On the Verification of the Multiverse

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Abstract

We outline a proposal for an experimental test of Everett's many-worlds interpretation of quantum mechanics that could potentially verify the existence of a multiverse. This proposal is based on a quantum field theory formulation of many-worlds through the path integral formalism and a careful choice of the vacuum state.

1 Introduction

The interpretation of quantum mechanics, in particular that of measurements, has been an area of contention since its early beginnings some 100 years ago. The many-worlds interpretation was first proposed by Hugh Everett in 1957 [1]. This interpretation, sometimes called the relative state formulation, asserts that there exists a universal wavefunction that is objectively real, and that there is no wave function collapse in measurements. This implies that all possible outcomes of measurements are physically realized in some universe [2], and that the evolution of the universe is rigidly deterministic. This classifies the many-worlds interpretation as a *multiverse theory.*

Everett's original formulation can be summarised in the relatively innocent looking statement that:

All isolated systems evolve according to the Schrödinger equation

$$i\hbar\frac{d}{dt}|\Psi\rangle = \hat{H}|\Psi\rangle, \tag{1}$$

where the emphasis is on the entire Universe evolving according to this equation since it is an isolated system. However, accepting this formulation implies as a corollary that there are no definite outcomes of quantum measurements – the so-called *wavefunction collapse* of the Copenhagen interpretation – since this would break the universal application of the postulate.

Below we will first discuss existing (limited) thoughts about possible tests of the many-worlds interpretation in Sec. 2, before we briefly recapitulate central point in the path integral formulation of quantum mechanics and quantum field theory in Sec. 3. This will be essential to our new proposal which can be found in Sec. 4, before we conclude in Sec. 5.

2 Current status of proposed tests

Few realistic tests of the many-worlds interpretation have been proposed to date. The problem has been that it is difficult to come up with a test where the different interpretations give different predictions. Perhaps the most famous proposal is a version of Schrödinger's Cat [3] coined the *quantum suicide test* [2]. In this experiment a superposed state is prepared, for example a spin-$\frac{1}{2}$ particle in the state

$$\frac{1}{\sqrt{2}}(|{\uparrow}\rangle + |{\downarrow}\rangle). \tag{2}$$

The (z-component of) the spin of the particle is measured $\hat{U}$, and an attached loaded gun is fired at the head of a nearby cat if the measurement is $|{\downarrow}\rangle$, and an attached unloaded gun is fired if $|{\uparrow}\rangle$ is measured.

The result of the experiment then seems to depend on the observer. The experimenter – safely hidden behind a concrete wall one supposes – hears randomly a bang from the fired gun or a click from the empty gun with equal probability. If repeated infinitely many times the expectation for the particular state given above is that we will have the same number of dead and alive cats.

From the point of view of the cat life is more interesting. In the traditional Copenhagen interpretation with the collapse of the wavefunction the cat will – if lucky – experience a click or two from the empty gun, then observe no more. However, in Everett's many-worlds interpretation the state of the trigger-gun-cat system after the first measurement is

$$\hat{U}\frac{1}{\sqrt{2}}(|{\uparrow}\rangle + |{\downarrow}\rangle) \otimes |\text{🐱}\rangle = \frac{1}{\sqrt{2}}\left(|{\uparrow}\rangle \otimes |\text{😺}\rangle + |{\downarrow}\rangle \otimes |\text{😵}\rangle\right). \tag{3}$$

The *observing* cat will then with 100% certainty hear the click of the unloaded gun, and this will repeat for all further repetitions of the experiment, although there will after a while be very many worlds with a non-observing cat. The cat can then after a significant number of such experiments exclude the Copenhagen interpretation of quantum mechanics at a great confidence level.

The observation has traditionally been that firstly, the job of the cat is not to be envied, and more important for physics, that it is an essential problem that the conclusion of the experiment can not convincingly be communicated to the rest of the world – not solely from the cats problems with communication – but that in the wast majority of worlds the cat is indeed dead.

3 Theoretical background

To set the scene for our new proposal we first discuss the formulation of quantum mechanics in terms of the path integral formalism, and the natural extension of this to quantum field theory.

