

Exploration of the Criteria for Graviton Detection from Relic Conditions

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Abstract. We will first of all reference a value of momentum, in the early universe. This is for 3+1 dimensions and is important since Wesson has an integration of this momentum with regards to a 5 dimensional parameter included in an integration of momentum over space which equals a ration of L divided by small l (length) and all this times a constant. The ratio of L over small l is a way of making deterministic inputs from 5 dimensions into the 3+1 dimensional HUP. In doing so, we come up with a very small radial component for reasons which due to an argument from Wesson is a way to deterministically fix one of the variables placed into the 3+1 HUP. This is a deterministic input into a derivation which is then **First** of all, we restate a proof of a highly localized special case of a metric tensor uncertainty principle first written up by Unruh. Unruh did not use the Roberson-Walker geometry which we do, and it so happens that the dominant metric tensor we will be examining is variation in δg_{tt} . We state that the metric tensor variations given by δg_{rr} , $\delta g_{\theta\theta}$ and $\delta g_{\phi\phi}$ are negligible contributions, as compared to the variation δg_{tt} . From there the expression for the HUP and its applications into certain cases in the early universe are strictly affected after we take into consideration a vanishingly small r spatial value how we define δg_{tt}

Key words, Massive Gravitons, Heisenberg Uncertainty Principle (HUP), Riemannian-Penrose Inequality

I. Introduction, why we analyse our HUP with very small radial r value. Here it comes directly from the 5th dimension

Wesson in [1], page 105 has the following result of how the momentum is affected by a 5 dimensional input (from the fifth dimension). In other words we have the following expression, namely

$$\int p_{\alpha} dx^{\alpha} = \pm \frac{h}{c} \cdot \frac{L}{\ell} \quad (1)$$

We will be defining what the momentum p_α is in our treatment of the early universe whereas the first five dimensional input values L here comes from the inverse of the square root of the cosmological constant

$$L \equiv \sqrt{\frac{3}{\Lambda}} \quad (2)$$

Whereas the term ℓ is equal to the Compton wavelength of a "particle" m for which

$$\ell \equiv \frac{h}{m \cdot c} \quad (3)$$

So if we use the reasoning done in [2] in the early universe, namely [2] we have that due to that documents page 2, formula (9)

$$\langle p \rangle = -\frac{\tilde{\beta}}{2m_p} \cdot \sqrt{\frac{v}{\pi G}} \cdot \frac{\ln t}{\varpi c} \quad (4)$$

And keeping in mind the Wesson 5 dimensional line element [1] given by

$$dS_{5\text{-dim}}^2 = \frac{L^2}{\ell^2} \cdot ds_{4\text{-dim}}^2 - \frac{L^4}{\ell^4} \cdot d\ell^2 \quad (5)$$

We get an infinitesimal r value due to from the 5th dimension fixing the value of to be very small in a deterministic fashion

$$\begin{aligned} \int p_\alpha dx^\alpha &= \pm \frac{h}{c} \cdot \frac{L}{\ell} = \pm \frac{h}{c} \cdot \sqrt{\frac{3}{\Lambda}} \frac{m_{particle}}{h} \\ &= \frac{r}{m_{pl}} \cdot \left(1 - \log \left[\frac{r}{\varpi \cdot c} \right] \right) \end{aligned} \quad (6)$$

$$\Leftrightarrow r \approx \varepsilon^+$$

This is in tandem with the value of z , as to red shift showing up in [3] and it shows how to obtain a very small radial value in a different manner, namely in a tiny scale factor due to an enormous z red shift as given in [3]

Quote

Note this comes from a scale factor, if $z \sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, but here note that the scale factor is not zero, so we do not have a space – time singularity.

End of quote

However that scale factor being very small, with enormous red shift is in tandem with Eq. (6). So go to the HUP

II. Recalling the argument from [3] as to the form of the early Universe HUP

Note this comes from a scale factor, if $z \sim 10^{55} \Leftrightarrow a_{scale-factor} \sim 10^{-55}$, i.e. 55 orders of magnitude smaller than what would normally consider, [3]

$$\begin{aligned} \left\langle (\delta g_{uv})^2 (\hat{T}_{uv})^2 \right\rangle &\geq \frac{\hbar^2}{V_{Volume}^2} \\ \xrightarrow{uv \rightarrow tt} \left\langle (\delta g_{tt})^2 (\hat{T}_{tt})^2 \right\rangle &\geq \frac{\hbar^2}{V_{Volume}^2} \end{aligned} \quad (7)$$

$$\& \delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$$

$$\delta t \Delta E \geq \frac{\hbar}{\delta g_{tt}} \neq \frac{\hbar}{2} \quad (8)$$

$$\text{Unless } \delta g_{tt} \sim O(1)$$

$$\delta g_{tt} \sim a^2(t) \cdot \phi \ll 1 \quad (9)$$

Then, there is no way that Eq. (9) is going to come close to $\delta t \Delta E \geq \frac{\hbar}{2}$.

