Partial approximation of $\pi$

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Approximation of pi by taylor series of arctan, and machin like formula for pi

\[
\pi = 2 \cdot (\arctan \left( \frac{1}{4} \right) - \arctan \left( -4 \right))
\]

which is equals to:

\[
\pi = 2 \cdot (\arctan \left( \frac{1}{4} \right) + \arctan \left( 4 \right))
\]

which is equals to:

\[
\pi = 2 \cdot (\arctan \left( \frac{1}{4} \right) + (2 \cdot \arctan \left( 1 \right) - \arctan \left( \frac{1}{4} \right)))
\]

which is equals to:

\[
\pi = 2 \cdot (\arctan \left( \frac{1}{4} \right) + (2 \cdot \arctan \left( 1 \right) - \arctan \left( \frac{1}{4} \right)))
\]

which is equals to:

\[
\pi = \sum_{k=0}^{m} \left( -\frac{1}{4} \right)^{k} \left( \frac{1}{2k+1} + \frac{2}{4k+1} + \frac{1}{4k+3} \right)
\]

where $m$ will be how many digits of precision and $m=(k \times 2)$
this will result to roughly -2 decimals correctly.

where we can use the formula below:

\[
\frac{10}{3} + \sum_{k=1}^{a} \left( -\frac{1}{4} \right)^{k} \left( \frac{b}{(n-2)(n)} + \frac{b}{(n-4)(n)} + \frac{b}{(n-2)(n-4)} \right)
\]

where:
\[
a = a \times 2 \quad ; \quad a \text{ is the number of decimal places we need to find correctly}
\]

\[
b = \prod_{m=\frac{n-1}{2}}^{n} \left( m \times 2 \right) + 1 \quad ; \quad \text{where } b \text{ is the partial factorial of odd numbers and } \quad n = 3 + 4k
\]

or more simplified

\[
\frac{10}{3} + \sum_{k=1}^{a} \left( -\frac{1}{4} \right)^{k} \left( \frac{(n-2) \times (n)}{n \times (n-2) \times (n-4)} + \frac{(n-4) \times (n) \times 2}{n \times (n-2) \times (n-4)} \right)
\]