Extended Proof of The Collatz Conjecture

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The Collatz Conjecture asks whether its function being repeated always reaches number 1. Its function is piecewise with two conditions. Any number entering the function repeatedly eventually becomes number 1. This paper shows a proof that extends to using specified functions.

Introduction

The function in the Collatz Conjecture, when repeated continuously, always reaches number one. The proof in this paper shows that there can be specified functions that are piecewise to determine the conditions of the function. This proof shows a step by step approach to prove the conjecture.

S is defined as a set of positive integers. Let A be a subset of S with the following property. It belongs to a set of positive integers defined by a function such that the range consists of positive integers. Let B be the set of S excluding the values from A. a is defined as a variable that belongs to set A and b belongs to B.

$$S = \{\mathbb{Z}^+\} \qquad B = S - A$$

$$A \subseteq S \qquad a \in A$$

$$A = \{x \mid P(x)\} \qquad b \in B$$

The function f(n) is defined as a piecewise function composed of functions g(n) and h(n). g(n) and h(n) are one-to-one correspondence functions. The domain of h(n) is greater than its range and the domain of g(n) is less than its range.

$$f(n) = \begin{cases} g(n), & n = a \\ h(n), & n = b \end{cases}$$

$$\mathrm{T} \equiv (g(a) > a) \wedge (h(b) < b)$$

i belongs to S. Let $H_i(b)$ be the function composition of h(b) such that it is composed of i functions. There exists b such that $H_i(b)$ = a. For all a, g(a) = b.

$$i \in S$$

$$Hi(b) = h1(b) \circ h2(b) \circ hi(b)$$

$$\forall a\ (g(a) = b)$$

$$\exists b\ (Hi(b) = a)$$

There exists only one j such that $j \in S$. There exists only one a and b such that $g(a) = b$ and $H_j(b) = a$. Therefore, the composition of these functions shows that $H_j(g(a)) = a$, meaning that the function gives the same number. A new function C(a) is defined as this function. A new function D(a) is defined as $H_i(g(a))$ and is equal to b.

$$\exists!\, j(j \in S)$$

$$\exists!\, \mathrm{a}\ \exists!\, \mathrm{b}\ ((g(a) = b) \wedge (Hj(b) = a))$$

$$\exists!\, \mathrm{a}\ \exists!\, \mathrm{b}\ (Hj \circ g(a) = a)$$

$$Hj \circ g(a) = C(a)$$

$$Hi \circ g(a) = b$$

$$Hi \circ g(a) = D(a)$$

m belongs to S. D(a) is repeated as a composition with $D(a_m) = a_{m+1}$. For the last function, $D(a_m) = a_1$. A system of equations can be constructed from the composition of D(a). The addition to the system of equations gives the same equation as if the value for the domain was the same as for the range. This is possible if the range of a_m was a_{m-1} excluding the last variable which becomes a_m. This is the same function as C(a).

$$m \in S$$

$$\begin{array}{rl}
(D(a_1) & = a_2) \\
(D(a_2) & = a_3) \\
\cdots & \cdots \\
+(D(a_m) & = a_1) \\
\hline
D(a_1) + D(a_2) + \cdots + D(a_m) & = a_2 + a_3 + \cdots + a_1
\end{array}$$

$$\begin{array}{rl}
(D(a_1) & = a_1) \\
(D(a_2) & = a_2) \\
\cdots & \cdots \\
+(D(a_m) & = a_m) \\
\hline
D(a_1) + D(a_2) + \cdots + D(a_m) & = a_1 + a_2 + \cdots + a_m
\end{array}
\qquad
\begin{array}{rl}
(C_1(a) & = a) \\
(C_2(a) & = a) \\
\cdots & \cdots \\
+(C_m(a) & = a) \\
\hline
mC(a) & = ma
\end{array}$$

To prove the conjecture it is proved that there is one number for which the function has the same domain and range. This is done by using a system of equations and another with switching variables from the range. When the systems of equations are added, they become equivalent. This means that they have the same solution for as many times as the function can be repeated. All numbers that enter the function always change, except for the solution. All the numbers that enter the functions that are not the solution always change when the function is repeated. When the function reaches the solution and repeats, the solution always stays the same.