

# Lorentz Volumes Paradox: Limits of Special Relativity in Application

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**ABSTRACT :** When applying the Lorentz transformations of the coordinates of space to different types of volumes such as the volume of a gas and the rational volume between the gas and its molecules in a thermodynamic system, the observer gets contradictory results in the same frame of reference, Although the contradictory results are for different types of volumes, but both types are thermodynamic variables, or in other words, both types have an effect on gas pressure, Thus, the observer gets contradictory results also in the same frame of reference for relativistic pressure, as a result, the Lorentz transformations for different volumes represent a new paradox in special relativity, this paradox is not a mistake in special relativity, but it reveals to us the limits of the special theory of relativity in application, and the reasons for the failure to combine special relativity with thermodynamics.

**Keywords:** Relativistic thermodynamics, Energy-time paradox, Inverse Relativity, Modified Lorentz transformations, Super-time, Negative space, Michael Girgis paradoxes.

## 1 INTRODUCTION

For more than a hundred years, physicists have faced a problem in combining special relativity [1] with thermodynamics and establishing relativistic thermodynamics [2], Where special relativity gave physicists more than one model of relativistic thermodynamics [3] each model includes different transformations of the variables of the thermodynamic system such as gas volume, gas pressure and absolute temperature of the gas, physicists have always been trying to reach a correct model of relativistic thermodynamics, or in other words trying to agree on a single model without contradictions, there was no attempt by physicists to find out the reasons for the failure of combining special relativity with thermodynamics, Or to ask if there are limits to special relativity in the application?, Do the limits of special relativity stop when combined with thermodynamics or not?, The first paper, titled The Energy-Time Paradox Limits of Special

Relativity in Application [4] revealed to us one of the reasons that lead to obtaining different transformations of temperature, i.e. contradictory results in the same frame of reference, are there, then, more paradoxes that arise when combining special relativity with thermodynamics that we can uncover? Do these paradoxes reveal to us the nature of the mistake that all physicists made when establishing relativistic thermodynamics?

## 2 METHODS

### 2-1 The thermodynamic System within Reference Frames

We assume that we have two reference frames S and [1] S' from orthogonal coordinate systems, each reference frame has an observer at the origin point O and O', and that the frame S' is moving at a uniform velocity  $V_S$  relative to the S frame in the positive direction of the x-axis, and we also assume that the frame of reference S' contains a closed thermodynamic system of ideal gas and in thermodynamic equilibrium, as shown in the following figure

