Negative energy form Wormhole obtained from First Principles and Compared with Tokamak GW

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Abstract:

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We revisit how we utilized how Weber in 1961 initiated the process of quantization of early universe fields to the issue of what was for a wormhole mouth. While the wormhole models are well understood, there is not such a consensus as to how the mouth of a wormhole could generate signals. We try to develop a model for doing so and then revisit it, the Wormhole while considering a Tokamak model we used in a different publication as a way of generating GW, and Gravitons

Keywords: Minimum scale factor, cosmological constant, space-time bubble, bouncing cosmologies , Tokamaks

I. Introduction to the Weber quantization procedure

The template of what we will be looking at will be a wormhole, using a wavefunction quantization procedure, **Using [1] [2] a statement as to quantization for a would-be GR term comes straight from**

$$\Psi_{Later} = \int \sum_{H} e^{(iI_H/\hbar)(t,t^0)} \Psi_{Earlier}\left(t^0\right) dt^0 \qquad (1)$$

The approximation we are making is to pick one index, so as to have'

$$\Psi_{Later} = \int \sum_{H} e^{(iI_H/\hbar)(t,t^0)} \Psi_{Earlier}\left(t^0\right) dt^0 \xrightarrow[H \to 1]{} \int e^{(iI_{H_{FIXED}}/\hbar)(t,t^0)} \Psi_{Earlier}\left(t^0\right) dt^0 \qquad (2)$$

This corresponds to say being primarily concerned as to GW generation, which is what we will be examining in our ideas, via using

$$e^{(iI_{H_{FIXED}}/\hbar)(t,t^0)} = \exp\left[\frac{i}{\hbar} \cdot \frac{c^4}{16\pi G} \cdot \int_{M} dt \cdot d^3 r \sqrt{-g} \cdot \left(\Re - 2\Lambda\right)\right] \qquad (3)$$

We will use the following, namely, if Λ is a constant, do the following for the Ricci scalar

$$\Re = \frac{2}{r^2} \qquad (4)$$

If so then we can write the following, namely: Eq.(3) becomes, if we have an invariant Cosmological constant, so we write $\Lambda \xrightarrow[all-time]{} \Lambda_0$ everywhere, then

$$e^{\left(iI_{H_{FIXED}}/\hbar\right)\left(t,t^0\right)} = \exp\left[\frac{i}{\hbar}\cdot\frac{c^4\cdot\pi\cdot t^0}{16G}\cdot\left(r-r^3\Lambda_0\right)\right] \quad (5)$$

Then, we have that Eq. (1) is re written to be

$$\begin{aligned}\Psi_{Later} &= \int\sum_{H} e^{\left(iI_H/\hbar\right)\left(t,t^0\right)}\Psi_{Earlier}\left(t^0\right)dt^0 \\ &\xrightarrow[at-wormhole]{} \int \exp\left[\frac{i}{\hbar}\cdot\frac{c^4\cdot\pi\cdot t^0}{16G}\cdot\left(r-r^3\Lambda_0\right)\right]\Psi_{Earlier}\left(t^0\right)dt^0\end{aligned} \quad (6)$$

From here on we will be reviewing what to put in the earliest version of the wormhole function

II. What to call the initial wave function in Eq. (6) Two candidates. First being the Hartle – Hawkings wave function

In order to do this we will re reviewing one candidate brought up in [2] first which is the Hartle Hawkiigs wavefunction. Then we will be considering what is done with a wave function from a completely different standard as referenced in section V.

First the Hartle Hawkings wave function

[3] [4] [5] [18] states a Hartle-Hawking wavefunction which we will adapt for the earlier wavefunction as stated in Eq. (6) so as to read as follows

$$\Psi_{Earlier}\left(t^0\right) \approx \Psi_{HH} \propto \exp\left(\frac{-\pi}{2GH^2}\cdot\left(1-\sinh\left(Ht\right)\right)^{3/2}\right) \quad (7)$$

Here, making use of Sarkar [6] we set, if say g_* is the degree of freedom allowed [6]

$$H = 1.66\sqrt{g_*}T_{temp}^2 \big/ M_{Planck} \quad (8)$$

III. Inputs into the temperature in Eq. (8), which is a huge issue. As well as initial time values

We will make the following approximation as far as temperature which will be

$$E(energy) = \frac{k_B T_{temperature}}{2} \Rightarrow T_{temperature} \approx \frac{2E(energy)}{k_B} \quad (9)$$