3.1 Path integral formulation in quantum mechanics

The essence of the path integral formulation of quantum mechanics is very much in line with the core of Everett's idea: the system will take on every possible configuration as it evolves from the initial to the final state; anything that can happen will happen. Each of these distinct histories can be thought of as a path through the space of all configurations that describe the state of the system (a Fock space).

In one dimension the path integral formulation, see *e.g.* [4], can be written as

$$\langle x, t | x_0, t_0 \rangle = N \int \mathcal{D}x \, e^{iS[x]} \,, \tag{4}$$

where the (square of the) bracket signifies the probability of finding the system in the state $|x\rangle$ at time t if it was in the state $|x_0\rangle$ at t_0. The normalisation constant N is an infinite constant that will cancel in physical quantities, while the measure is defined as

$$\int \mathcal{D}x = \int_{x_0(t_0)=x_0}^{x(t)=x} \mathcal{D}x(t') \equiv \lim_{n\to\infty} \int \prod_{k=1}^{n} dx_k \,, \tag{5}$$

where $\mathcal{D}x$ implies the sum over all paths $x(t')$ with a given boundary condition.[1]

This can be generalised to an n-point Green's function

$$G^{(n)}(x_1, \ldots, x_n) = \langle 0 | \hat{T}(\hat{x}(t_1) \ldots \hat{x}(t_n)) | 0 \rangle = N \int \mathcal{D}x \, x(t_1) \ldots x(t_n) \, e^{iS[x]} \,, \tag{6}$$

relating to the vacuum state $|0\rangle$ of the system. Here $\hat{T}$ is the time-ordering operator.

3.2 Path integral formulation in quantum field theory

In quantum field theory the same n-point Green's function can be written as the field correlator

$$G^{(n)}(x_1, \ldots, x_n) = \langle 0 | \hat{T}(\phi(x_1) \ldots \phi(x_n)) | 0 \rangle \,. \tag{7}$$

Replacing the position operators in Eq. (6) with fields we write

$$G^{(n)}(x_1, \ldots, x_n) = N \int \mathcal{D}\phi \, \phi(x_1) \ldots \phi(x_n) \, e^{iS[\phi]} \,. \tag{8}$$

To make the path integral calculations simpler we make use of a *generating functional*. This is defined as the vacuum amplitude in the presence of a source $J(x)$,

$$\mathcal{Z}[J] \equiv \langle 0 | 0 \rangle_J = N \int \mathcal{D}\phi \, \exp\left(iS[\phi] + i \int d^4x \, J(x) \phi(x) \right) \,, \tag{9}$$

[1] This is also formally infinite, but will not appear in physical quantities so we dispense with the technicalities here.

where we require the vacuum state to be normalized, *i.e.* $\langle 0|0\rangle = 1$, giving

$$N^{-1} = \int \mathcal{D}\phi \, \exp\big(iS[\phi]\big) \,. \tag{10}$$

Generalising to a number of fields ψ_k, and including also gravity and an expanding universe with cosmological constant Λ, we can write down the complete generating functional as

$$\mathcal{Z}[J] = N \int \mathcal{D}\hat{g} \prod_k \mathcal{D}\psi_k \, \exp\left[iS[\psi_k] + \int d^4x \sqrt{-\hat{g}} \left(\frac{-\hat{R} + 2\Lambda}{16\pi G} \right) + i \int d^4x \, J\psi_k \right] . \tag{11}$$

To go from the generating functional to the Green's function is then straight forward, we simply apply the *functional derivative*. This gives

$$G^{(n)}(x_1, \ldots, x_n) = (-i)^n \frac{\partial^n \mathcal{Z}[J]}{\partial J(x_1) \cdots \partial J(x_n)}\Big|_{J=0} \,. \tag{12}$$

In practical calculations this generating functional corresponds to all possible Feynman diagrams, also those that do not describe scatterings. Therefore, it is convenient to define a generating functional that corresponds to *connected* Feynman diagrams. This is defined as

$$\mathcal{W}[J] \equiv -i \ln \mathcal{Z}[J] \,, \tag{13}$$

giving the connected Green's function $G_c^{(n)}$:

$$G_c^{(n)}(x_1, \ldots, x_n) = (-i)^{n+1} \frac{\partial^n \mathcal{W}[J]}{\partial J(x_1) \cdots \partial J(x_n)}\Big|_{J=0} \,. \tag{14}$$