$$S' \rightarrow x' y' z' t'$$

$$S \rightarrow x y z t$$

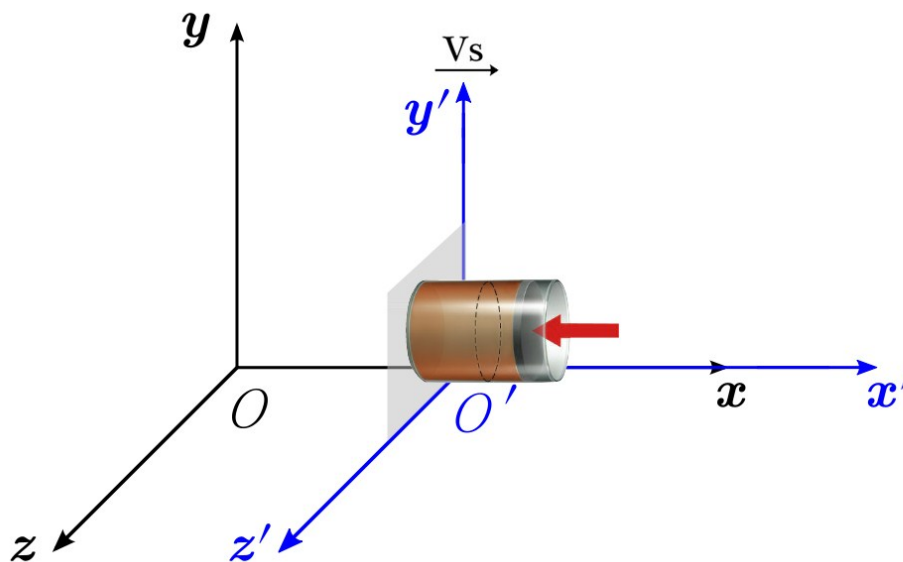


Figure 1:6

## 2-2 Transformation of Gas Volume as a Thermodynamic Variable

The observer O' observes the thermodynamic system variables relative to the reference frame S', such as the amount of gas or the number of molecules  $N$ , the gas volume  $V_{\alpha_0}$  which is the volume of the space that contains the gas, the gas pressure  $P_{\alpha_0}$  and the absolute temperature of the gas  $T_{\alpha_0}$ , we use the vector symbol  $\vec{\alpha}_0$  for both volume, pressure, and temperature, even though they are non-vector scalar quantities, because the vector  $\vec{\alpha}_0$  represents the instantaneous displacement vector of gas molecules that is related to the previous scalar quantities, this vector also represents the first observation conditions or the observation conditions of special relativity

In order to determine here if the relativistic gas volume observed by the observer O relative to the reference frame S in the first observation conditions, i.e. according to special relativity, is a relativistic variable or a constant, we must obtain the conversion of the gas volume from the reference frame S' to the S frame, but in order to get the gas volume conversion, we first need to get the dimensional conversion volume, so we use the Lorentz transformations for the coordinates of the place [2] [1] with the vector symbol used in the second paper [5], i.e. in the following formula

$$x'_{\alpha_0} = \gamma (x_{\alpha} - V_s t_{\alpha}) \quad (1.2)$$

$$y'_{\alpha_0} = y_{\alpha} \quad (2.2)$$

$$z'_{\alpha_0} = z_{\alpha} \quad (3.2)$$

Where  $\gamma$  Lorentz factor

$$\gamma = \frac{1}{\sqrt{1 - \frac{V_s^2}{c^2}}} \quad (5.2)$$

Because the geometric dimensions of the thermodynamic system shape can be represented on the coordinates of the reference frame S', with length periods on each coordinate, so we write the previous set of equations in the following formula [6]

$$\Delta x'_{\alpha_0} = \gamma (\Delta x_{\alpha} - V_s \Delta t_{\alpha}) \quad (1.6)$$

$$\Delta y'_{\alpha_0} = \Delta y_{\alpha} \quad (2.6)$$

$$\Delta z'_{\alpha_0} = \Delta z_{\alpha} \quad (3.6)$$

Because the measurement of the length of the period between two points on the x-coordinate is at the same time and therefore,  $\Delta t_{\alpha} = 0$ , by substituting for this in the previous equation and rearranging the previous set of equations, we get the following

$$\Delta x_{\alpha} = \Delta x'_{\alpha_0} \gamma^{-1} \quad (4.6)$$

$$\Delta y_{\alpha} = \Delta y'_{\alpha_0} \quad (5.6)$$

$$\Delta z_{\alpha} = \Delta z'_{\alpha_0} \quad (6.6)$$

The previous set of equations represents the transformation of length periods for each coordinate from the reference frame S' to the frame S according to special relativity or in the first observation conditions, through it, we can obtain the gas volume transformation from one reference frame to another, because the thermodynamic system is cylindrical, see figure 1:6, Thus, the volume of gas  $V'_{\alpha_0}$  that the observer O' observes is the volume of a cylinder relative to the reference frame S'

$$V'_{\alpha_0} = \pi r'^2_{\alpha_0} h'_{\alpha_0} \quad (7.6)$$

Where  $r'_{\alpha_0}$  is the radius of the base of the cylinder, and because the base is in the-  $y'z'$  plane as shown in figure 1:6, So the diameter of the cylinder is equal to  $2r'^2_{\alpha_0} = \Delta y'^2_{\alpha_0} + \Delta z'^2_{\alpha_0}$ , and  $h'_{\alpha_0}$  is the height of the cylinder, which is parallel to the  $x'$ -axis, and therefore  $h'_{\alpha_0} = \Delta x'_{\alpha_0}$  See figure 1:6, Substituting for this in the previous equation, we get the volume of the cylindrical space in which the gas diffuses by the coordinates of the reference frame S'

$$V'_{\alpha_0} = \pi \frac{1}{2} (\Delta y'^2_{\alpha_0} + \Delta z'^2_{\alpha_0}) \Delta x'_{\alpha_0} \quad (8.6)$$