Whereas we will be from here, using that as input into the Eq. (7) while determining how to obtain $E(energy)$. To do this, note that in a wormhole, we have if the wormhole as a charge in the mouth that7][8]

$$T_{temperature} \approx \frac{(Q)^2}{2\pi k_B r_0^2} \quad (10)$$

To which then we need to discuss what would be the charge, Q, for a wormhole mouth. To see this [7] use an applied electric field we can write as:

$$Q = \frac{E(electric-field)\cdot r^2}{\sqrt{g_{tt}\cdot g_{rr}}} \quad (11)$$

We will go to the Visser values of the denominaror of Eq. (11) next,. From [9] we are picking the simple version of the items read from the Schwartzshield metric

$$
\begin{aligned}
g_{tt} &= -\exp\left(2\Phi\left(r\right)\right) \\
g_{rr} &= \frac{1}{\left(1-\frac{b(r)}{r}\right)} \\
b(r) &= r_0 \cdot \left(\frac{r}{r_0}\right)^{1/\omega}
\end{aligned}
\tag{12}
$$

Here, the value of b(r) has been vastly simplfied from extremely mathematical treatments of these functions.

But , for the record, r_0 is the MINIMUM width of the "throat" of the wormhole, and b as presented is the "shape function " of the funnel of the worm hole. Whereas, the term $\Phi\left(r\right)$ is the so called redshift function. We can and do take the liberty of stating results from [10] which has the following values for the redshift function

$$
\begin{aligned}
g_{tt} &= -\exp\left(2\Phi\left(r\right)\right) \\
g_{rr} &= \frac{1}{\left(1-\frac{b(r)}{r}\right)} \\
b(r) &= r_0 \cdot \left(\frac{r}{r_0}\right)^{1/\omega} \\
\Phi\left(r\right) &= \left\{ const, \quad or \, \frac{1}{r} \right.
\end{aligned}
\tag{13}
$$

In our case, it is simplest to use

$$
\Phi\left(r\right) = const \tag{14}
$$

The consequence of ding this is that the energy, as given by Eq. (10) is NEGATIVE. This negative energy , due to a negative temperature is stunning but defacto stablizes the wormhole, as seen in [11], whereas our result about the temperature T and then the Energy resulting from T <0 can be held to be in fidulity with the results of [12] where we can compare out results, with negative if the minimum width of the wormhole "mouth" is of the order of Planck length.

What is the consequence of having our negative temperature value ?

Go to the value of H in Eq, (8). We find that the Hartle – Hawkings wave function is unchanged and will not be altered by our procedures, since the value of H is proportional to ths quare of the temperature, so if we have an evaluation of Eq. (8) at or about the throat of the wormhole, we will NOT seen a change in evolutionary behavior

Needless to state, we will be assuming that the time, initially will be of Planck time, especially if the generation of Gravitons is about the value of Planck length, i.e. next to the smallest time in the wormhole throat of about Planck time.

In doing so,m we could make the following observation, namely this would probably be the rate of graviton production,

First of all if we had the temperature where we could see say a production of Planck sized black holes, going through the transversable worm hole, we could say based on the following value of M , for generaelized mass in the neighborhood of the throat, i.e. we can go to LOBO et al, [13] on page 125

$$\rho_{\tilde{\alpha}}(r) = \frac{M}{\left(4\pi\tilde{\alpha}\right)^{3/2}} \cdot \exp\left(-\frac{r^2}{4\tilde{\alpha}}\right)$$
$$\Rightarrow M = \left(4\pi\tilde{\alpha}\right)^{3/2} \rho_{\tilde{\alpha}}(r) \cdot \exp\left(\frac{r^2}{4\tilde{\alpha}}\right) \quad (15)$$

So, if this is true, assuming some non commutative geometry, let us assume a way to obtain $\rho_{\tilde{\alpha}}(r)$. I.e.

what if we had a radial dimension of the wormhole throat as of the order of Planck lenth ? If so then we could to first order write

$$M = \left(4\pi\tilde{\alpha}\right)^{3/2} \rho_{\tilde{\alpha}}(r) \cdot \exp\left(\frac{r^2}{4\tilde{\alpha}}\right)$$
$$\xrightarrow[\tilde{\alpha}=r^2\approx\ell_p^2\to 1]{} M \propto \rho_{\tilde{\alpha}}(r) \approx E(energy) = \frac{(Q)^2}{4\pi r_0^2} \quad (16)$$

If we can asusme this, then it is not unreasonble to have the absolute value of the mass, as close to say 1000 planck mass, with due to radiation decay 1/1000 of value , i.e. Planck sized black holes.Say produced 2-3 per second,   so if one had 3000 gravitons produced per second, as measured on Earth, one would likely have 2-3 black holes, of mass of about 10^-5 grams per black hole, producing say 10^57 gravitons, produced per black hole of mass about 10^-62 grams per black hole[14][15] [16]

$$\Gamma \approx \exp\left(\omega_{signal} / T_{temperature}\right) \quad (17)$$

To do this we would have to look at the absolute value of the energy and temperature, i.e. then obtaining Whereas we have from [16] a probability for "scalar" particle production from the wormhole given by

$$\Gamma \approx \exp\left(-E / T_{temperature}\right) \quad (18)$$

IV. What we simplified from and this is the shape function of the wormhole included for completeness of the record

Whereas we are doing a major simplification of the material which is in Lobo's book [13]