III. How we can justifying writing very small $\delta g_{rr} \sim \delta g_{\theta\theta} \sim \delta g_{\phi\phi} \sim 0^+$ values.

To begin this process, we will break it down into the following coordinates [3]

In the $rr, \theta\theta$ and $\phi\phi$ coordinates, we will use the Fluid approximation,

$$T_{ii} = \text{diag}(\rho, -p, -p, -p) [3] \text{ with}$$

$$\begin{aligned}
\delta g_{rr} T_{rr} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot r^2}{V^{(4)}} \right| \xrightarrow{a \rightarrow 0} 0 \\
\delta g_{\theta\theta} T_{\theta\theta} &\geq - \left| \frac{\hbar \cdot a^2(t)}{V^{(4)} (1 - k \cdot r^2)} \right| \xrightarrow{a \rightarrow 0} 0 \\
\delta g_{\phi\phi} T_{\phi\phi} &\geq - \left| \frac{\hbar \cdot a^2(t) \cdot \sin^2 \theta \cdot d\phi^2}{V^{(4)}} \right| \xrightarrow{a \rightarrow 0} 0
\end{aligned} \tag{10}$$

IV. After doing this, how can we obtain values of δg_{tt}

We will put in different values of the scalar potential and make comments as to what this pertains to in terms of early universe physics

The first one will be using a scalar field from inflaton physics, as presented by Padmanabhan, [4]. For the record, Dr. Tony Scott has communicated his disapproval of involving the Padmanabhan potential to the author in communications, but this will be presented as one of the possible choices

First we have from [2]

$$V(\phi) = V_0 \exp\left(-\frac{\lambda\phi}{m_p}\right) \leftrightarrow V_0 \exp\left(-\sqrt{\frac{16\pi G}{3}} \cdot \phi\right) \tag{11}$$

And also [2] [5]

$$\begin{aligned}
a(t) &= a_{initial} t^\nu \\
\Rightarrow \phi &= \ln \left(\sqrt{\frac{8\pi G V_0}{\nu \cdot (3\nu - 1)}} \cdot t \right)^{\sqrt{\frac{\nu}{16\pi G}}} \\
\Rightarrow \dot{\phi} &= \sqrt{\frac{\nu}{4\pi G}} \cdot t^{-1} \\
\Rightarrow \frac{H^2}{\dot{\phi}} &\approx \sqrt{\frac{4\pi G}{\nu}} \cdot t \cdot T^4 \cdot \frac{(1.66)^2 \cdot g_*}{m_p^2} \approx 10^{-5}
\end{aligned} \tag{12}$$

Where we can put in the values of Eq. (12) into Eq. (9). We can write an expression for V_0 from [10], page 153 taking the form of if the denominator is the e fold value of inflation,

$$V_0^{1/4} = \frac{.022 m_p}{\sqrt{q N_{e\text{-folds}}}} \tag{13}$$

And using the Starobinsky model, plus [2] [6] [7] to get a value of L

And so we obtain if we have a scale factor behaving as in (12)

$$\begin{aligned}
\Lambda &\approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2N + \frac{\gamma \cdot (3\gamma-1)}{8\pi G \cdot \tilde{t}^2}\right]}{\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x} + \left(6 \cdot \left(\frac{\ddot{a}}{a} + \left(\frac{\dot{a}}{a}\right)^2\right)\right) \Big|_{t=\tilde{t}} \\
&\approx \frac{-\left[\frac{V_0}{3\gamma-1} + 2V_0 \cdot (1 - \exp[-q \cdot \phi/m_p])^2 + \frac{\gamma \cdot (3\gamma-1)}{8\pi G \cdot \tilde{t}^2}\right]}{\frac{1}{\kappa} \int \sqrt{-g} \cdot d^3x} \\
&\quad + 6 \cdot \frac{-t \cdot \gamma \cdot (3\gamma-1)}{m_p G} \cdot \sqrt{\frac{1}{8\pi}} \\
&\quad + \frac{48\pi G}{3} \cdot \left[V_0 \cdot (1 - \exp[-q \cdot \phi/m_p])^2\right]
\end{aligned} \tag{14}$$

This can be put into the value of Eq. (9), If we presume Planck time, then if the value of Eq. (9) is very small which is frequently a result, we will have a very large value for change in Energy, which would in its own way confirm the enormous value of M initially confirmed as forming which is in [7] via the relationship of change in energy E will be proportional to the very large value of M so initially formed

V. Conclusion: Comparing with the other assumed early uncertainty principles

What is in Eq. (9) plus the inputs into Eq. (14) put into Eq. (9) which influences Eq. (8) should be compared with