As for the observer O, he observes the gas volume  $V_{\alpha}$  relative to the reference frame S in the first observation conditions in the same mathematical formula according to the principle of special relativity [1]

$$V_{\alpha} = \pi \frac{1}{2} (\Delta y^2_{\alpha} + \Delta z^2_{\alpha}) \Delta x_{\alpha} \quad (9.6)$$

Substitute from 4.6, 5.6, 6.6, to 9.6

$$V_{\alpha} = \pi \frac{1}{2} (\Delta y_{\alpha_0}^2 + \Delta z_{\alpha_0}^2) \Delta x_{\alpha_0} \gamma^{-1} \quad (10.6)$$

Substitute from 8.6 to 10.6

$$V_{\alpha} = V_{\alpha_0} \gamma^{-1} \quad (11.6)$$

Equation 11.6 shows that the observer O observes the relativistic volume of the gas contracting or decreasing relative to the reference frame S with the increase in the speed of the reference frame  $V_s$ , as a result of the length contraction, because the thermodynamic system contains an ideal gas, then the observer O' can obtain the relation between volume, pressure, absolute temperature and the amount of gas relative to the reference frame S' through the general equation of the ideal gas [2]

$$P_{\alpha_0} V_{\alpha_0} = N k T_{\alpha_0} \quad (12.6)$$

Where  $k$  is a Boltzmann constant and  $N$  is a constant number because the system is closed, as mentioned above

$$\frac{P_{\alpha_0} V_{\alpha_0}}{T_{\alpha_0}} = N k \quad (13.6)$$

And because the gas volume is a thermodynamic variable, therefore, the relativistic gas volume contraction will have an effect on the relativistic gas pressure when the relativistic temperature is kept constant, and using the same mathematical formula, the observer O obtains the relation between volume, pressure and temperature relative to the reference frame S according to the principle of special relativity

$$\frac{P_{\alpha} V_{\alpha}}{T_{\alpha}} = N k \quad (14.6)$$

Substitute from 13.6 to 14.6

$$\frac{P_{\alpha} V_{\alpha}}{T_{\alpha}} = \frac{P_{\alpha_0} V_{\alpha_0}}{T_{\alpha_0}} \quad (15.6)$$

Because we have assumed higher that the relativistic temperature is constant, therefore, the conversion of the temperature from the reference frame S' to the frame S is according to the following equation

$$T_{\alpha} = T_{\alpha_0} \quad (16.6)$$

Substitute from 11.6, 16.6 to 15.6

$$\frac{P_{\alpha} V_{\alpha_0} \gamma^{-1}}{T_{\alpha_0}} = \frac{P_{\alpha_0} V_{\alpha_0}}{T_{\alpha_0}} \quad (17.6)$$

$$P_{\alpha} = P_{\alpha_0} \gamma \quad (18.6)$$

We conclude from equation 18.6 that the relativistic gas pressure observed by the observer O relative to the reference frame S increases with the increase in the speed of the reference frame  $V_s$ , as a result of the relativistic volume contraction, assuming that the relativistic temperature remains constant.

### 2-3 Transformation of Rational Volume as a Thermodynamic Constant

But as the volume of the gas contracts, the volumes of the molecules of the ideal gas also contract, because the contraction of the volume results from the contraction of the length on the x-dimension according to the Lorentz transformations, that is, everything that has length on the x-dimension also contracts, Thus, the volume of each gas molecule is subjected to a contraction equation similar to the gas volume contraction equation and is written in the following formula

$$\hat{V}_{\alpha} = \hat{V}_{\alpha_0} \gamma^{-1} \quad (19.6)$$

Where  $\hat{V}_{\alpha_0}$  is the volume of one molecule of the gas,  $\hat{V}_{\alpha}$  the relativistic molecule volume, by multiplying both sides of the equation by the number of gas molecules  $N$  we get the conversion of the sum of the volumes of gas molecules from the S' frame to the reference frame S

$$N \hat{V}_{\alpha} = N \hat{V}_{\alpha_0} \gamma^{-1} \quad (20.6)$$

Divide Equation 20.6 by Equation 19.6

$$\frac{V_{\alpha}}{N \hat{V}_{\alpha}} = \frac{V_{\alpha_0} \gamma^{-1}}{N \hat{V}_{\alpha_0} \gamma^{-1}} \quad (21.6)$$

$$\frac{V_{\alpha}}{N \hat{V}_{\alpha}} = \frac{V_{\alpha_0}}{N \hat{V}_{\alpha_0}} \quad (22.6)$$

