 3, then it is turned into an inequality.

In addition, as has been proved that E in $A^X+D^E\neq 2^ZS^Z$ is greater than E in $A^X+D^E=2^ZS^Z$, so we get that E in $A^X+D^E\neq 2^ZS^Z$ is greater than or equal to 3, therefore, there are $A^X+D^E\neq 2^ZS^Z$ under the known constraints.

Or rather, E in $A^X+D^E=2^ZS^Z$ can only be equal to 1 or 2, since we have supposed $X\geq 3$ and $Z\geq 3$ before this. Yet, for D^E within $A^X+D^E\neq 2^ZS^Z$ under the known constraints, after you consider $2^{Z-1}S^Z$ as the center of symmetry, if A^X

and D^E lie not on two symmetric positions, then D^E can be any odd number out of the symmetry with A^X; if A^X and D^E lie on two symmetric positions, then it allows only $E\geq3$ under the prerequisites of $X\geq3$ and $Z\geq3$. If not, it can lead to $A^X+D^E=2^ZS^Z$.

For the inequality $A^X+D^E\neq2^ZS^Z$, substitute B for D, since both B and D can be every positive odd numbers; in addition, substitute Y for E, where $E\geq3$, also $Y\geq3$, then we get $A^X+B^Y\neq2^ZS^Z$ under the known constraints.

In this proof, if B^Y is one of O^V with $V\geq3$, then A^X is surely one of $O^{1\sim2}$, or A^X and B^Y are two of $O^{1\sim2}$. And yet, conclusions concluded finally from there two cases are only one and the same with $A^X+B^Y\neq2^ZS^Z$ under the known constraints.

To sum up, the author has proven $A^X+B^Y\neq2^ZS^Z$ with S=J+2 under the known constraints.

By the preceding way, we can continue to prove that when O=J+4, J+6… up to each of odd numbers more than and equal J+4, there are $A^X+B^Y\neq2^Z(J+4)^Z$, $A^X+B^Y\neq2^Z(J+6)^Z$… up to general $A^X+B^Y\neq2^ZO^Z$ under the known constraints.

8. Proving $A^X+2^Y\neq C^Z$ Under the Known Constraints

In this chapter, the author is going to prove $A^X+2^Y\neq C^Z$ under the known constraints by reduction to absurdity, *ut infra*.

***Proof*·** According to $A^X+B^y=2^{K+1}$ and $A^X+O^E=2^{K+2}$ under the known constraints except for Y and E, and Y and E=1 or 2, in the chapter 6, we can let $O_1^M+O_2^L=2^Y$, where O_1 and O_2 are positive odd numbers, the exponents

M and $Y\geq3$, and the exponent $L=1$ or 2.

Assume that there are $A^X+2^Y=C^Z$ under the known constraints, then there are $A^X+O_1^M+O_2^L=C^Z$, i.e. $A^X+O_1^M=C^Z-O_2^L$.

Since there are $A^X+O_1^M\neq2^S$ where $S\geq3$, according to proven $A^X+B^Y\neq2^Z$ under the known constraints in the chapter 6, then there only are $C^Z-O_2^L\neq2^S$, i.e. $O_2^L+2^S\neq C^Z$. It is obvious that the inequality $O_2^L+2^S\neq C^Z$ derived from the assumption does not hold water, because there are always $O_2^L+2^S=C^Z$ due to $O_2\geq1$, $C\geq3$, $S\geq3$, $Z\geq3$ and $L=1$ or 2. Therefore, the assumption is wrong.

As thus, there only are $A^X+O_1^M+O_2^L\neq C^Z$, i.e. $A^X+2^Y\neq C^Z$.

So far, the author has proved $A^X+2^Y\neq C^Z$ under the known constraints.

9. Proving $A^X+2^YO^Y\neq C^Z$ Under the Known Constraints

The proof in this chapter is similar to that in chapter 8. Namely prove $A^X+2^YO^Y\neq C^Z$ under the known constraints by reduction to absurdity, *ut infra*.

Proof· According to $A^X+B^Y=2^ZJ^Z$ and $A^X+D^E=2^Z(J+2)^Z$ under the known constraints except for Y and E, and Y and E=1 or 2, in the chapter 7, we can let $O_3^M+O_4^L=2^YO^Y$, where O_3, O_4 and O are positive odd numbers, the exponents Y and $M\geq3$, and the exponent $L=1$ or 2.

Assume that there are $A^X+2^YO^Y=C^Z$ under the known constraints, then there are $A^X+O_3^M+O_4^L=C^Z$, i.e. $A^X+O_3^M=C^Z-O_4^L$.

Since there are $A^X+O_3^M\neq2^UO_n^U$ where O_n is an odd number ≥3, and the exponent $U\geq3$, according to proven $A^X+B^Y\neq2^ZS^Z$ under the known constraints, in the chapter 7, then there only are $C^Z-O_4^L\neq2^UO_n^U$, i.e.

$O_4^L+2^UO_n^U \neq C^Z$. It is obvious that the inequality $O_4^L+2^UO_n^U \neq C^Z$ derived from the assumption does not hold water, because there are always $O_4^L+2^UO_n^U = C^Z$ due to $O_4 \geq 1$, $O_n \geq 3$, $C>3$, $U \geq 3$, $Z \geq 3$, and L=1 or 2.

Therefore, the assumption is wrong.

As thus, there only are $A^X+O_3^M+O_4^L \neq C^Z$, i.e. $A^X+2^YO^Y \neq C^Z$.

Thus far, the author has proved $A^X+2^YO^Y \neq C^Z$ under the known constraints.

10. Make A Summary and Reach the Conclusion

To sum up, the author has already proved every kind of $A^X+B^Y \neq C^Z$ under the necessary constraints in chapters 6, 7, 8 and 9, where A, B and C are two odd numbers and one even number without common prime factor.

In addition to this, the author has given examples to have proved $A^X+B^Y=C^Z$ under the necessary constraints in chapter 3, where A, B and C have at least one common prime factor.

By this token, By making a comparison between $A^X+B^Y=C^Z$ and $A^X+B^Y \neq C^Z$ under the necessary constraints, we can reach the conclusion that an indispensable prerequisite of the existence of $A^X+B^Y=C^Z$ under the necessary constraints is the very which A, B and C must have a common prime factor.

The proof was thus brought to a close. As a consequence, Beal's conjecture is tenable.

P.S. Proving Fermat's Last Theorem from Proven Beal's Conjecture

Fermat's last theorem is a special case of Beal's conjecture, [3]. If Beal's conjecture is proved to be true, then let X=Y=Z, so $A^X+B^Y=C^Z$ are going to be changed to $A^X+B^X=C^X$.

Furthermore, you divide three terms of $A^X+B^X=C^X$ by greatest common divisor of these three terms, then you will get a set of solution of positive integers without common prime factor.

It is obvious that the conclusion is in contradiction with proven Beal's conjecture. As thus, we have proved Fermat's last theorem by reduction to absurdity as easy as pie.

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