4 Outline of a new test

As we saw in Sec. 2, the essential difficulty of current proposed test lies in the problem of a single observer verifying the many-worlds interpretation being very improbable in any given world. We want to point out here that a distinction occurs in the many-worlds interpretation when we take into account that the number of observers is not uniquely predicted. In the Copenhagen interpretation each measurement is weighted simply by its quantum mechanical probability, but in the many-worlds description there should also be a weighting by the number of observations.

In the modern cosmological understanding of the Universe as expanding, and dominated by a vacuum energy density, the lifetime of the Universe has been estimated to be exceedingly long, giving $t_{\max} < 10^{60}$ yr [5] in the most conservative case. Taking this into account one concludes that all possible events, including those with extremely low probability, will occur. For the present discussion, one of the most interesting of those unlikely events would be the spontaneous appearance of observers from quantum fluctuations of the vacuum, surrounded by an environment suitable for observation. It is then natural to ask what the influence of these spontaneously created observers is.

To answer that question we simplify the example in Sec. 2 to consider instead the spontaneous decay of an unstable particle (the cat). For concreteness we calculate the decay of a scalar particle into two other scalars with order one couplings. As we saw in the previous section, in a quantum field theory formulation in terms of the path integral, the vacuum state enters in the generating functional. We now join these two concepts in the idea that as parts of the Universe, the observers, have no choice but to consider themselves as quantum objects, and thus as part of the state.

Allowing for the fluctuation of observers in the vacuum, assuming a uniform probability density of observers per (four-) volume unit in an expanding Universe, the decay rate can be calculated using Eqs. (11) and (14), following the normal prescription for decay rates in quantum field theory, and a modification of the generating functional to generate a (normalized) vacuum with observers

$$|0\rangle = \otimes_k |\text{😸}\rangle_k, \tag{15}$$

where the direct product is taken over a ensemble of observers in the local volume generated by the uniform probability distribution.[2]

We then arrive at a lifetime for the particle which does not follow the normal exponential law with the standard lifetime τ_0 for the decay, but instead has an expected lifetime

$$\tau = \frac{\hbar}{\Gamma} = \tau_0 \exp\left(\frac{V_4(t)}{V_4(0)}\right), \tag{16}$$

that depends on the fluctuating observers through a relative volume factor

$$V_4(t) = c\int_0^t dt'\, a^3(t'), \tag{17}$$

which is the four-volume of the Universe at a time t given by the scale factor $a(t)$ in general relativity. Here we have taken $t = 0$ to be the present time.

While the enhancement in lifetime is exponential, the effect is still very small because the Hubble time that sets the scale of the effect is a very large number. Assuming $a(t) = a_0 e^{Ht}$ in the present vacuum energy dominated universe, we have

$$\frac{V_4(t)}{V_4(0)} = e^{3Ht} - 1 = e^{3t/t_H} - 1 \simeq \frac{3t}{t_H}, \tag{18}$$

where the Hubble time is $t_H = 4.55 \cdot 10^{17}$ s.

Nonetheless, for very long lived particles with large relativistic gamma factors it may be experimentally feasible to search for a deviation in the exponential decay law. Ignoring the possibility of an unstable proton, the longest lived candidate would be the neutron. With gamma factors of the order of 100–1000 achievable in modern particle accelerators it would be necessary to detect relative deviations in the predicted lifetime of the order of 10^{-11}.

[2]We are also considering other assumptions for the vacuum state (work in progress).

5 Conclusion

Through a re-formulation of the many-worlds interpretation in quantum field theory, and a deliberate choice of the vacuum state of the Universe, we have shown that the many-worlds interpretation of quantum mechanics may have observable consequences for measurements of the decays of long lived particles. If such lifetime measurements can be carried out to the necessary precision it may lead to a dramatic verification of the existence of the multiverse in the near future.

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References

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