The right side of the equation 22.6 represents the rational volume  $\bar{V}'_{\alpha_0}$ , which is a rational number that represents the ratio between the volume of the gas and the sum of the volumes of gas molecules that exist in the thermodynamic system, while the left side represents the rational volume conversion  $\bar{V}'_{\alpha}$  or relativistic rational volume

$$\bar{V}'_{\alpha} = \bar{V}'_{\alpha_0} \quad (23.6)$$

Equation 23.6 shows us that the transformation of the rational volume from the reference frame S' to the frame S is constant, This means that it is not subject to Lorentz contraction, although this type of volume results from the Lorentz transformations of the coordinates of the place and its elements are subject to the Lorentz contraction, but is this type of volume also considered a thermodynamic variable? In other words, does the change in the rational volume lead to a change in pressure also when the temperature is constant, by returning to the ideal gas equation and dividing both sides of the equation by the sum of the volumes of molecules, i.e. is, by the amount  $N \hat{V}'_{\alpha_0}$

$$P'_{\alpha_0} \frac{V'_{\alpha_0}}{N \hat{V}'_{\alpha_0}} = \frac{N k}{N \hat{V}'_{\alpha_0}} T'_{\alpha_0} \quad (24.6)$$

$$P'_{\alpha_0} \bar{V}'_{\alpha_0} = \frac{N k}{N \hat{V}'_{\alpha_0}} T'_{\alpha_0} \quad (25.6)$$

Because  $k$  is a constant, as we mentioned earlier,  $\hat{V}'_{\alpha_0}$  is also a specific constant with the type of gas used in the thermodynamic system, Therefore, we can reduce both constants to only one constant that has the symbol  $k_v$ , which is a hypothetical specific constant, and we write an ideal gas equation with the following formula

$$P'_{\alpha_0} \bar{V}'_{\alpha_0} = k_v T'_{\alpha_0} \quad (26.6)$$

The last equation represents another formula of the ideal gas equation, which combines the rational volume with the pressure and the absolute temperature, and we conclude from this equation that when the temperature remains constant, the gas pressure is inversely proportional to the rational volume, and this means that the rational volume is also a thermodynamic variable, we can understand the matter when the observer moves the piston in the thermodynamic system and changes the volume of space for the gas, we find that the rational volume changes directly as well, because both the sum of the volumes of molecules and their number remain constant during this thermodynamic process

$$\frac{P'_{\alpha_0} \bar{V}'_{\alpha_0}}{T'_{\alpha_0}} = k_v \quad (27.6)$$

But if the rational volume is also thermodynamic variable, and therefore the relativistic rational volume also has an effect on the relativistic pressure under the same conditions as above, i.e., when the temperature and the amount of gas are constant, and using the same mathematical formula, observer O obtains the relation between rational volume, pressure and temperature relative to the reference frame S according to the principle of special relativity, and by following the same previous steps in equations 15.6, 16.6, 17.6, we get to the following equation

$$P_{\alpha} = P'_{\alpha_0} \quad (28.6)$$

Equation 28.6 shows us that the relativistic gas pressure observed by the observer O relative to the reference frame S is constant with the increase in the speed of the reference frame  $V_s$ , as a result of the stability of the relativistic rational volume. It is a result that contradicts the previous result in equation 18.6 despite adherence to the same conditions

Therefore, we conclude from all of the above that gas volume and rational volume are thermodynamic variables, although they are different types of volumes, and therefore the different transformations of these volumes represent a paradox in the Lorentz transformations of space or in special relativity in general. It also follows another paradox in relativistic thermodynamics, where we cannot, with different transformations of volumes, determine if the relativistic pressure increases or remains constant with the increase in the speed of the reference frame.