See this as to what we simplified. We include it in for completeness of the record

$$b(r) = \left[r_0^{\frac{\gamma-1}{\gamma}} + \gamma \cdot \frac{\left(8\pi G\right)^{\frac{\gamma-1}{\gamma}}}{\tilde{\tilde{\omega}}^{1/\gamma}} \cdot \left(r^3 - r_0^3\right) \right]^{\frac{\gamma}{\gamma-1}} \xrightarrow[r\to r_0]{} r_0 \quad (19)$$

Whereas the b coefficient in the case of NON commutative geometry is chosen [13]

$$b(r)=\frac{2r_s}{\sqrt{\pi}}\cdot\widehat{\gamma}\left(\frac{3}{2},\frac{r^2}{4\tilde{\alpha}}\right)$$

$$\equiv\frac{2r_s}{\sqrt{\pi}}\cdot\left(\frac{r^2}{4\tilde{\alpha}}\right)^{3/2}\cdot\tilde{\Gamma}(3/2)\cdot e^{-3/2}\cdot\sum_{k=0}^{\infty}\left(\frac{\left(\frac{r^2}{4\tilde{\alpha}}\right)^k}{\tilde{\Gamma}((3/2)+k+1)}\right) \tag{20}$$

$$dS^2=-\exp(-2\Phi(r))dt^2+\frac{dr^2}{1-b(r)/r}+r^2\cdot\left(d\theta^2+\left(\sin^2\theta\right)d\varphi^2\right) \tag{21}$$

What we did was to take the simplest case versions of the shape function and other things to keep this from becoming a biblical length text

V. Reviewing a different initial wave function. This one from Kieffer.

Notice the terms for the H factor, and from here we will be making our prediction, where we make the following estimate as to frequency of a signal. That is, if the energy, E , has the following breakdown

$$H=1.66\sqrt{g_*}T_{temp}^2/M_{Planck}$$
$$\Rightarrow E\approx k_B T_{Temp}\approx\hbar\cdot\omega_{signal} \tag{22}$$
$$\Rightarrow\omega_{signal}\approx\frac{k_B\cdot\sqrt{M_{Planck}H}}{\hbar\cdot\sqrt{1.66\sqrt{g_*}}}$$

Eq. (22) would imply an initial frequency dependence,. What we are doing next is to strategize as to understand the contribution of the cosmological constant in this sort of problem. I.e. the way to do it would be to analyze a Kieffer "dust solution" as a signal from the Wormhole.

$$\Delta\omega_{signal}\Delta t\approx 1 \tag{23}$$

If so then we can assume, that the time would be small enough so that

$$\Delta t\approx\frac{\hbar\sqrt{1.66\sqrt{g_*}}}{k_B\cdot\sqrt{M_{Planck}H}} \tag{24}$$

If Eq. (24) is of a value somewhat close to t, in terms of general initial time, we can write[17]

$$\psi_{\tilde{n},\lambda}(t,r)\equiv\frac{1}{\sqrt{2\pi}}\cdot\frac{\tilde{n}!\cdot(2\lambda)^{\tilde{n}+1/2}}{\sqrt{(2\tilde{n})!}}\cdot\left[\frac{1}{(\lambda+i\cdot t+i\cdot r)^{\tilde{n}+1}}-\frac{1}{(\lambda+i\cdot t-i\cdot r)^{\tilde{n}+1}}\right] \tag{25}$$

Here the time t would be proportional to Planck time, and r would be proportional to Planck length, whereas we set

$$\lambda\approx\sqrt{\frac{8\pi G}{V_{volume}\hbar^2 t^2}}\xrightarrow{G=h=\ell_{Planck}=k_B=1}\sqrt{\frac{8\pi}{t^2}}\equiv\frac{\sqrt{8\pi}}{t} \tag{26}$$

Then a preliminary emergent space-time wave function would take the form of

$$\psi_{\tilde{n},\lambda}(\Delta t, r) \equiv \frac{1}{\sqrt{2\pi}} \cdot \frac{\tilde{n}! \cdot \left(2 \cdot \sqrt{8\pi} \cdot (\Delta t)^{-1}\right)^{\tilde{n}+1/2}}{\sqrt{(2\tilde{n})!}} \cdot \left[\begin{array}{c} \dfrac{1}{\left(\sqrt{8\pi} \cdot (\Delta t)^{-1} + i \cdot \Delta t + i \cdot r\right)^{\tilde{n}+1}} \\ -\dfrac{1}{\left(\sqrt{8\pi} \cdot (\Delta t)^{-1} + i \cdot \Delta t - i \cdot r\right)^{\tilde{n}+1}} \end{array} \right] \quad \textbf{(27)}$$

