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## Direct Proof of Fermat's Last Theorem

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### Abstract

In 1994 the mathematician Andrews Wiles proved, using concepts of modern mathematics, namely the Modular Elliptic Curves, the Fermat's Last Theorem. That demonstration is long and it is understandable to mathematic specialists, moreover, these mathematical concepts were not known at the time when Fermat lived, so he could not prove it through this road. I thought that there might be another simpler proof which would use the properties of algebraic equations and inequalities. These mathematical concepts were known at the time in which Fermat lived and which, therefore, could apply for the proof of the theorem of which he wrote, in the margin of a page of the book of Arithmetica of Diophantus he was reading, to have found a demonstration "wonderful," but that he had not could to write it to the narrowness of the margin itself. I propose, therefore, the following demonstration that directly uses the mathematical properties of algebraic equations and inequalities that are understood by all those who study algebra.

### Theorem

**It is possible to divide a power, with an integer base of degree  $\mathbf{n}$ , into the sum of two integer powers, of the same degree, only if  $\mathbf{n}$  is equal to 2.**

### Demonstration

$$1) - X^n + Y^n = Z^n$$

$X, Y, Z, n$  are integers, it's obvious that  $X < Y < Z$  and that  $X + Y > Z$ ; we also have that

$$2) - \frac{X^n + Y^n}{Z^n} = 1 \quad \text{and more}$$

$$3) - \frac{X^n}{Z^n} > 0 < \frac{1}{2} \quad \text{and also}$$

$$4) - \frac{Y^n}{Z^n} > 0 > \frac{1}{2} \quad \text{we raising to } \mathbf{1/n}, \mathbf{3)} \text{ and } \mathbf{4)}, \text{ we have}$$

$$5) - \frac{X}{Z} > 0 < \left(\frac{1}{2}\right)^{1/n} \quad \text{and yet}$$

$$6) - \frac{Y}{Z} > 0 > \left(\frac{1}{2}\right)^{1/n} \quad \text{and so adding the } \mathbf{5)} \text{ and the } \mathbf{6)} \\ \text{we have}$$

$$7) - \frac{X + Y}{Z} > 1 < 2 \left(\frac{1}{2}\right)^{1/n} \quad \text{the value of the second member of the} \\ \text{inequality, for any value of } \mathbf{n},$$

it has infinite solutions, always  $< 2$ , therefore it is indeterminate, but being  $X < Y < Z$ , and  $X+Y > Z$  as indicated in number 1), the second member, equally, is always  $< 2$ , therefore we can write that

8) -  $\frac{X+Y}{Z} > 1 < 2$  being the first members of the inequalities 7) and 8), equal, we must also have the equality of the second members and thus we must find the value of  $n$  that satisfies them, so we can write

9) -  $2 \left(\frac{1}{2}\right)^{1/n} = 2$  and develop  $(2^{n-1})^{1/n} = 2$  we raise the first member to  $n$ .

that is  $[(2^{n-1})^{1/n}]^n = 2$  develop and we obtain  $2^{n-1} = 2$  if we put

to the exponent  $n = 2$  we develop and obtain the equality  $2 = 2$

therefore the exponent  $n=2$  satisfies the equality and it is the only exponent that satisfies the 9). Any other value of  $n > 2$  does not satisfy equality 9), in fact there would be increasing values and therefore greater than 2.

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We have therefore found also the exponent **2** that satisfies the inequalities **7)**. From what has been said above, let's now check and proceed with the substituting the exponent **n** with **2** on the second member of inequality **7)** and then we raise the first and second members to the exponent **2** and we have

$$10) - \left( \frac{X+Y}{Z} \right)^2 > 1 < \left[ 2 \left( \frac{1}{2} \right)^{1/2} \right]^2 \text{ we develop} \\ \text{and we obtain}$$

$$11) - \frac{X^2 + Y^2 + 2 * X * Y}{Z^2} > 1 < [ ( 2^{2-1} )^{1/2} ]^2 \text{ and thus}$$

$$\frac{X^2 + Y^2 + 2 * X * Y}{Z^2} > 1 < 2 \text{ the numerator of the first member of}$$

the inequality **is the square of a binomial** of which: **X<sup>2</sup> + Y<sup>2</sup>** are the two powers, of the same degree, and that are equal to the first member of the equation **1)** and **2\*X\*Y** is the product relating to the **binomial**. We delop and obtain the equation **2)** **from which it is clear that**

$$12) - \frac{X^2 + Y^2}{Z^2} = 1 \quad \text{and} \quad \frac{2 * X * Y}{Z^2} < 1 \quad \text{that is to say}$$

it is possible to divide a power, with an integer base of degree  $\mathbf{n}$ , into the sum of two integer powers, of the same degree, only if  $\mathbf{n}$  is equal to  $\mathbf{2}$ . Therefore we can write the equation  $\mathbf{1}$ ), with integers:

$$13) - X^2 + Y^2 = Z^2$$

**Q. E. D.**

## References

- [1] A. Wiles - Modular elliptic curves and Fermat's Last Theorem  
<http://math.stanford.edu/~lekheng/flt/wiles.pdf>
- [2] Simon Singh, L'ultimo teorema di Fermat, Milano, Rizzoli, 1999
- [3] Wikipedia: Fermat's Last Theorem