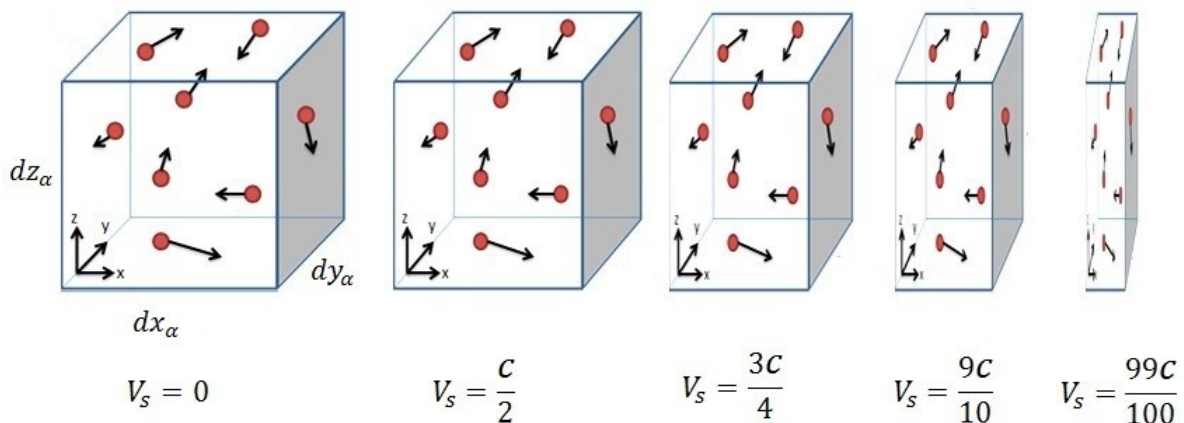




Figure 2:6 represents the effect of Lorentz contraction for a differential volume element or an infinitesimal volume of the thermodynamic system, as it shows us that the volumes of gas molecules do not fill the volume of the space in which the gas diffuses even with volume contraction at high speeds close to the speed of light, which shows the constancy of volume relativistic rational between them

### 3 RESULTS

When applying the Lorentz transformations of the coordinates of space to the volume of gas in a thermodynamic system, we get a contraction in the relativistic volume of the gas in one of the reference frames, and because the volume of the gas is a thermodynamic variable, Thus, the relativistic volume contraction leads to an increase in the relativistic pressure under the assumption that both the relativistic temperature and the amount of gas remain constant, but when applying the Lorentz transformations to another type of volume, which is the rational volume or the ratio between the volume of gas to the volume of gas molecules in a thermodynamic system, we get a constant relativistic rational volume, It is also a thermodynamic variable, Thus, the stability of the relativistic rational volume will lead to the stability of the relativistic pressure under the same assumptions as above, which is the result of a contradiction in the first result, in other words, we cannot determine if the relativistic pressure of the gas is increasing or constant with an increase in the speed of the reference frame, and therefore the Lorentz transformations of volumes in a thermodynamic system represent a new paradox in special relativity called Lorentz volumes paradox, although it is transformations of different types of volumes, but both volumes ( gas volume and rational volume ) are thermodynamic variables that have an effect on the relativistic pressure of a gas under the same conditions, this paradox does not represent a mistake in special relativity, but only reveals to us the limits of special relativity in application, and the reasons for the failure to combine special relativity with thermodynamics

## 4 DISUSSIONS

The Lorentz volumes paradox shows us that physicists were not wrong in all the mathematical models that were established for relativistic thermodynamics [11] [10] [9] [8] where all their models adhere to the concepts and results of special relativity, but the mistake was trying to merge itself, or in other words, exceeding the limits of special relativity in the application, because its limits stop at thermodynamics.

The Lorentz volumes paradox is the second paradox beside the first paradox of the energy and time that arises when merging special relativity with thermodynamics, as both paradoxes confirm that special relativity is not suitable for merging with thermodynamics and because of these paradoxes, it is possible to obtain a diverse number of transformations of relativistic variables and therefore a diverse number from the mathematical models of relativistic thermodynamics, and there is no possibility to know which transformations are the correct ones [3]

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