Following Uncertainty principle [8] [9] [10] [11]

$$\begin{aligned}
\Delta t \geq \frac{\hbar}{\Delta E} + \gamma t_p^2 \frac{\Delta E}{\hbar} &\Rightarrow (\Delta E)^2 - \frac{\hbar \Delta t}{\gamma t_p^2} (\Delta E) + \frac{\hbar^2}{\gamma t_p^2} = 0 \\
\Rightarrow \Delta E &= \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \left(1 + \sqrt{1 - \frac{4\hbar^2}{\gamma t_p^2 \cdot \left(\frac{\hbar \Delta t}{2\gamma t_p^2}\right)^2}}\right) = \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \left(1 \pm \sqrt{1 - \frac{16\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2}}\right) \tag{15}
\end{aligned}$$

$$\begin{aligned}
\Delta E &\approx \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \left(1 \pm \left(1 - \frac{8\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2}\right)\right) \\
\Rightarrow \Delta E &\approx \text{either } \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \frac{8\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2}, \text{ or } \frac{\hbar \Delta t}{2\gamma t_p^2} \cdot \left(2 - \frac{8\hbar^2 \gamma t_p^2}{(\hbar \Delta t)^2}\right)
\end{aligned} \tag{16}$$

A point by point comparison of these values should be the next objective of a research project. Furthermore the items brought up in references [12] [13] [14] [15] [16] will be able to be vetted provided that we make the comparison between Eq. (8) and Eq. (9) with Eq. (15) and Eq. (16) in a rigorous manner.

In particular, I would look forward to eventual experimental verification, if the early universe HUP were really understood of investigating the great ideas brought up by Corda in [12]

Determination of that would be exciting experimental gravitational physics

V. Determining detection and LIGO , and other possible detectors playing a role in data analysis , as a conclusion to this inquiry

Using that rule, we could assume 10^{122} gravitons, as actually being generated from primordial conditions with say of this number, say at most about 10^{21} Planck sized black holes being formed due to the uncertainty principles just elucidated, initially. This by Eq. (17) and Eq. (18) would in the following upper bound of about 1 Hertz for primordial black holes

Then we can use inputs from [17] [18] [19] where we are assuming the energy is an input into Delta E (change in energy) as to our above uncertainty principle relations. So then we use for energy,

$$\begin{aligned}
 E_{BEC-Graviton} &\approx \frac{k_B T_{BH}}{2} \approx \frac{k_B \times 10^{-5} \times T_P}{2} \\
 \Rightarrow \omega_{BEC-Graviton} &\propto 10^{-5} \times 10^{43} \text{ Hz} \approx 10^{38} \text{ Hz} \\
 \Rightarrow \omega_{BEC-Graviton-to-CMBR} &\approx 10^{38} \times 10^{-3} \text{ Hz}
 \end{aligned} \tag{17}$$

Whereas we would have simple primordial black hole – primordial black hole given by a redshift, of $z \sim 10^{25}$, that this would mean a present value of frequency as having a maximum value of about 1 Hz. This comes from [17][18] and [19]

The non-zero entropy would come from a massive graviton as seen by using the relationship [20]

$$\begin{aligned}
 m &\approx \frac{M_P}{\sqrt{N_{gravitons}}} \\
 M_{BH} &\approx \sqrt{N_{gravitons}} \cdot M_P \\
 R_{BH} &\approx \sqrt{N_{gravitons}} \cdot l_P \\
 S_{BH} &\approx k_B \cdot N_{gravitons} \\
 T_{BH} &\approx \frac{T_P}{\sqrt{N_{gravitons}}}
 \end{aligned} \tag{18}$$

Here we have that the mass of a graviton is related to small m in Eq. (18) . Also the Energy of a graviton would be linked to creation of the mass of a black hole, which according to [17] [18] [19] and a massive redshift down from primordial conditions i.e. the initial high frequency GW and gravitons would be detectable about for a massive graviton , where we consider a massive graviton as given by Novello [21] as given by

$$m_g = \frac{\hbar \cdot \sqrt{\Lambda}}{c} \quad (19)$$

Also the Hawking argument as to the probability of finding a universe with Λ being a given value[22][23]

$$P(\Lambda) \sim \exp(-2S_E(\Lambda)) \approx \exp\left(\frac{3\pi M_P^2}{\Lambda}\right) \quad (20)$$

If Eq. (20) holds with almost infinite probability, and if we can confirm that, as well as the reality of Eq. (18) holding and Eq (19) being the same as m in Eq. (18) we have a working schematic as far as graviton generation in relic conditions which is linkable to LIGO and other detectors.

The presence of the graviton's mass should indeed generate a longitudinal component in the LIGO response function which could be, in principle, detected if LIGO and the other interferometers will improve their sensitivity in the next few years. This is stressed, for example, [24] [in Int. Jour. Mod. Phys. D 18, 2275 (2009)]

This matter of a longitudinal component in the LIGO response function, combined with the presence of non-zero entropy at the start of the universe, may be confirmable in the next several years.

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