Abstract:

I have previously shown that the exponential version of the gravitational time dilation (GTD) equation (first given by Einstein in 1907) is incorrect, because it is inconsistent with the outcome of the twin paradox. I then gave a corrected version of the GDT equation which IS consistent with the outcome of the twin paradox, and which is also consistent with the co-moving-inertial-frames (CMIF) simultaneity method. In this brief paper, I describe an experimental test of my GTD equation that might be feasible to conduct.

Section 1. Einstein’s Exponential GTD Equation

In Einstein’s 1907 paper [https://einsteinpapers.press.princeton.edu/vol2-trans/319], Einstein stated that the GTD equation is

\[ R(g) = \exp(g \cdot L), \]

where “g” is the force per unit mass exerted by the uniform gravitational field, and “L” is the constant distance (in the direction of the field) between two stationary clocks. The quantity “R” is the ratio of the tic rates of the two clocks: the clock which is higher in the field (farther from the source of the field) will run “R” times faster than the other clock.

According to the Equivalence Principle, we can then also say that when there are no gravitational fields (i.e., in a Special Relativity scenario), two clocks which are initially unaccelerated, and which are separated by a constant distance “L”, and which are then simultaneously accelerated with an acceleration “A” (in the direction of their separation), then the rate ratio “R” is

\[ R(A) = \exp(A \cdot L). \]

The leading clock runs “R” times faster than the trailing clock. Note that for constant “A” and “L”, the rate ratio “R” DOES NOT VARY WITH TIME.

I showed in https://vixra.org/abs/2109.0076 that the above exponential equation is inconsistent with the outcome of the twin paradox. Specifically, if the traveling twin (he) changes his velocity instantaneously at his turnaround, the exponential equation says that the home twin (she) will be INFINITELY old after his turnaround, and when the twins are reunited. That isn’t true: both of their ages are finite at the reunion. Thus the exponential equation for “R” is incorrect. (And the exponential equation is also incorrect for the case where the turnaround is not instantaneous, but is just very quick.)
Section 2. My GTD Equation

The corrected rate ratio equation is

\[ R(A) = 1 + L A \text{sech}_\text{sqrd}(\theta) \],

where sech_sqrd( ) is the square of sech( ), the hyperbolic secant. (The hyperbolic secant is the reciprocal of the hyperbolic cosine, cosh, which is more likely to be available in tables). Cosh(\theta) can also be calculated using the equation

\[ \cosh(\theta) = \{ \exp^\theta + \exp^{-\theta} \} / 2. \]


The quantity “\theta” in the above equation for R(A) is

\[ \theta(t) = A t, \]

where “t” specifies how long the constant acceleration “A” has been going on, since it abruptly started from zero acceleration at time zero. So we really should write the “R” equation as

\[ R(A,t) = 1 + L A \text{sech}_\text{sqrd}(\theta(t)). \]

A plot of R(A,t) versus “t” is given in section 4 of https://vixra.org/abs/2201.0015 for the case A = 1 ls/s/s (about 40 g’s) and L = 7.52 ls.

The R(A,t) equation produces quite different qualitative results than those produced by the exponential R(A) equation. The exponential R(A) equation says that if the constant acceleration “A” goes on for an essentially infinite time, the rate ratio NEVER CHANGES … i.e., the leading clock keeps ticking faster than the trailing clock by the same ratio, forever. And for large “A”, that constant ratio is HUGE! In contrast, the new R(A,t) equation says that, as “t” goes to infinity, R(A,t) approaches 1.0. I.e., the two clocks eventually tic at essentially the same rate. That is quite a qualitative difference, which might be observable experimentally.

The above results are derived and explained more thoroughly in https://vixra.org/abs/2201.0015 and in https://vixra.org/abs/2206.0133.

Section 3. A Proposed Experimental Test of My GTD Equation

Charged particles can be accelerated to speeds that are a large fraction of the speed of light, by exposing them to very large electric fields. So we can start with a stationary pair of them, separated by the distance “L”, and then switch on a very strong uniform electric field (in the direction of their separation) that will accelerate both of them at the same rate, and maintain their separation at “L”.

But how can we use each particle as a “clock”? We might be able to accomplish that by using UNSTABLE charged particles … particles that have a known average lifetime (before they decay into uncharged particles that won’t accelerate in the electric field). That way, if the leading particle is ageing faster than the trailing particle, the leading particle will (on average) decay quicker than the trailing particle, which might be observable. That might allow an experimental way to verify or falsify my GDT equation.