We would take the real part of the Equation (27) and call this as from [17]. This would be with the same frequency as in the Hartle Hawkings wavefunction, and would be for delta t approximately Planck time whereas r would be initially of Planck radius. Right at the mouth of a wormhole

VI. Briefly referring to the behavior of a tokamak, as far as simulating early universe Gravitational waves, and Gravitons

This is from [18] , and we are assuming a Plasma fusion burning temperature of about 100 MeV

Then the power for the Tokamak is

$$P_{\Omega}\Big|_{Tokamak\ toroid} \leq \frac{\xi^{1/8} \cdot \tilde{\alpha}}{\mu_0 \cdot e_j \cdot R} \times \frac{\left(T_{\text{Tokamak temperature}}\right)^{9/4}}{(.87)^{5/4}} \quad (28)$$

Then, per second, the author derived the following rate of production per second of a $10^{-34} eV$ graviton, as, if $\tilde{a} = R/3$

$$n\Big|_{massive-gravitons/\sec ond} \propto \frac{3 \cdot \hbar \cdot e_j}{\mu_0 \cdot R^2 \cdot \xi^{1/8} \cdot \tilde{\alpha}} \times \frac{\left(T_{\text{Tokamak}-temperature}\right)^{1/4}}{\lambda^2_{Graviton} \bullet m_{graviton} \cdot c^2 \cdot (.87)^{5/4}} \sim 1/\lambda^2_{Graviton} scaling \quad (29)$$

If there is a fixed mass for a massive graviton, the above means that as the wavelength decreases, that the number of gravitons produced between plasma burning temperatures of 30 to 100 KeV changes . See [19,20,21,22,23,34]

VII. Linkage to the Big bang. The Frequency would decrease as by 10^-25 taking into account inflation and redshift which would mean incredibly high GW frequencies generated by the Tokamak

Further elaboration of this matter in the experimental detection of experimental data sets for massive gravity lies in the viability of the expression derived , $h \sim 10^{-27}$ for a GW detected 5 meters above a Tokamak represents the decrease in strain, by a factor of about 100, whereas in the center of the Tokamak, we would have, say, $h_{2nd-term} \sim 10^{-26} - 10^{-27}$. We would have the situation in a tokamak as given in [18] that the strain value, h, would be modest, as given and would be sensitive to detection whereas we do not know yet as how to calculate the strain for GW emanating from a wormhole mouth. This is a detail which has to be completed.

VIII. **Concluding a comparison between the Tokamak and the worm hole models**

As seen in Eq. (29) there would be a LOT more gravitons produced per second by the Tokamak, as of about the same small mass of gravitons in the same mass range.

Frequency of the worm hole and the Tokamak would be different, as the tiny wormhole mouth would not be in the center of the universe, with its down scaling of 10^-25 or so, for Tokamak Frequencies in order to come across the frequencies found on Earth for the big bang. The redshift values for the worm hole would likely be only about 10^-3 at most, for redshift values, whereas Eq. (22) would refer to GW generated by the wormhole. These wormhole frequencies would have to be red shifted down only about 10^-3 instead of the enormous value as to how to have frequencies of a Tokamak scaled downward as to be a predictor – corrector for assumed frequencies of relic GW

For what is is worth this is the frequency scaling to keep in mind for the Tokamak

$$\left(1+z_{initial-era}\right) \equiv \frac{a_{today}}{a_{initial-era}} \approx \left(\frac{\omega_{Earth-orbit}}{\omega_{initial-era}}\right)^{-1} \qquad (30)$$

$$\Rightarrow \left(1+z_{initial-era}\right)\omega_{Earth-orbit} \approx 10^{25}\,\omega_{Earth-orbit} \approx \omega_{initial-era}$$

We would see at most only about a 10^-3 scaling down of energy for the Wormholes, with black holes generated several per second as to what is stated in energy would still have to be considered along the lines of

$$\langle E\rangle_{\kappa=n,\lambda} = \frac{(\kappa=n)+1/2}{\lambda} \xrightarrow[\lambda\approx 1/\hbar\omega]{} \hbar\omega\cdot\left((\kappa=n)+1/2\right) \qquad (31)$$

What we can do, is to ascertain the last step would be to make a cosmology wavefunction in a sense partly related to the simple harmonic oscillator,. We would have to keep in mind Eq. (26) as well and keep in mind the redshift issues brought up

The references 25 – 34 before are meant to be informative issue related docments which could futher a comparsion of the physics os the two situations

The tokamaks would NOT involve black holes. The worm hole would likely induce the production of black holes. However, in the early universe we would likely see the production of millions of micro black holes which would declay, after the initiation of the big bang.

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